A Theory of Public Debt Overhang

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Abstract

In this paper we analyze the effects of unsustainable public debt on technology choice and economic growth. Unsustainable public debt undermines the credibility of government policy because the government will do whatever necessary to postpone fiscal consolidation, as an incumbent government inevitably falls from power upon implementing fiscal consolidation. We show that the lack of commitment makes firms' choice of technology inefficient. Fiscal consolidation can restore credibility and high growth in the baseline model, while with a different policy setting in the modified model fiscal consolidation may not be able to restore credibility and growth if it is implemented too late.

Keywords:

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1 Introduction

Countries occasionally experience one or more decades of recession, which Kehoe and Prescott (2007) call a "great depression." The Japanese economy has suffered from deflation and slow or negative economic growth for two decades since the 1990s. What causes a great depressions is typically a decade-long slowdown of productivity (see papers in Kehoe and Prescott, 2007), while it is still a puzzle what factors cause productivity slowdowns in many cases.

One noteworthy hypothesis raised recently by Reinhart, Reinhart and Rogoff (2012) is the adverse effects of public debt overhang. They found that countries with large public

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debt tend to experience persistent stagnation. They show that out of 26 episodes of high public debt in advanced economies 23 cases were associated with more than a decade of low economic growth. They found that public debt/GDP ratios above 90 percent are associated with an average annual growth rate 1.2 percent lower than in periods with debt below 90 percent of GDP. They argue that as this relationship is nonlinear these episodes indicate that causality runs from large public debt to slowdown of economic growth. They also found that in 11 out of 26 high-debt episodes, real interest rates are not higher than in low-debt periods.

In this paper we propose simple models that replicate the findings of Reinhart, Reinhart and Rogoff (2012), and explain how decade-long recession occurs in periods of high debt/GDP ratios. In our model, unsustainable debt undermines the credibility of government's commitment on policy actions. People expect that the government is forced to default on its debt or to implement fiscal consolidation in finite time, which forces an incumbent government out of office. As a government wants to maximize its tenure, it will do whatever is required to extend its tenure. In particular, it will not hesitate to renege on its commitment if necessary. This lack of commitment on government policy induced by unsustainable debt undermines the expected profitability of new technology and discourages firms from adopting new technology. As firms choose inefficient technology, the economic growth rate and interest rate fall unless fiscal consolidation is implemented. In a finite period of time, debt default or fiscal consolidation inevitably take place and the incumbent government falls from power. Once a new government is established as a result of fiscal consolidation, the length of the new government's tenure becomes infinite. In the baseline model, the new government has no incentive to renege on its commitments because the length of its tenure is already maximized and indefinite. As the commitment problem is solved by fiscal consolidation, high growth and a high interest rate are restored. With a different policy setting, however, if the remaining debt is too large, the new government has incentive to renege on its commitments and the economy stays on a low-growth path as we show in the second model. This implies that fiscal consolidation may have little effect on restoring economic growth if it comes too late.

Related literature: Several empirical studies find that public debt/GDP ratios above a certain threshold lowers economic growth. In addition to Reinhart, Reinhart and Rogoff (2012) and Reinhart and Rogoff (2010), Checherita-Westphal and Rother (2012) and

Baum, Checherita-Westphal and Rother (2013) identify the same phenomenon in data from 12 euro area countries. These findings are new in that previous literature does not show that public debt has a significant effect on economic growth, while it does show the negative effects of government spending and budget deficit on economic growth (Barro and Sala-i-Martin, 1995; Fischer 1991). Reinhart, Reinhart and Rogoff's (2012) findings that public debt overhang tends to lower both economic growth and the interest rate is puzzling because the textbook argument of crowding out (see for example Galí, López-Salido and Vallés, 2007, and standard macroeconomics textbooks, such as Romer 2011) implies that expansionary and inefficient fiscal policy is associated with high interest rates. Interest rates are low in 11 episodes of public debt overhang including Japan's lost decades. Non-Keynesian effects (Giavazzi and Pagano, 1990; Bertola and Drazen, 1993; Perotti, 1999) are also related to this finding on public debt overhang: the non-Keynesian effects of expansionary fiscal policy lead to low consumption due to expectations of a one-time tax distortion in the future. As the non-Keynesian effects explain the short-term phenomenon, the finding that public debt overhang continues for decades is still puzzling. Our theory is related to the theoretical literature on public debt and economic growth pioneered by Diamond (1960). Saint-Paul (1992), Brauninger (2005), Futagami, Hori and Ohdoi (2010), and Arai, Kunieda and Nishida (2012) find that an increase in debt lowers economic growth. The slowdown of growth is basically driven by crowding-out and the interest rate rises when growth slows down. Our theory of public debt overhang is a political economy model in line with Acemoglu and Robinson (2005) and Acemoglu (2009, chapters 22 and 23) in that the incumbent government's political incentive plays a crucial role in generating an inefficient outcome.

