Economic Growth with Mechanization of the Production Process [†]

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Abstract: This paper considers endogenous technological changes of inputs from labor to capital in operations of a production process, i.e., a kind of mechanization of the production process. While retaining the property of decreasing returns in capital, we can show that because of complementary relationships between the accumulation of capital and the mechanization of the production process, long-run growth is possible through the accumulation of capital only. It involves no growth of total factor productivity. Furthermore, our model can partly present a microfoundation of constant-elasticity-of-substitution production function. Investigating our model empirically, we confirm both technological progress and our technological change.

keywords: Capital Accumulation, Mechanization of a Production Process, Total Factor Productivity, Envelope, Elasticity of Substitution

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1. Introduction

Quah (1996), among others, explained by empirical analysis that since World War II, income distribution throughout the world has become bipolar between growing countries and stagnating countries. Many analyses have shown both theoretically and by growth regressions that to explain international income gaps, the accumulation of physical capital per unit of labor is one of the most crucial factors.

However, when we consider a production process in which there are several operations to produce goods, and we differentiate between operations in which labor is the main input and operations in which capital such as machines is the main input, we observe that there would be positive relationships not only between output and the amount of capital per unit of labor but also between output and the proportion of operations in which capital is the main input. In developing countries, the capital per labor is low, and labor is widely used in production processes to produce not only agricultural goods but also manufacturing goods. In developed countries, the capital per labor is high, and capital rather than labor is widely used in production processes to produce both types of good. Furthermore, the utilization of labor in some operations has given way to capital, i.e., a high degree of mechanization would be observed in developed countries, while this seems to be difficult in developing countries.[1]

The neoclassical growth model and many endogenous growth theories investigate

[1] For example, in the beginning of modern times in Japan, female labor was mainly used to produce clothes but now machines are used in almost all operations.

how quantitative changes in inputs of capital and labor influence economic growth. However, few studies investigate how the mechanization of the production process occurs and it influences economic growth, and why that change has made progress in developed countries but not in developing countries.[2]

This paper considers not only quantitative changes in inputs of labor and capital but also the mechanization of the production process. We investigate how quantitative changes in inputs relate to the mechanization of the production process, and how that mechanization influences economic growth. Smith (1789) suggested that the division of labor promotes the use of machines, which greatly facilitates labor and reduces dependence on it, and which in turn induces labor productivity to increase. Similar to Smith (1789), the process innovation of the production process in our model, i.e., the changes of some operations away from labor to capital can be considered as technological change in which labor costs are saved by introducing machines into some operations, and labor productivity rises.

Considering the mechanization of the production process in which an input of labor is changed to capital in some operations of the production process, this paper becomes to be able to explain the following three points.[3]

First, while retaining the property of decreasing returns in capital, long-run

[2] Although Zeira (1998) considered technological change where an input of labor is changed to capital for production of intermediate goods, an explanation of the three points mentioned below becomes possible in our model.

[3] Our model allows changes of some operations not only from labor to capital but also from capital to labor.

growth with no change of total factor productivity (TFP) is possible through the accumulation of capital only. Young (1994) among others found that the accumulation of physical capital, but not the growth of TFP, mainly contributed to the high economic growth of the Asian Newly Industrialized Economies (NIEs). Although the question of whether or not those countries could maintain such high growth has been controversial, those countries have had sustained economic growth, except for temporary negative shocks. Furthermore, Kumar and Russell (2002) estimated a worldwide production function nonparametrically, which has two inputs of labor and physical capital, and exhibits constant-returns-to-scale technology. They found that capital deepening could explain both growth and international divergence, and technological progress was insignificant.

In our model, complementary relationships between the accumulation of capital and the mechanization of the production process emerge endogenously, and there are two cases where those relationships continue eternally and temporally, i.e., an economy will be able to attain long-run growth and an economy will be caught in a growth trap. Furthermore, because our production function through market equilibrium consists of the envelope of the Cobb-Douglas type, the growth rate of output per labor does not involve the growth of TFP. Therefore, this paper can present a theoretical explanation for the empirical findings mentioned above.[4]

[4] In our own empirical studies, we could confirm the mechanization of the production process, and the growth of TFP depended on the interval of years. Although we do not deny technological progress to explain economic growth, in our theoretical model, we focus on technological change

Second, we present a microfoundation of a production function with a constant elasticity of substitution between labor and capital. Assuming the constant-returns-to-scale technology, through market equilibrium, our production function includes a constant-elasticity-of-substitution (CES) production function in which the value of elasticity of substitution is larger than unity, and as a special case, equals unity, i.e., a Cobb-Douglas specification. The elasticity of substitution can be represented by the parameter that indicates the rate of decreasing the relative usage of capital to labor among the operations of the production process. Arrow, Chenery, Minhas and Solow (1961) mathematically derived a CES production function which have the properties of constant returns to scale and of constant elasticity of substitution between labor and capital including the Leontief type and the Cobb-Douglas type, because they found the varying degrees of substitutability in different types of production. Our model can present partly a microfoundation how a production function with the constant-returns-to-scale technology becomes the CES type.

Third, considering the mechanization of the production process, we try to explain empirically the international differences in economic growth rates. Using the data of Penn World Table (Mark 5.6a) constructed by Summers and Heston (1991), we estimate the growth regression implied by the mechanization of the production process occurring as a result of the accumulation of capital per labor. We confirm that the proportion of operations in which capital is input is described by the capital per labor of an economy. In addition, similar to the empirical findings of

that does not involve the change of TFP.

Kumar and Russell (2002), technological progress would be insignificant to explain economic growth of 57 countries from the years 1965 to 1990. However, as shown in our own empirical studies, using the pooled data with five-year intervals, we found significant growth of TFP for some years, because the estimate of the variance of error terms became smaller than from the years 1965 to 1990. There is a possibility that the significance of the growth of TFP changes with respect to the size of the intervals.

Our estimation results also imply that the elasticity of substitution between labor and capital of our sample would be larger than unity. Therefore, this result is the same as Duffy and Papageorgiou (2000), who found that the estimate of elasticity of substitution was larger than unity, and therefore the Cobb-Douglas specification was rejected. In addition, our empirical studies show that it would be more difficult to change some operations away from labor to capital in stagnated countries than in growing countries. We may be able to partly explain international differences in economic growth by the difficulty of mechanizing the production process.

The outline of our model is as follows. We presume a closed economy, which is constituted by households and firms that are perfectly competitive.[5] A firm produces a single good through a production process in which the total number of op-

[5] A closed economy is assumed to investigate the dynamics that involve the accumulation of capital and the mechanization of the production process. Using international data on saving and investment rates, Weil (2004) concluded that economies including developed and developing countries seem to be still closed to capital flows but not to be perfectly open.

erations is constant. We assume the constant-returns-to-scale production function. In each operation, only capital or only labor is inputted. We call an operation 'labor operations' when labor is inputted 'capital operations' when capital is inputted. We consider the technological level of the production process, which is measured by the proportion of the number of capital operations to that of total operations.[6] A firm determines not only which type of input, labor or capital, should be used in each operation, but also the quantity of the input to be used.

In equilibrium, labor is used in some operations, while capital is used in other operations. The proportion of capital operations to the total operations is determined by the ratio of wage rate to interest rate. The accumulation of capital causes the ratio of wage rate to interest rate to increase. The increase in the ratio of factor prices results in an urge not only to input more capital than labor, but also to increase the proportion of capital operations. The increase of the proportion of capital operations causes more accumulation of capital, and in turn, the accumulation of capital raises the proportion of capital operations. Therefore, we can see a certain complementary relationship between the accumulation of capital and the mechanization of the production process that is measured by the increase in the proportion of capital operations. If these relationships never cease, long-run growth

[6] Nakamura and Nakamura (2003) introduced the qualitative level of the production process. However, they considered not only physical capital but also human capital and assumed an external effect of human capital to explain the bipolarization of economic growth. Therefore, the growth of TFP appeared in their model. This paper shows that with no growth of TFP, long-run growth is possible through physical capital accumulation only.

will become possible.