2 Benchmark model

2.1 The case without public debt overhang

We consider the case without public debt as a benchmark. The economy is a continuous-time AK model in which capital stock does not depreciate. There are a representative consumer and a government, while the incumbent government is replaced if it introduces a certain policy unfavorable to the consumer, as described later. The consumer owns N firms, where N is a large integer. Firms borrow capital stock from the consumer and produce consumer goods. We assume that there are two production technologies, A and

B. If a firm with capital stock k_t adopts technology A, it can produce Ak_t at time t, and if the firm adopts technology B, it can produce Bk_t, where

$$0 < B < A.$$

We assume that the government can impose "output tax" τ_{kt} only on output produced by technology A, where the government can choose $0 \leq \tau_{kt} \leq 1$. We also assume that an output tax is not costly for the government in that it does not lose political power when it imposes output tax.

After-tax revenue of a firm is $(1 - \tau_{kt})Ak_t$ if it adopts technology A, while it is Bk_t if it adopts technology B. For convenience, we define r_H and r_L by

$$r_H = A \quad \text{and} \quad r_L = B.$$

Government's objective: In this economy, the government's ultimate objective is to maximize its tenure. The government can credibly honor any promise as long as breaking it does not extend its tenure. In the benchmark case where there is no public debt, the tenure of the government is already indefinite and no more extention is possible. As imposition of an output tax does not affect the length of tenure, government can credibly commit to set

$$\tau_{kt} = 0.$$

We also assume that in the benchmark case with no public debt the government has no tax revenue and no expenditures from the beginning. The government simply does nothing in the benchmark case.

Firm's objective: As $\tau_{kt} = 0$ the firm's profit maximization is described as follows, given the gross rental price of capital, R_t:

$$\max_{y} \; y - R_t k_t,$$

$$\text{s.t. } y \in \{Ak_t, \; Bk_t\}.$$

Consumer's objective: Given R_t, the consumer maximizes the discounted present value of his utility:

$$\max_{c_t, \dot{k}_t} \int_0^{\infty} e^{-\rho t} \ln c_t,$$

$$\text{s.t. } c_t + \dot{k}_t = R_t k_t + X_t,$$

where ρ is the subjective rate of time preference,

$$\rho > 0,$$

and X_t is a transfer from the government.

Equilibrium without public debt: In the case where there is no public debt, the government can commit to $\tau_{kt} = 0$ and firms choose technology A. Given this technology choice, the equilibrium outcome is the same as the textbook AK model:

$$\begin{aligned}
&R_t = A,\\
&X_t = 0,\\
&k_t = k_0 e^{(A-\rho)t},\\
&c_t = \rho k_t,\\
&\zeta_t = \frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = A - \rho.
\end{aligned}$$

2.2 Case with public debt overhang

We assume that there exists outstanding debt b_0 at the initial point $t = 0$, and that there are no taxes at this point. Without fiscal consolidation, the government has no revenue and no expenditures. Thus the government budget is equal to the law of evolution of debt:

$$\dot{b}_t = r_t b_t,$$

where r_t is the market rate of interest. This debt is obviously unsustainable and agents in this economy expect that fiscal consolidation, which we will describe shortly, will occur at $t = T$. Given expectations T, the consumer's problem for $0 \leq t < T$ is

$$\begin{aligned}
&\max_{c_t, \dot{k}_t, \dot{b}_t} \int_0^\infty e^{-\rho t} \ln c_t,\\
&\text{s.t.} \quad c_t + \dot{k}_t + \dot{b}_t = R_t k_t + r_t b_t + X_t.
\end{aligned}$$