The β convergence occurs as a result of the accumulation of capital and the mechanization of the production process. Because our model retains the property of diminishing returns to capital, the growth rate of output per labor converges either to a constant rate which is described by our model's parameters in the case of long-run growth or to zero.[7] Therefore, this paper has the implications of both the neoclassical growth model and endogenous growth theories.

We investigate the conditions for the eternal complementary relationships between the accumulation of capital and the mechanization of the production process, i.e., of long-run growth. In an economy where the proportion of capital operations is intrinsically difficult to increase, similar to the neoclassical growth model with no technological progress, the economy will converge to the steady state, i.e., the economy will be eventually caught in a growth trap. On the other hand, in an economy where the proportion of capital operations is intrinsically easy to increase, there is no steady state. In such cases, the supplementary relationships between the accumulation of capital and the mechanization of the production process never halt, and long-run growth will become possible. To explain the differences in economic growth rates between stagnating and growing economies, the difficulty of changing some operations away from labor to capital becomes a crucial factor in our model.

[7] Barro and Sala-i-Martin (1995), Basu and Weil (1998), and Howitt (2000) considered technological transfer among countries and derived the conditional convergence in their endogenous growth theories. We explain these papers in sections 3 and 4.

Technological change in our model, when represented by the mechanization of the production process, differs essentially from technological progress in endogenous growth theories, for example, Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Technological progress, which means the increase of TFP, implies an upward shift in the frontier of the production function. However, in our model, the mechanization of the production process does not involve the growth of TFP. When the proportion of capital operations increases, the production functions that have constant proportions of capital operations slide in a northeast direction. Because the optimal proportion of capital operations becomes a function of the capital per labor of the economy, our production function consists of the envelope of the production functions with constant proportions of capital operations. As a result, economic growth is explained through the growth of the capital per labor only, with no change of TFP. This would seem to be consistent with the empirical findings not only of Young (1994) and Krugman (1994) that the growth of TFP contributed little to the high economic growth of the Asian NIEs, but also of Kumar and Russell (2002) that both growth and international divergence were driven mainly by capital deepening.

The rest of the paper is organized as follows. Section 2 sets up the model. In Section 3, we consider the possibility that long-run growth occurred as a result of the complementary relationships between the accumulation of capital and the mechanization of the production process. In Section 4, we investigate the relationships between the mechanization of the production process and the change of TFP. In

Section 5, taking the elasticity of substitution between labor and capital into account, the growth regression implied by our model is investigated empirically. In Section 6, we present conclusions and remarks.

2. The model

In a closed economy, there exist households and firms that are perfectly competitive. The goods are produced by using a fixed number of operations normalized to unity. We assume that in each operation, only capital or only labor is inputted. The goods are used for consumption and accumulation of capital. At any moment of time, households supply their labor to firms and determine their consumption and saving levels. The income of households consists of wages and the interest on their savings.

2.1 The firms

The goods are produced by using a fixed number of operations normalized to unity. The production function of the representative firm is assumed to be of the continuous Cobb-Douglas type at any moment:

$$\ln Y_t = \int_0^1 \ln z_t(i) di, \tag{1}$$

where Y_t is the output of goods at time t, and $z_t(i)$ is the effective input of capital or labor of operation i at time t.

To minimize the cost, a firm determines not only which type of input, labor or capital, should be used in each operation, but also the quantity of the inputs.

Although both the type of input and the quantity of input are simultaneously determined, for simplicity of explanation, we first see the choice of the type of input. Given the choice of the type of input in each operation, we consider which quantities of inputs are used.

The effective unit of input in the i-th operation is denoted as follows:

$$z_t(i) = \eta(i)l_t(i) \quad or \quad \theta(i)x_t(i), \tag{2}$$

where $l_t(i)$ and $x_t(i)$ are the inputs of labor and capital at time t in operation i respectively. $\theta(i)$ and $\eta(i)$ represent the efficiency of the inputs of labor and capital respectively in operation i in which those functions take nonnegative values.

We assume that the operations exist continuously from an operation in which capital is relatively more effectively used than labor to an operation in which capital is relatively less efficiently used than labor. Arranging the operations, compared to labor, from an operation in which capital is easily used to an operation in which capital is hardly used and defining the function $\Psi(i) \equiv \theta(i)/\eta(i)$, which indicates the relative efficiency of inputs of capital to labor, this function is assumed to have the following properties:

$$\Psi(i) \geq 0, \quad \Psi'(i) \leq 0, \quad i \in [0, 1], \tag{3}$$

where $\Psi(0) > 0$, and $\Psi(i) = 0$ implies that the efficiency of the input of capital is zero, i.e., capital is unable to be used.[8]

[8] In section 3, we show that long-run growth will be able to be attained, even if $\lim_{i \to 1} \Psi(i) = 0$, i.e., $\lim_{i \to 1} \theta(i) = 0$.

Given the wage rate w_t and the interest rate r_t, the average cost of production in the i-th operation becomes $w_t/\eta(i)$ when labor is the input, or $(r_t+\delta)/\theta(i)$ when capital is the input where δ is the depreciation rate of capital. The factor prices determine the ratios of capital and labor operations.

We show the relationships between the factor prices and the proportion of capital operations within total operations. In Figure 1, the horizontal axis indicates the index of operations and the vertical line represents the relative cost of production in the i-th operation, which is measured by the ratio of the input costs of labor and capital.

We can see that capital is the input in the operations $[0, a_t]$, because the ratio of labor cost to capital cost is larger than unity. Therefore, labor is used in the remaining operations $(a_t, 1]$. The increase of the wage rate relative to the interest rate urges the upward shift of the curve denoted as CC. It causes the increase in the ratio of capital operations and therefore the decrease of the ratio of labor operations. The increase in the proportion of capital operations (decrease in proportion of labor operations) means that a capital-intensive (labor-intensive) technology is chosen.

Using the optimal proportion of capital operations a_t, the production function (1) is rewritten as follows:

$$\ln Y_t = \int_0^{a_t} \ln \theta(i) x_t(i) di + \int_{a_t}^1 \ln \eta(j) l_t(j) dj. \tag{4}$$

The total cost of a firm can be described as follows:

$$C_t = \int_0^{a_t} (r_t + \delta) x_t(i) di + \int_{a_t}^1 w_t l_t(j) dj. \tag{5}$$

Given the constraint (4) of production technology, a firm chooses the quantities of inputs, $x_t(i)$, $i \in [0, a_t]$, and $l_t(j)$, $j \in (a_t, 1]$. To solve this problem, we consider the following Lagrangian function:

$$\mathcal{L}_t \equiv \int_0^{a_t} (r_t + \delta) x_t(i) di + \int_{a_t}^1 w_t l_t(j) dj$$

$$+\mu_t \left\{ \ln Y_t - \int_0^{a_t} \ln \theta(i) x_t(i) di - \int_{a_t}^1 \ln \eta(j) l_t(j) dj \right\},$$

where μ_t is the Lagrangian multiplier.

We have the first-order conditions:

$$x_t(i) = x_t \equiv \frac{\mu_t}{r_t + \delta}, \quad i \in [0, a_t],$$

$$l_t(j) = l_t \equiv \frac{\mu_t}{w_t}, \quad j \in [a_t, 1]. \tag{6}$$

The ratio of capital operations must satisfy the following first-order condition:

$$(r_t + \delta) x_t(a_t) - w_t l_t(a_t) - \mu_t \left\{ \ln \theta(a_t) x_t(a_t) - \ln \eta(j) l_t(a_t) \right\} = 0.$$

Using equations (6), we have relationships between the proportion of capital operations, the ratio of input quantities of labor to capital, and the ratio of the rental rate to the wage rate:

$$\Psi(a_t) = \frac{l_t}{x_t} = \frac{r_t + \delta}{w_t}. \tag{7}$$

We can confirm that the decrease of rental rate relative to the wage rate implies not only more inputs of capital relative to labor but also an increase in the proportion of capital operations.

Inserting equation (6) into equations (4) and (5), we obtain the following cost function:

$$C_t = \frac{(r_t+\delta)^{a_t} w_t^{1-a_t}}{\Omega(a_t)} Y_t, \quad \textit{where} \quad \ln \Omega(a_t) \equiv \int_0^{a_t} \ln \theta(i) di + \int_{a_t}^1 \ln \eta(j) dj. \tag{8}$$

Normalizing the price of goods to unity, the equilibrium condition for the goods market yields the relationships between the ratio of capital operations and relative factor prices:

$$\frac{(r_t+\delta)^{a_t} w_t^{1-a_t}}{\Omega(a_t)} = 1. \tag{9}$$

Furthermore, using equations (7) and (9), we can derive the relationships between the proportion of capital operations and the interest rate:

$$\ln(r_t+\delta) = \ln \Omega(a_t) + (1-a_t) \ln \Psi(a_t), \tag{10}$$

$$\textit{where} \quad \frac{dr_t}{da_t} = (r_t+\delta)(1-a_t)\frac{\Psi'(a_t)}{\Psi(a_t)} \leq 0.$$

We see that the interest rate is negatively related to the proportion of capital operations. This implies that the decrease of interest rate widens the proportion of capital operations, i.e., the usage of capital.

2.2 The households

Each household consists of one person with a unit of labor that is supplied inelastically to a firm. We write the number of households at time t as L_t and assume the growth rate of the number of households as n. The utility function of the representative household is specified as follows:

$$U \equiv \int_0^\infty u(c_t) \exp(-\rho t) dt, \quad u'(c) > 0, u''(c) < 0, \tag{11}$$

where c_t is consumption per household at time t and ρ is the time preference rate.

At each moment, the households obtain wages by supplying their labor and earnings interest on savings. These incomes are allocated to consumption and additional savings. We can represent a household's budget constraint as:

$$\dot{s}_t = r_t s_t + w_t - c_t - n s_t, \tag{12}$$

where s_t is savings per household.[9]

At every moment, the household determines its consumption to maximize the utility function (11) under the budget constraint (12) and the initial savings s_0. From the first-order condition of this problem, we obtain the following differential equation:

$$-\left(\frac{c_t u''(c_t)}{u'(c_t)}\right)\frac{\dot{c}_t}{c_t} = r_t - \rho - n. \tag{13}$$

Then, to choose one equilibrium path, we impose the transversality condition:

$$\lim_{t\to\infty} \lambda_t s_t = 0,$$

where λ_t is the costate variable of s_t.

We also impose a non-Ponzi-game condition ruling out the possibility that a household's debt will explode.

3. The possibility of long-run growth

[9] Because we focus on the role of physical capital to economic growth, human capital is not considered. However, following Galor and Moav (2002), if we assumed the concavity of the accumulation of human capital, the physical capital per labor adjusted by human capital would accumulate, and therefore the mechanization of the production process would occur.

In this section, we show how the technology level, as measured by the proportion of capital operations, can be described by the capital per labor and then, investigate the possibility of long-run growth.

The equilibrium conditions for capital and labor markets are respectively as follows:

$$X_t = \int_0^{a_t} x_t(i)di = a_t x_t, \tag{14}$$

$$L_t = \int_{a_t}^1 l_t(j)dj = (1-a_t)l_t, \tag{15}$$

where X_t represents the stock of capital in the economy.

Using equations (7), (14) and (15), we can relate the capital per labor denoted as k_t to the proportion of capital operations:

$$k_t = \frac{a_t}{1-a_t}\frac{1}{\Psi(a_t)}. \tag{16}$$

Differentiating k_t with respect to a_t yields

$$\frac{dk_t}{da_t} = k_t \left\{ \frac{1}{a_t} + \frac{1}{1-a_t} - \frac{\Psi'(a_t)}{\Psi(a_t)} \right\} > 0.$$

Because the function $\Psi(a_t)$ is invertible, the ratio of capital operations can be represented by an increasing function of capital per labor:

$$a_t = a(k_t), \quad a'(k_t) > 0, \quad a(0) = 0, \quad \lim_{k_t \to \infty} a(k_t) = 1. \tag{17}$$

The technology measured by the proportion of capital operations is described by the capital per labor of the economy. In an economy where the amount of capital per labor is not sufficiently accumulated, the proportion of capital operations is low, i.e., labor is used widely in the production process. Even if no capital existed, contrary to the neoclassical growth model with a Cobb-Douglas production function, the production of all operations would be performed only with labor, and therefore output would not become zero. On the other hand, in an economy with a high level of capital per of labor, the proportion of capital operations is high, and capital is used widely. If an infinite quantity of capital per labor existed, production of all operations would be performed only with capital.

In our model, the extent of capital operations implies the share of capital income within total income. Therefore, we can recognize that the increase of capital per labor implies an increase in the share of capital income. This would be consistent with the empirical findings of Duffy and Papageorgiou (2000) that by estimating a CES production function, they obtained an estimate of the constant elasticity of substitution between labor and capital that was larger than unity. It implies that the share of capital income increases with the capital per labor.[10]

Using equations (10) and (17), we can represent the interest rate as the function of capital per labor in the economy:

$$r_t = r(k_t), \quad r'(k_t) \leq 0, \quad \lim_{k_t \to 0} r(k_t) = \overline{R} - \delta,$$

$$\textit{where} \;\; \overline{R} \equiv \Omega(0)\Psi(0). \tag{18}$$

[10] Furthermore, Solow (1958) noted his reservations about the presumption of constant shares of labor and capital.

In addition, we can prove the following inequality:

$$|r'(k_t)| \leq \frac{(1-a_t)(r_t+\delta)}{k_t} = \left|\frac{d^2 y_t}{dk_t^2}\right|_{given \quad a_t},$$

where $y_t \equiv Y_t/L_t$ is the output per labor.

Similarly to the neoclassical model, the interest rate depends negatively on the capital per labor. However, compared with the case of a Cobb-Douglas production function that would correspond to the case where the proportion of capital operations is exogenously given in our model, the marginal product of capital decreases less when the capital per labor increases. Because the accumulation of capital promotes an increase in the proportion of capital operations, i.e., the mechanization of the production process, this effect weakens the decrease of the marginal product of capital resulting from the accumulation of capital.

The assumption of a closed economy implies that at any moment, the net savings must equate to the accumulation of capital. Because we define the initial stock of savings as the initial stock of capital, the equilibrium condition of the asset market is that $s_t = k_t$ at any moment. From the budget constraints of households (12) with equations (14) and (15), the dynamics of capital per labor is derived as follows:

$$\frac{\dot{k}_t}{k_t} = \frac{r_t+\delta}{a_t} - \frac{c_t}{k_t} - (n+\delta). \tag{19}$$

A steady state is defined where both consumption and capital per labor become constant over time:

$$\dot{c}_t = \dot{k}_t = 0, \tag{20}$$

where consumption capital per labor at the steady state are noted as c and k, respectively.

Substituting this definition into equation (13), we show that the capital per labor in the steady state must satisfy the following equation:

$$r(k) = \rho + n. \tag{21}$$

The right-hand side of this equation is represented by the time preference rate and the population growth rate that are both exogenous and constant. This can be considered as a criterion whether or not the accumulation of capital per labor continues. The left-hand side of this equation represents the interest rate, i.e., the marginal product of capital. As shown in Figure 2, it decreases with the increase of capital per labor. However, contrary to the neoclassical growth model, when the capital per labor tends to infinity, the interest rate would not become zero. We present the following two cases that $r(k) < \rho+n$ for some range of k and $r(k) > \rho+n$ for any k.[11]

As shown in the curve noted as AA of Figure 2, if the following inequality holds for some k,

$$r(k_t) < \rho + n, \tag{22}$$

then there will be a steady state in this economy. We can show that the steady state is a saddle point. The economy converges to the steady state, and long-run growth

[11] Although there is also a possibility that labor but not capital exists at the steady state, i.e., that labor is used eternally in all operations, we do not consider this case because of its improbability. This case can be excluded by the inequality, $\lim_{k_t \to 0} r(k_t) > \rho + n$.

becomes impossible. The convergence process in our model is qualitatively similar to the neoclassical model. If countries take the same values of the parameters, the income levels converge to a common value. However, because of the mechanization of the production process, the income per labor at the steady state is larger than predicted by the neoclassical model.