Two taxes and fiscal consolidation: We assume that government has two tax instruments to finance public debt: the lump-sum tax on the consumer and/or the output tax on output produced by technology A. If the incumbent government implements the lump-sum tax it forces itself out of power. In other words, if the lump-sum tax is implemented at T, the tenure of the incumbent government is terminated at T; and the new government obtains tax revenues τ_t at $t \geq T$ without additional political cost, where τ_t is chosen by the government at t such that

$$0 \leq \tau_t \leq \tau.$$

The government can impose output tax, $\tau_{kt} A k_t$, on the firms who used technology A without losing power, where the government can choose

$$0 \leq \tau_{kt} \leq 1.$$

We assume that output produced by technology A is taxable, while output produced by technology B is not taxable; and that the government can implement an output tax at t after observing firms' technology and production at t.

Tenure T of incumbent government: We ask the limit of time when sustainability of government debt is restored by implementing a lump-sum tax. We can show the following lemma:

Lemma 1. *Given that the lump-sum tax τ_t is implemented at time $t = T$, the transversality condition (TVC) for the consumer is satisfied from T on if and only if*

$$r_H b_T \leq \tau,$$

where $\tau_t = \tau$ if $b_t > 0$ and $\tau_t = 0$ if $b_t = 0$.

(Proof) Debt evolves by $\dot{b}_t = r_t b_t - \tau_t$ for $t \geq T$, where the interest rate r_t should be either r_H or r_L and $\tau_t = \tau$ if $b_t > 0$ and $\tau_t = 0$ if $b_t = 0$. Since r_t and τ_t are constant if $b_t > 0$, the solution to this differential equation is

$$b_{t+T} = \max \left\{ 0, \ \frac{\tau}{r} + \left(b_T - \frac{\tau}{r} \right) e^{rt} \right\}.$$

The TVC for the consumer is

$$\lim_{t \to \infty} e^{-rt} b_t = 0,$$

which is equivalent to $b_T \leq \frac{\tau}{r}$. The interest rate for $t \geq T$ is $r = r_H$, given that the TVC is satisfied. This is shown as follows: since the TVC is satisfied, the tenure of the (new) government is indefinite, and the government has no incentive to impose an output tax because tax revenue from the output tax cannot extend the tenure; so it can credibly commit to set $\tau_{kt} = 0$ and firms choose technology A for $t \geq T$, leading the interest rate to $r = r_H$. Therefore, the TVC is equivalent to

$$r_H b_T \leq \tau.$$

(end)

Define T by $r_H b_T = \tau$, where b_t follows $\dot{b}_t = r_L b_t$, for $0 \leq t \leq T$.

Lemma 2. *Incumbent government's tenure is limited by* $0 \leq t \leq T$.

(Proof) We assume and justify later that $r_t = r_L$ for $0 \leq t \leq T$. The consumer accepts to buy government bond b_t for $t > T$ if and only if TVC is satisfied, i.e., the lump-sum tax, τ, is introduced at T. As we assume that the incumbent government leaves office upon implementing the lump-sum tax, it inevitably goes out of power at T. Even if the incumbent government does not implement a lump-sum tax at T, it goes out of political power as default on the government debt inevitably occurs because the consumer refuses to buy government bond at T. Thus, in any case, the tenure of the incumbent can never be longer than T. We justify that $r_t = r_L$ for $0 \leq t \leq T$ in the following lemma.
(end)

Lack of commitment: Firms choose technology B due to political economy distortion (Acemoglu, 2009) for $0 \leq t \leq T$. This is from the following setup of our model:

- At every point in time t, the government decides whether to introduce an output tax after observing firms' choices at time t of technology, A or B,
- The government cannot precommit not to introduce an output tax,
- We restrict the equilibrium to being the Markov Perfect Equilibrium (MPE).

Given these conditions we have the following lemma:

Lemma 3. *Given that the tenure of the incumbent government is expected to end at a certain time T, firms choose technology B for* $0 \leq t < T$.

(Proof) If firms choose technology A, the government imposes an output tax and sets $\tau_{kt} = 1$ to maximize tax revenue, because it can extend its tenure T by obtaining a positive amount of tax revenue at t.[1] Anticipating this, all firms choose technology B at all $t \in [0, \ T]$.

(end)

[1] In a continuous time model, tax revenue at point t is infinitesimally small and does not affect the length of tenure T. Thus, rigorously speaking we cannot say that the government strictly prefers implementing an output tax at t for $0 \leq t < T$. We can conclude, however, that the government strictly prefers implementing an output tax by the following argument: We assume that if firms choose technology A at t they cannot change technology until $t + \Delta$, where Δ $(\ll 1)$ is a very short time interval. We also assume that capital stock that is allocated to one firm cannot be reallocated to other firms; in other words, capital stock of

Equilibrium with public debt overhang: Given the initial values k_0 and b_0, the equilibrium is determined as follows. $T = \arg\max_t \; b_t$ subject to $\dot{b}_t = r_L b_t$ and $r_H b_t \le \tau$. Firms choose technology B for $0 \le t < T$. For $t \ge T$ technology A is dominant and the interest rate is r_H as we saw in the proof of Lemma 1.

$$r_t = \begin{cases} r_L = B, & \text{for } 0 \le t < T, \\ r_H = A, & \text{for } t \ge T, \end{cases}$$

$$b_t = \begin{cases} b_0 e^{r_L t}, & \text{for } 0 \le t < T, \\ b_T, & \text{for } t \ge T, \end{cases}$$

$$k_t = \begin{cases} k_0 e^{(r_L - \rho)t}, & \text{for } 0 \le t < T, \\ k_T e^{(r_H - \rho)(t-T)}, & \text{for } t \ge T, \end{cases}$$

$$c_t = \rho k_t,$$

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = \begin{cases} r_L - \rho, & \text{for } \quad 0 \le t < T, \\ r_H - \rho, & \text{for } \quad t \ge T, \end{cases}$$

Therefore, the growth rate and interest rate are low before fiscal consolidation, while they are high after fiscal consolidation. An output tax is not imposed at any time in equilibrium.

3 Modified model with multiple equilibria

We showed in the previous section that fiscal consolidation can restore high growth and high interest rate in the baseline model. This result crucially depends on the setting of fiscal policy. We consider a different setting of fiscal policy and show that fiscal consolidation does not necessarily restore the credibility of government commitment. We show that fiscal consolidation may not be able to restore high growth and a high interest rate if it is implemented too late. The setting of the modified model is as follows.

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each firm k_t must satisfy $k_s \ge k_t$ for all $s \ge t$. Given these assumptions suppose a firm who owns capital stock $\frac{k_t}{N}$ chooses technology A at time t and goes back to technology B at time $t + \Delta$. If the government imposes output tax from t on, government debt $b_{t'}$ at $t' = t + \Delta$ becomes

$$b_{t'} \approx b_t + \left(r_t b_t - A \frac{k_t}{N} \right) \Delta < b_t + r_t b_t \Delta.$$

At t' the amount of debt $b_{t'}$ is lower by approximately $A \frac{k_t}{N} \Delta$ if the government imposes an output tax versus not imposing an output tax. Thus the tenure is longer by $\kappa\Delta$ with an output tax versus without it, if firms adopt technology A, where $\kappa > 0$ is a positive constant. Therefore, given that Δ is a positive constant, we can conclude that the government strictly prefers implementing an output tax if one or more firms choose technology A.

Technology: We consider an AK model similar to the baseline model. There are two technologies, A and B. Output tax, τ_k, on output produced by technology A and B has already been implemented. If firms adopt technology A they produce Ak_t, while they need to pay output tax $\tau_k Ak_t$ to the government and political rent γk_t ($\gamma > 0$) to the consumer, where γk_t represents political rent associated with the necessary education of workers and/or investment in infrastructure to utilize new technology A. If firms adopt technology B they produce Bk_t, while firms need to pay output tax $\tau_k Bk_t$. Firms that use technology B do not pay political rent. The parameters satisfy

$$0 < A - \gamma < B < A. \tag{1}$$

Fiscal policy: The government can pay subsidy, $g_t k_t$, to firms that use technology A. This subsidy $g_t k_t$ represents redistribution associated with public education and/or public investment in infrastructure for new technology. Given that the government implements fiscal policy (τ_k, g_t), the revenue of a firm that uses technology A is

$$[(1 - \tau_k)A - \gamma + g_t]k_t,$$

while the revenue of a firm that uses technology B is

$$(1 - \tau_k)Bk_t.$$

Thus a profit-maximizing firm chooses technology A if

$$g_t \geq \gamma - (1 - \tau_k)(A - B). \tag{2}$$

Conditions (1) and (2) imply that tax revenue decreases if the government pays $g_t k_t$ to firms that use technology A:

$$\tau_k Ak_t - g_t \leq \tau_k B.$$

We make the following crucial assumption:

Assumption 1. The government decides $g_t \in [0, \infty)$ after observing firms' choice of technology A or B at time t.