As explained above, the ratio of interest rate to wage rate decreases because of the accumulation of capital per labor, and it encourages not only further accumulation of capital but also the increase of the proportion of capital operations. Therefore, complementary relationships between the accumulation of capital and the mechanization of the production process exist in our model. In an economy where equation (22) holds, this complementary relationships weaken as the capital per labor accumulates, and sooner or later, the growth rate of output per labor converges to zero. However, if equation (22) does not hold, i.e., there is no steady state, an economy will be able to attain long-run growth with the eternal complementary relationships.

Now, to consider an economy of long-run growth, we impose the following restriction on the function $\theta(i)$, which indicates the efficiency of the input of capital in the i-th operation:

$$\theta(i) \geq \overline{\theta}(i) \equiv A_x(1-i)^{\epsilon}, \quad i \in [0,1], \quad \textit{where} \;\; \epsilon \equiv \ln \frac{A_x}{\rho+n+\delta}, \tag{23}$$

where A_x is the shift parameter of the capital operations in the production process.

We do not impose any further restrictions on $\eta(i)$, which represents the efficiency of the input of labor in the i-th operation. In Figure 3, we depict the functions $\theta_1(i)$ and $\theta_2(i)$, which satisfy the above restriction. On the other hand, in the cases where the efficiency of the input of capital is sufficiently low in some operations such as $\theta_3(i)$ and $\theta_4(i)$, the above restriction is unable to be satisfied. As explained below, for long-run growth, the efficiency of the input of capital must satisfy equation (23).

Arranging equation (10) with equation (23), we derive the following inequalities:

$$\ln(r_t + \delta) = \ln \Omega(a(k_t)) + (1 - a(k_t)) \ln \Psi(a(k_t))$$

$$= \int_0^{a(k_t)} \ln \theta(i) di + \int_{a(k_t)}^1 \ln \eta(j) dj + (1 - a(k_t)) \{\ln \theta(a(k_t)) - \ln \eta(a(k_t))\}$$

$$\geq \ln A_x + \int_0^{a(k_t)} \epsilon \ln(1-i) di + \int_{a(k_t)}^1 \ln \eta(j) dj + (1 - a(k_t)) \{\epsilon \ln(1 - a(k_t)) - \ln \eta(a(k_t))\}$$

$$= \ln A_x - \epsilon a(k_t) + \ln \eta(1) - \ln \eta(a(k_t)) - \int_{a(k_t)}^1 \frac{d \ln \eta(j)}{d \ln j} dj.$$

Furthermore,

$$\lim_{k_t \to \infty} \ln(r_t + \delta) \geq \ln A_x - \epsilon = \ln(\rho + n + \delta),$$

because of $\lim_{k_t \to \infty} a(k_t) = 1$.

This implies that the following inequality holds:

$$\lim_{k_t \to \infty} r(k_t) \geq \rho + n.$$

If the condition in equation (23) is satisfied, we can describe the relationships between the interest rate and the capital per labor such as the curve noted as BB in Figure 2. In this economy, there is no steady state. The accumulation of capital and the growth of consumption never halt, because of the eternal supplementary relationships between the accumulation of capital and the mechanization of the production process.

Barro and Sala-i-Martin (1997), Basu and Weil (1998), and Howitt (2000) considered how economic growth is influenced by technological transfer among countries. They derived the β convergence in their endogenous growth models. In our model, the accumulation of capital and the mechanization of the production process cause the β convergence and in the case of long-run growth, the growth rate of output per labor converges to a constant rate which is described by the structural parameters. If countries take the same values of the parameters, the growth rates of those countries converge to a common rate. However, depending on the initial capital, the income levels differ eternally. Furthermore, as seen above, because of the complementary relationships between the accumulation of capital and the mechanization of the production process, the speed of convergence becomes time-varying and slower than predicted by the neoclassical model.[12]

To explain easily the implication of equation (23), we specify the efficiency of the input of capital as $\theta(i) = A_x(1-i)^\gamma$ where γ takes a nonnegative value. The parameter γ represents the rate of increasing difficulty of changing from labor operations to capital operations. A larger value of the parameter γ means a lower value

[12] Howitt (2000) considered technological transfer in a multicountry Schumpeterian growth model. He found that because R&D is positively correlated with investment rates, the capital's share of GDP is less and therefore, the convergence speed is slower than predicted by the neoclassical model.

of the efficiency of capital and implies that it becomes more difficult to change some operations away from labor to capital.

In an economy where $\gamma > \epsilon$, i.e., where the inequality of equation (23) does not hold, the difficulty of changing from labor to capital operations increases rapidly with an increasing proportion of capital operations. Sooner or later, the proportion of capital operations becomes constant. On the other hand, in an economy where $\gamma \leq \epsilon$, i.e., where equation (23) holds, the difficulty of increasing the proportion of capital operations rises slowly. The accumulation of capital causes the mechanization of the production process which in turn, it urges further accumulation of capital. These complementary relationships continue eternally, and therefore long-run growth can be attained. At the limit, all operations would be performed only with capital, and the production function would become the AK type.[13]

Furthermore, we can see that in equation (23), the value of ϵ depends negatively on the time preference rate ρ, the population growth rate n, and the depreciation rate δ, and positively on the productivity A_x of capital operations. Smaller values of ρ, n, and δ and a larger value of A_x imply higher possibilities of long-run growth. The efficiency of capital but not of labor is crucial for long-run growth, because in our model, the mechanization of the production process is essential for economic growth. Therefore, a high efficiency of labor would not contribute to attainment of long-run growth.

[13] However, the production function at the limit would depend not only on the shift parameter A_x but also on the difficulty of the mechanization of the production process γ.

4. The relationships between TFP and long-run growth

Technological change in our model essentially differs from technological progress in endogenous growth theories, such as those of Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992). Technological progress in their models means the upward shift of a production frontier. That shift appears in the growth of TFP that is unable to be measured by the accumulation of capital per labor to explain economic growth. On the other hand, in our model, differentiating equation (4) with respect to time and using equation (16), the growth rate of output per labor can be described as follows:

$$\frac{\dot{y}_t}{y_t} = a(k_t)\frac{\dot{k}_t}{k_t}. \tag{24}$$

We can see that the growth rate of output per labor does not involve the growth of TFP. In other words, our technological change, which is represented in the change of the proportion of capital operations, is unable to be considered by the change of TFP.

In Figure 4, we illustrate our technological change, i.e., the mechanization of the production process. When the proportion of capital operations increases, the production functions having the constant proportions of capital operations, $a_1 < a_2 < a_3$, slide in a northeast direction. The production function with the constant proportion of capital operations becomes the Cobb-Douglas type. Because in equilibrium the optimal proportion of capital operations is determined to correspond with the capital per labor, k_0, k_1 and k_2 of the economy, our production function consists of the envelope, P_0, P_1 and P_2 of the production functions with constant proportions of capital operations, a_0, a_1 and a_2. As a result, economic growth is explained only through the accumulation of capital per labor, with no change of TFP.[14]

As inferred in Figure 4, if the increase of the proportion of capital operations did not correspond with the increase of capital per labor, for example, the capital per labor increased from k_0 to k_1 with the change of proportion of capital per labor from a_0 to a_2 because of technological transfer, the output per labor would fall lower than the level our model predicted, and furthermore, there would be also possibilities that the income per labor with a_2 is smaller than with a_0, because the capital per labor is k_1 but not k_2. Therefore, as a result of technological diffusion from a developed country, if capital came to be used in further operations of a developing country, there might be a negative effect on economic growth.[15]

With our technological change, we can explain the economic growth of Asian NIEs. In our model, compared with the neoclassical model, higher accumulation of

[14] Nakamura (2004) tried to explain theoretically and empirically the bipolarization of economic growth through the accumulation of capital relative to labor alone. In his model, because the accumulation of capital at the previous time appeared as technological progress, an economy could intertemporally show increasing returns to capital. Therefore, there was a possibility of multiple steady states.

[15] This implication is similar to Acemoglu and Zilibotti (2001) that considered a model where technologies that are imported from the North to the South are relatively skill-complementary, while the South employs unskilled workers requiring more labor-complementary technologies. In their model, productivity differences between the North and the South arose even in the absence of barriers to technology transfer.

capital can be attained because of the mechanization of the production process. As seen above, this type of technological change does not appear in the growth of TFP. Krugman (1994) and Young (1994) explained that high economic growth of Asian NIEs was mainly driven not by technological progress but rather by high capital accumulation. Our model could explain the sustained economic growth of those countries, because with no growth of TFP, long-run growth is possible through the accumulation of capital only.[16]

5. The empirical analyses

In this section, using the data of the Penn World Table, we undertake some empirical estimation of the growth of output per labor to investigate the empirical validity of our model, especially, whether or not the proportion of capital operations can be described by the capital per labor. We specify the efficiency of the inputs of capital and labor as follows: $\theta(i) = A_x(1-i)^\gamma$ and $\eta(i) = A_l i^\gamma$ where A_l is the shift parameter of the labor operations. The parameter γ indicates the difficulty of changing some operations away from labor to capital. With this specification, the production function can be written as follows:

$$\ln Y = \int_0^{a_t} \ln A_x(1-i)^\gamma x_t(i)di + \int_{a_t}^1 \ln A_l j^\gamma l_t(j)dj.$$

As proved in the Appendix, the elasticity of substitution between labor and

[16] Basu and Weil (1998) considered that when a country has a high rate of investment, a low growth of TFP could be implied because it would be unable to reap benefits of technological transfer sufficiently.

capital of our production function becomes $\gamma^{-1}+1$, which takes a value larger than or equal to unity.[17] A higher value of γ, which implies it to be more difficult to change from labor operations to capital operations, means a lower value of the elasticity of substitution. When γ takes an infinite value, i.e., a firm is unable to choose the ratio of operations between capital and labor, the elasticity of substitution becomes unity, which is the Cobb-Douglas type. That is, our production function includes a CES production function that the value of elasticity of substitution is larger than unity, and as a special case, equals unity.

In equilibrium, the growth rate of output per labor of this production function can be described as follows:

$$\frac{\dot{y}_t}{y_t} = \frac{k_t^{1/(1+\gamma)}}{(A_x/A_l)^{-1/(1+\gamma)} + k_t^{1/(1+\gamma)}} \frac{\dot{k}_t}{k_t}. \tag{25}$$

Even using a CES production function, we can derive the same equation. The term $(A_x/A_l)^{-1/(1+\gamma)}$ corresponds with the ratio of distribution parameters of labor to capital of a CES production function.

We estimate the relationships of the growth rates between GDP per labor and capital per labor. We change γ to $g \equiv (1+\gamma)^{-1}$, because our model includes the case that γ takes an infinite value. Approximating equation (25) and assuming the constant term C in the growth regression to investigate whether or not the changes in TFP exist, the growth rate of output per labor of i-th country from time t to

[17] If we assumed the imperfect capital market that a firm must borrow capital at an interest rate higher than the marginal product of capital, we would be able to consider the case that the elasticity of substitution between labor and capital takes a value smaller than unity.

time $t + m$ is written with the error term in the following log form:

$$\ln y_{i,t+m} - \ln y_{i,t} = C + \frac{k_{i,t}^{g}}{D + k_{i,t}^{g}} (\ln k_{i,t+m} - \ln k_{i,t}) + u_{i,t}, \tag{26}$$

where C represents the growth rate of TFP, $D \equiv (A_x/A_l)^{-1/(1+\gamma)}$, $y_{i,t}$, $k_{i,t}$ and $u_{i,t}$ are the output per labor, the capital per labor, and the error term of i-th country at time t respectively.

The significance of the parameter C implies the existence of the change in TFP, i.e., the accumulation of capital with the mechanization of the production process would not be able to explain economic growth completely. If the parameter g is significantly positive, we will be able to confirm that mechanization of the production process occurred as a result of the accumulation of capital. If neither of the parameters g and C is significant, it will imply the Cobb-Douglas production function with no technological progress.

Fifty-seven countries were chosen as the full sample. Although the available countries included two major oil-producing countries, Iran and Venezuela, in accordance with convergence literature, these two countries were excluded. That is, $i = 1, 2, \cdots, 57$. The sample is shown in Table 1. First, considering long-run growth, the commencing and the final years were chosen as $t = 1965$ and $t + m = 1990$. With the assumption of $u_{i,t} \sim i.i.d.N(0, \sigma^2)$, the estimation of equation (26) was performed by applying the maximum likelihood estimation.

The estimation results are shown in the left-hand side of Table 2. The estimate of the constant term was close to zero, and the estimate of D took an insignificant positive value. The estimate of the parameter g was significantly positive at the 5% significance level. The mean and standard deviation of the estimates of $a(k_{i,t})$ among the countries took the values 0.584 and 0.10, respectively. We could see that the proportions of capital operations among countries differed largely. Furthermore, we could confirm that the proportion of capital operations is a concave function of the capital per labor of the economy and that economic growth from the years 1965 to 1990 of our sample countries can be explained by the accumulation of capital along with the mechanization of the production process. This would be consistent with the empirical findings of Kumar and Russell (2002) that economic growth was mainly explained by the accumulation of capital and technological progress was insignificant. Furthermore, the significantly positive estimate of g would imply that the elasticity of substitution between labor and capital is larger than unity. Estimating a CES production function with cross-country data for 82 countries, Duffy and Papageorgiou (2000) found that the estimate of the elasticity of substitution was significantly larger than unity. Therefore, our model could present a microfoundation of their empirical finding.

In addition, we investigated whether the parameter g differed between high- and low-income countries. We sorted 57 countries according to their GDP per labor in 1990 and, as shown in Table 1, divided those countries into high-income (HI) and low-income (LI) groups. Korea was chosen as the split because of the sample size and the economic implications. The sizes of the HI and LI groups are 29 and 28 respectively. The HI group includes developed countries and Asian NIEs. The LI group comprises mainly African countries, Central and South American countries, and Asian low-income countries. Applying the ML estimation, the estimation results are shown in the middle of Table 2. The parameters g_h and g_l are of the HI and LI groups respectively. Similar to the estimation results of no structural change of g, the estimate of growth of TFP was close to zero, and the estimate of D was also insignificant. The estimate of g_h but not of g_l took the significant positive value. This means that the mechanization of the production process was confirmed in the HI group but not in the LI group. Therefore, the production functions of the developing countries might be close to the Cobb-Douglas type with no technological progress.

Not only Matsuyama (1992) but also Galor and Mountford (2003) explain that international trade has been a major cause of the great divergence of economic growth rates, because according to comparative advantage, a country specializing in production of manufacturing goods has had sustained economic growth, and a country specializing in production of agricultural goods has stagnated. We could guess that a production function to produce manufacturing goods has a higher value of the elasticity of substitution between labor and capital than that for agricultural goods. In our sample, the high-income countries might specialize to produce those manufacturing goods in which the mechanization of the production process is less difficult than for the agricultural goods whose production the low-income countries might specialize in.