Fiscal consolidation: Similar to the benchmark model, the government can implement the lump-sum tax τ on consumers to restore TVC. Once the lump-sum tax is implemented at T, the tenure of the incumbent government is terminated at T and the new government obtains tax revenue τ for $t \geq 0$.

Lack of commitment: We can show the following lemma.

Lemma 4. *Given that the tenure of the incumbent government is expected to end at a certain T $(< \infty)$, all firms choose technology B for $0 \leq t \leq T$.*

(Proof) The government decides whether to pay $g_t k_t$ after firms choose a technology. Suppose that firms choose technology A at t $(< T)$. Government tax revenue will be $\tau_k A k_t$ if it does not pay subsidy $g_t k_t$, while revenue will be $(\tau_k A - g_t)k_t$ if it pays the subsidy. As the ultimate goal of the incumbent government is to maximize the length of its tenure, it does not pay the subsidy $g_t k_t$ if firms choose technology A.[2] Anticipating this, all firms choose technology B for $t \in [0,\ T]$.

Evolution of government debt: We assume that the initial value of government debt b_0 is large such that

$$b_0 > \frac{\tau_k B k_0}{\rho} > 0.$$

Given that firms choose technology B before fiscal consolidation, debt evolves by

$$\dot{b}_t = r_t b_t - \tau_k B k_t.$$

Using the fact that $k_t = k_0 e^{(r_L - \rho)t}$ in the equilibrium path where all firms adopt technology B, the solution to the above differential equation is

$$b_t = \frac{\tau_k B k_0}{\rho} e^{(r_L - \rho)t} + \left(b_0 - \frac{\tau_k B k_0}{\rho} \right) e^{r_L t}.$$

Since b_0 is large, debt diverges at the rate r_L unless the lump-sum tax is imposed.

Optimization problems: The consumer solves

$$max \int_0^\infty e^{-\rho t} \ln c_t,$$

$$\text{where} \quad c_t + \dot{k}_t + \dot{b}_t = R_t k_t + r_t b_t + X_t.$$

[2] The same critique as in footnote 1 applies here. To justify the argument in the proof of Lemma 4, we assume as follows: Once firms choose technology A at time t, they cannot change technology for a short interval Δ; and once the government sets g_t at t it must pay $g_t k_s$ for at least $s \in [t, t + \Delta]$ to firms that use technology A. Suppose that firms believe at t that the government will pay $g k_s$ $s \geq t$ if they choose technology A at t. Then they actually adopt technology A at t. Observing firms' choice of technology at t, the government decides whether to pay $g k_s$ for $t \leq s \leq t + \Delta$. If the government does not pay the subsidy, the amount of outstanding debt at $t' = t + \Delta$ is lower by approximately $g k_t \Delta$ than in the case where it pays the subsidy. Thus the tenure is strictly longer in the case where the government does not pay the subsidy than in the case where it pays. Since the ultimate goal of the incumbent government is to extend the tenure, it optimally chooses not to pay the subsidy $g k_s$ $(s \in [t,\ t + \Delta])$ even if firms choose technology A at t.

Firms choose technology A or B to maximize profits, given the rental price of capital R_t:

$$\max \ \left[\max \ \{(1-\tau_k)A - \gamma + g_t, \ \ (1-\tau_k)B\} - R_t, \ \ 0\right].$$

The incumbent government solves the following problem:

$$\max_{g_t} T,$$
$$\text{s.t.} \ \begin{cases} \dot{b}_t = r_t b_t - (\tau_k A - g_t)k_t \mathbf{1}(g_t) - \tau_k B k_t(1-\mathbf{1}(g_t)), \\ g_t \geq \gamma - (A-B), \\ \text{ and TVC,} \end{cases}$$

where $\mathbf{1}(g_t) = 1$ if $g_t > 0$ and $\mathbf{1}(g_t) = 0$ if $g_t = 0$, and TVC is the consumer's transversality condition that determines the upper bound of T, which we describe below.