Furthermore, we investigated especially Hong Kong, Korea, Taiwan and Japan, i.e., east Asian countries of high economic growth. We allowed for structural change of the parameter g between the four east Asian countries and the other countries. The right-hand side of Table 2 shows the estimation results, in which g_a and g_o represent the parameters of the east Asian countries and of the other countries respectively. Although the estimate of growth of TFP took a little larger value than the estimation reported above, it was insignificant because of the large estimate of variance of error terms. We obtained a higher estimate of g_a than of g_o. We could guess that in these four countries, the mechanization of the production process is more flexible than in the other countries, and therefore these countries have been able to catch up with the developed countries. Young (1994) and Krugman (1994) reported that the accumulation of capital, but not the growth of TFP, contributed mainly to the high economic growth of the Asian NIEs. In those countries, the mechanization of the production process that has not appeared in the growth of TFP might contribute to the high economic growth.

Next, we considered the estimation using the pooled data with a five-year interval:

$$\ln y_{i,t+5} - \ln y_{i,t} = C_t + \frac{k_{i,t}^g}{D + k_{i,t}^g} (\ln k_{i,t+5} - \ln k_{i,t}) + u_{i,t}, \tag{27}$$

where $t = 1965, 1970, \cdots, 1985$, $i = 1, 2, \cdots, 57$, and $u_{i,t} \sim N(0, \sigma_t^2)$.

Not only the constant terms but also the variances of error terms of each year were taken into account. The sample size was 285. The left-hand side of Table 3 shows the estimation results of no structural change of g. We obtained the highly significant estimate of g. The estimate of D was also significantly positive. The growth of TFP for the years 1980 and 1985 were significantly negative and positive respectively. Compared to the estimation using the data of the years from 1965 to 1990, because the estimate of the variance of error terms became smaller, i.e., the fitness of the growth regression improved, we had some significant estimates of the change of TFP.

The middle of Table 3 shows the estimation results of allowing for structural change of g between the HI and LI groups. The estimates of g not only of the HI but also of the LI groups became significant and took similar values. In the beginning of the sample, the accumulation of capital with mechanization of the production process might be seen in the LI group, although using the sample of the years from 1965 to 1990, this could not be confirmed. In addition, the right-hand side of Table 3 shows the estimation results of allowing for structural change of g between the east Asian countries and the other countries. Similar to the estimation results of the years from 1965 to 1990, the estimate of g_a was larger than of g_o. In this estimation, the growth of TFP for the years 1965, 1980 and 1985 were significant. Kumar and Russell (2002) found insignificant technological progress by comparing their estimation of the years between 1965 and 1990. However, our empirical studies show that the estimate of the variance of error terms changed largely with the length of the intervals and therefore the significance of technological progress would depend on the length of the intervals.

6. Concluding remarks

Considering the endogenous technological change that was described by the mechanization of the production process, we derived supplementary relationships between the accumulation of capital and technological change that do not involve the growth of TFP. We showed that capital accumulates more than predicted by the neoclassical model and specified the conditions of the structural parameters to attain long-run growth, especially focusing on the difficulty of changing some operations from labor to capital operations. With the specific functions that represent the efficiency of the inputs of labor and capital, we proved that the difficulty of changing in some operations away from labor to capital implies a constant elasticity of substitution between labor and capital such that the value is larger than unity, and as a special case, equals unity. Furthermore, by our own empirical analyses, we found some favorable results for our model, i.e., that the proportion of capital operations is a concave function of the capital per labor and that in the high-income countries, capital and labor are more substitutable than in the low-income countries.

To explain the empirical findings obtained, we suggested that the high-income countries and the low-income countries might specialize in producing manufacturing goods and agricultural goods respectively, and therefore in the high-income countries, the mechanization of the production process is less difficult than in the low-income countries. However, we did not theoretically prove that specialization. In the future, we intend to explain how an economy specializes.

Appendix

This appendix shows that our production function includes a CES production function in which the value of elasticity of substitution between labor and capital is larger than unity, and as a special case, equals unity.

Using equations (7) and (16), we can derive the following relationships between the capital per labor and the relative cost:

$$k_t \equiv \frac{X_t}{L_t} = \frac{\tilde{a}\left((r_t+\delta)/w_t\right)}{1-\tilde{a}\left((r_t+\delta)/w_t\right)} \frac{1}{(r_t+\delta)/w_t},$$

where $\tilde{a}\left((r_t+\delta)/w_t\right) \equiv \Psi^{-1}\left((r_t+\delta)/w_t\right)$.

Therefore, the elasticity of substitution between labor and capital can be written as

$$-\frac{d\ln(X_t/L_t)}{d\ln\left((r_t+\delta)/w_t\right)} = \frac{-1}{1-\tilde{a}\left((r_t+\delta)/w_t\right)} \frac{d\ln\tilde{a}\left((r_t+\delta)/w_t\right)}{d\ln\left((r_t+\delta)/w_t\right)} + 1.$$

Now, we specify the function $\Psi(i)$ as

$$\Psi(i) = \left(\frac{1-i}{i}\right)^{\gamma}, \quad \gamma > 0. \qquad (A1)$$

This yields

$$\frac{r_t+\delta}{w_t} = \left(\frac{1-a_t}{a_t}\right)^{\gamma},$$

or

$$a_t = \tilde{a}\left((r_t+\delta)/w_t\right) = \frac{1}{1+\left((r_t+\delta)/w_t\right)^{\frac{1}{\gamma}}}.$$

Hence,

$$\frac{d\ln\tilde{a}\left((r_t+\delta)/w_t\right)}{d\ln\left((r_t+\delta)/w_t\right)} = -\frac{1}{\gamma}[1-\tilde{a}\left((r_t+\delta)/w_t\right)].$$

Therefore, with the specification (A1), the elasticity of substitution between labor and capital becomes

$$\frac{1}{\gamma} + 1.$$

This means that, if we assume (A1), the elasticity of substitution is constant and larger than unity. Moreover, it converges to unity as $\gamma \to \infty$.

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Table 1. Country classification

HI group		LI group	
Korea, Rep.	Portugal	Malawi	Madagascar
Mexico	Greece	Kenya	Zambia
Taiwan	Japan	Nigeria	Zimbabwe
Hong Kong	Israel	Sierra Leone	Ivory Coast
Ireland	Denmark	India	Honduras
Iceland	New Zealand	Philippines	Jamaica
Spain	Austria	Bolivia	Sri Lanka
U.K.	Finland	Paraguay	Thailand
Sweden	Norway	Morocco	Peru
Germany, West	Australia	Dominican Rep.	Guatemala
France	Italy	Panama	Turkey
Netherlands	Belgium	Ecuador	Yugoslavia
Switzerland	Canada	Colombia	Mauritius
U.S.A.	Luxembourg	Chile	Argentina
		Syria	

Note: The sample sizes of the full sample, the HI and LI groups are 57, 28 and 29 respectively.

Table 2. Estimation of equation (26) (for the years from 1965 to 1990)

$\hat{C}$	-0.008	$\hat{C}$	0.010	$\hat{C}$	0.074
	(0.02)		(0.028)		(1.83)
$\hat{D}$	9.057	$\hat{D}$	5.418	$\hat{D}$	10.665
	(1.10)		(1.03)		(1.19)
$\hat{g}$	0.305	$\hat{g}_h$	0.248	$\hat{g}_a$	0.387
	(5.86^{**})		(3.87^{**})		(10.84^{***})
		$\hat{g}_l$	0.203	$\hat{g}_o$	0.259
			(1.78)		(4.76^{**})
$\hat{\sigma}^2$	0.256^2	$\hat{\sigma}^2$	0.254^2	$\hat{\sigma}^2$	0.237^2
L.L.F.	49.21	L.L.F.	49.64	L.L.F.	53.56

Note: $\hat{C}$, $\hat{D}$, $\hat{g}$, $\hat{g}_h$, $\hat{g}_l$, $\hat{g}_a$, $\hat{g}_o$, and $\hat{\sigma}^2$ are the respective estimates. The figure in () is the value of the Wald test which has a χ^2 distribution with one degree of freedom under the null hypothesis that each parameter has a value of zero. $*$, $**$ and $***$ indicate significance at the 10, 5 and 1% levels respectively. L.L.F. is the maximum log of the likelihood function. The sample size is 57.

Table 3. Estimation of equation (27) (five-year interval)

$\hat{C}_{1965}$	0.021	$\hat{C}_{1965}$	0.021	$\hat{C}_{1965}$	0.029
	(1.58)		(2.03)		(4.21**)
$\hat{C}_{1970}$	0.003	$\hat{C}_{1970}$	0.003	$\hat{C}_{1970}$	0.010
	(0.02)		(0.02)		(0.19)
$\hat{C}_{1975}$	0.011	$\hat{C}_{1975}$	0.011	$\hat{C}_{1975}$	0.016
	(0.61)		(0.72)		(1.63)
$\hat{C}_{1980}$	−0.052	$\hat{C}_{1980}$	−0.052	$\hat{C}_{1980}$	−0.050
	(13.34***)		(13.67***)		(13.09***)
$\hat{C}_{1985}$	0.027	$\hat{C}_{1985}$	0.027	$\hat{C}_{1985}$	0.027
	(3.38*)		(3.20*)		(3.58*)
$\hat{D}$	7.784	$\hat{D}$	7.945	$\hat{D}$	6.936
	(3.51*)		(3.93**)		(5.87**)
$\hat{g}$	0.263	$\hat{g}_h$	0.265	$\hat{g}_a$	0.309
	(15.04***)		(20.04***)		(13.09***)
		$\hat{g}_l$	0.267	$\hat{g}_o$	0.226
			(12.13***)		(14.67***)
$\hat{\sigma}^2_{1965}$	0.087^2	$\hat{\sigma}^2_{1965}$	0.087^2	$\hat{\sigma}^2_{1965}$	0.084^2
$\hat{\sigma}^2_{1970}$	0.136^2	$\hat{\sigma}^2_{1970}$	0.136^2	$\hat{\sigma}^2_{1970}$	0.137^2
$\hat{\sigma}^2_{1975}$	0.087^2	$\hat{\sigma}^2_{1975}$	0.87^2	$\hat{\sigma}^2_{1975}$	0.086^2
$\hat{\sigma}^2_{1980}$	0.105^2	$\hat{\sigma}^2_{1980}$	0.105^2	$\hat{\sigma}^2_{1980}$	0.104^2
$\hat{\sigma}^2_{1985}$	0.107^2	$\hat{\sigma}^2_{1985}$	0.107^2	$\hat{\sigma}^2_{1985}$	0.105^2
L.L.F.	506.11	L.L.F.	506.11	L.L.F.	509.61

Note: $\hat{C}_t$, $\hat{\sigma}_t^2$, $t = 1965, 1970, 1975, 1980, 1985$, $\hat{D}$, $\hat{g}$, $\hat{g}_h$, $\hat{g}_l$, $\hat{g}_a$, and $\hat{g}_o$ are the respective estimates. The figure in () is the value of the Wald test which has a χ^2 distribution with one degree of freedom under the null hypothesis that each parameter has a value of zero. $*$, $**$ and $***$ indicate significance at the 10, 5 and 1% levels respectively. L.L.F. is the maximum log of the likelihood function. The sample size is 285.