Transversality condition: In this model, there are two distinct transversality conditions that correspond to distinct expectations on the equilibrium path after implementation of the lump-sum tax. After the lump-sum tax is implemented, the transversality condition must be satisfied. (Otherwise there is no equilibrium after T.) There are two expectations on technology adoption after implementation of the lump-sum tax: technology A or B. If technology A is adopted the interest rate will be r_H and tax revenue will be $(\tau_k A - g)k_t$, while if technology B is adopted the interest rate will be r_L and the revenue will be $\tau_k B k_t$. Therefore, given that the lump-sum tax is imposed at T, debt evolves by

$$\dot{b}_t = r b_t - \tau - \Gamma k_t, \qquad \text{for} \quad t \geq T,$$

where $(r, \ \Gamma) = (r_H, \ \tau_k A - g)$ if technology A is dominant for $t \geq T$ or $(r, \ \Gamma) = (r_L, \ \tau_k B)$ if technology B is dominant for $t \geq T$, and k_t evolves by

$$k_t = k_T e^{(r-\rho)(t-T)}, \qquad \text{for} \quad t \geq T.$$

The solution to the above differential equation is given by

$$\begin{aligned} b_{t+T} =& x + y e^{(r-\rho)t} + (b_T - x - y)e^{rt}, \\ \text{where} \ \ x =& \frac{\tau}{r}, \\ y =& \frac{\Gamma k_T}{\rho}. \end{aligned}$$

The consumer's transversality condition is

$$\lim_{t\to\infty} b_{t+T} e^{-rt} = 0,$$

which is equivalent to

$$b_T \leq x + y,$$

where $x+y = B_E(T)$ in the equilibrium where technology A is dominant and $x+y = B_L(T)$ in the equilibrium where technology B is dominant, where

$$B_E(T) = \frac{\tau}{r_H} + \frac{(\tau_k A - g)k_T}{\rho},$$
$$B_L(T) = \frac{\tau}{r_L} + \frac{\tau_k B k_T}{\rho},$$

where $k_T = k_0 e^{(r_L - \rho)T}$. Note that $B_E(T) < B_L(T)$ for any T. We define T_E by $b_T = B_E(T)$ and T_L by $b_T = B_L(T)$, where $b_T = b_0 e^{r_L T}$. It is obvious that

$$T_E < T_L.$$

The tenure of the incumbent government is terminated at either T_E or T_L, depending on macroeconomic expectations (ie, expectations by the consumer and firms) on the dominant technology after fiscal consolidation. There are two equilibrium paths, the good equilibrium and the bad equilibrium, where in the good equilibrium dominant technology is A after fiscal consolidation and in the bad equilibrium it is B.

Good equilibrium: As agents expect that technology A is dominant after fiscal consolidation, the lump-sum tax should be imposed at T_E. Otherwise the government defaults on its debt because the consumer refuses to buy government bonds at T_E. As the lump-sum tax τ is imposed at T_E, the credibility of fiscal policy is restored because TVC is satisfied for $t \geq T_E$ on the premise that the new government pays subsidy gk_t and the dominant technology is A. Thus firms choose technology A and the new government pays gk_t to firms for $t \geq T_E$, where $g = \gamma - (1 - \tau_k)(A - B)$. Note that the new government has no incentive to renege on the promise to pay gk_t, because breaking a promise does not extend its tenure, which is already indefinite.

The equilibrium path is determined by

$$r_t = \begin{cases} r_L = B, & \text{for } 0 \le t < T_E, \\ r_H = A, & \text{for } t \ge T_E, \end{cases}$$

$$b_t = \begin{cases} \tau_k B k_0 \rho^{-1} e^{(r_L-\rho)t} + (b_0 - \tau_k B k_0 \rho^{-1}) e^{r_L t}, & \text{for } 0 \le t < T_E, \\ \tau r_H^{-1} + (\tau_k A - g) k_T \rho^{-1} e^{(r_H-\rho)t}, & \text{for } t \ge T_E, \end{cases}$$

$$k_t = \begin{cases} k_0 e^{(r_L-\rho)t}, & \text{for } 0 \le t < T_E, \\ k_T e^{(r_H-\rho)(t-T)}, & \text{for } t \ge T_E, \end{cases}$$

$$c_t = \rho k_t.$$

The growth rate (ζ_t) and interest rate (r_t) are both high after fiscal consolidation, while they are low before fiscal consolidation in the good equilibrium path.