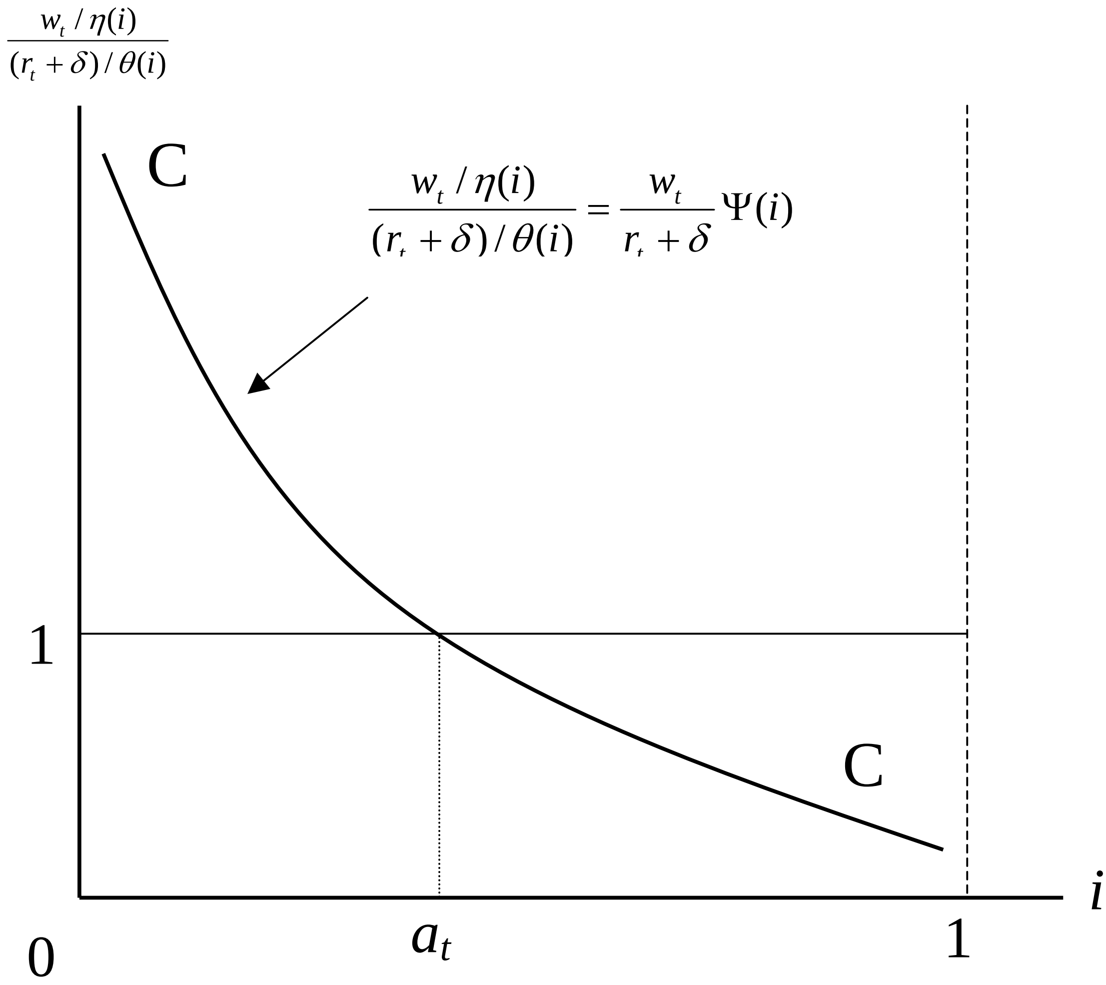

Figure 1. Relative cost in operation *i* of the production process

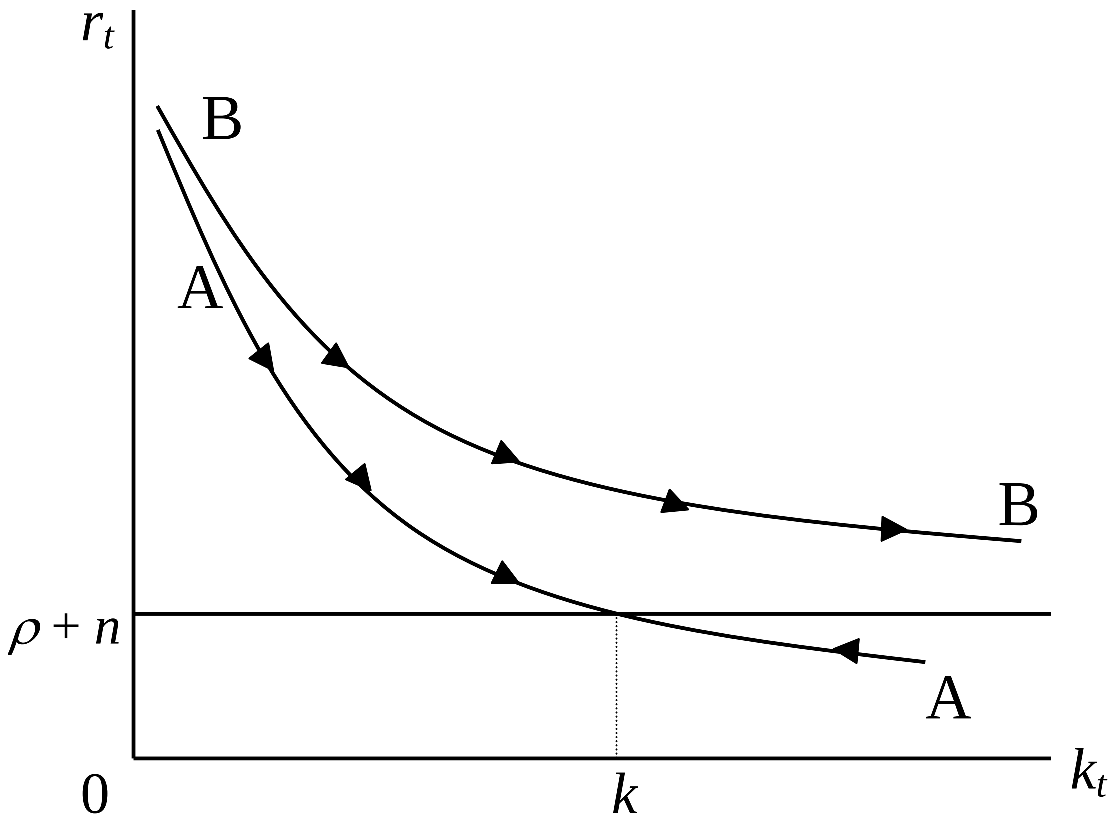

Figure 2. Steady state and long run growth

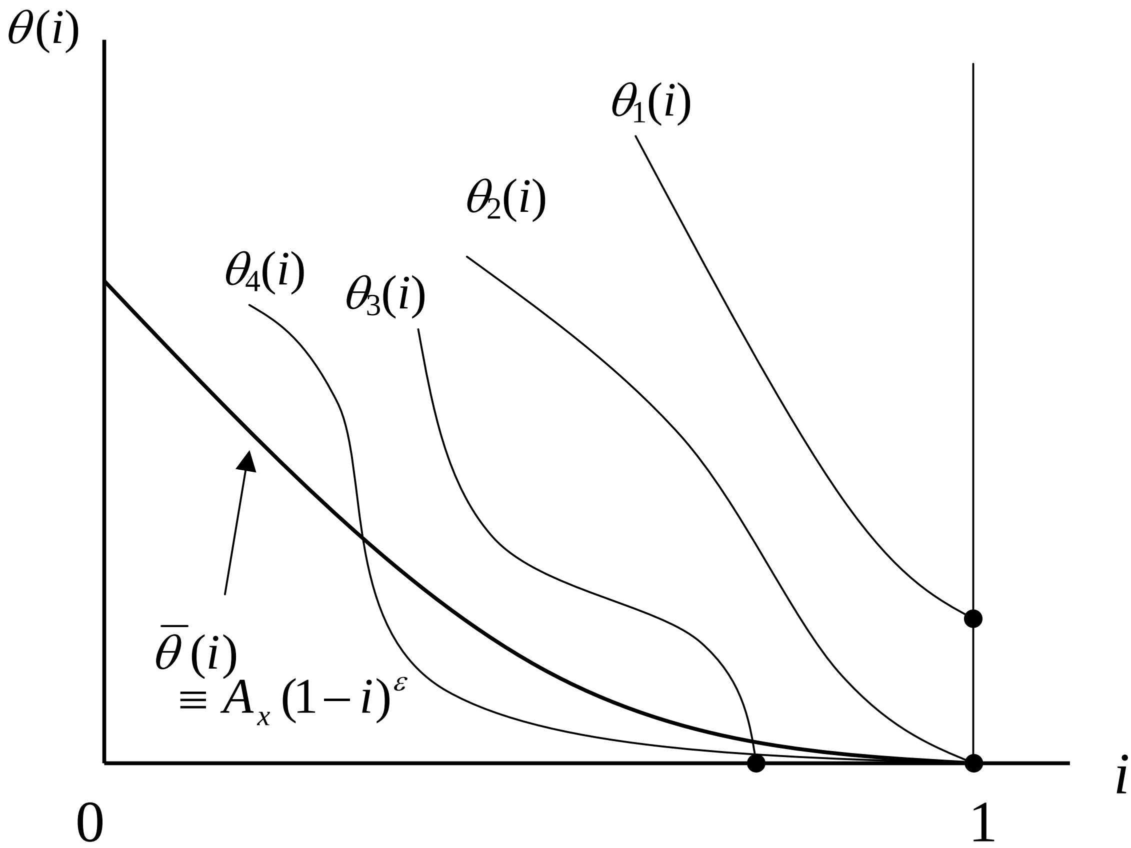

Figure 3. Illustration of $\theta(i)$ and $\bar{\theta}(i)$

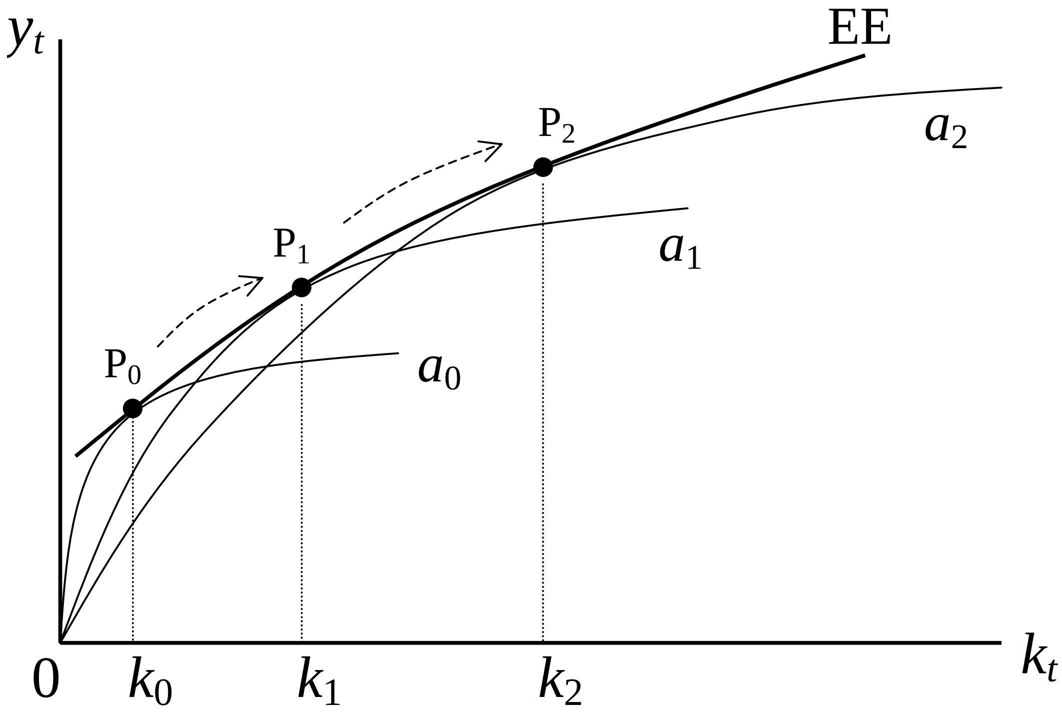

Figure 4. Relationships between our aggregate production function and the Cobb-Douglas type