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = \begin{cases} r_L - \rho, & \text{for} \quad 0 \le t < T_E, \\ r_H - \rho, & \text{for} \quad t \ge T_E. \end{cases}$$

Bad equilibrium: As agents expect that technology B is dominant after fiscal consolidation, the lump-sum tax is imposed at T_L. Otherwise the government defaults on debt because the consumer refuses to buy government bonds at T_L. In any case the tenure of the incumbent government is terminated at T_L. As TVC is satisfied for $t \ge T_L$ on the premise that the new government does not pay gk_t and dominant technology is B, the government cannot pay subsidy gk_t. This is shown in the following lemma.

Lemma 5. *We assume that the government knows that firms choose A iff it pays subsidy $g = \gamma - (1-\tau_k)(A-B)$. Still the new government cannot pay subsidy gk_t and the dominant technology is B for $t \ge T_L$ if the lump-sum tax τ is implemented at T_L.*

(Proof) By definition of T_L, TVC is satisfied for $t \ge T_L$ and the tenure of the new government is indefinite if it does not pay subsidy gk_t and the dominant technology is B. TVC holds on the premise that $r_t = r_L$ and tax revenue is $\tau_k B k_t$. We assume that once the government sets g_t at t, it must pay $g_t k_s$ for $t \le s \le t + \Delta$ to firms that use technology A. If the new government pays $g_t k_t$ where $0 < g_t < g = \gamma - (1-\tau_k)(A-B)$ then dominant technology is B and TVC is violated because government revenue is strictly smaller. Thus the government never pays $g_t k_t$ where $0 < g_t < g$. If the government pays gk_t then technology A becomes dominant. In this case the interest rate becomes $r_t = r_H$ and government revenue becomes $(\tau_k A - g)k_t$, which is smaller than $\tau_k B k_t$. Given these values TVC is violated because $r_t = r_H$ is higher than r_L and tax revenue $(\tau_k A - g)k_t$ is

lower than $\tau_k B k_t$. In this case the new government immediately goes out of power at T_L because the consumer does not buy government bonds. Anticipating this, the government never pays gk_t even if firms choose technology A at $t \geq T_L$. And anticipating that the government will not pay gk_t, all firms choose technology B at $t \geq T_L$ if the lump-sum tax τ is implemented at T_L.
(end)

The bad equilibrium is given by

$$
\begin{aligned}
& r_t = r_L = B, \quad \text{for } t \geq 0, \\
& b_t = \begin{cases} \tau_k B k_0 \rho^{-1} e^{(r_L-\rho)t} + (b_0 - \tau_k B k_0 \rho^{-1}) e^{r_L t}, & \text{for } 0 \leq t < T_L, \\ \tau r_L^{-1} + \tau_k B k_T \rho^{-1} e^{(r_L-\rho)t}, & \text{for } t \geq T_L, \end{cases} \\
& k_t = k_0 e^{(r_L-\rho)t}, \quad \text{for } t \geq 0, \\
& c_t = \rho k_t.
\end{aligned}
$$

The growth rate (ζ_t) and interest rate (r_t) are both low even after fiscal consolidation in the case where fiscal consolidation comes too late (T_L).

$$\frac{\dot{c}_t}{c_t} = \frac{\dot{k}_t}{k_t} = \zeta_t = r_L - \rho, \qquad \text{for} \quad t \geq 0.$$

4 Conclusion

In this paper we analyzed the effect of unsustainable public debt on technology choice and economic growth. We have shown that unsustainable debt undermines the credibility of government policy because the government will do whatever possible to postpone fiscal consolidation, as the incumbent government inevitably goes out of power upon implementing fiscal consolidation. We have shown that the lack of commitment makes firms' choice of technology inefficient. Fiscal consolidation can restore credibility and high growth in the baseline model, while with a different policy setting in the modified model fiscal consolidation may not be able to restore credibility and growth if it is implemented too late.

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