# Social Security Reform and Childcare Support*

Takashi Oshio

*Tokyo Gakugei University*

***Abstract***

*This paper examines how social security reform and childcare support affect fertility and social welfare, based on a simple overlapping generations model with endogenous fertility. In an open economy with no altruism, introducing a childcare subsidy is the second-best solution under an aging population. However, in a closed economy and/or assuming the household's altruistic bequests, childcare support is not necessarily desirable and the case that curtailing a pay-as-you-go social security system reduces social welfare cannot be ruled out. In addition, we show that social security reform and childcare have different effects on the transition process to a new steady state.*

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*Correspondence to*:  Takashi Oshio, Tokyo Gakugei University, 41-1 Nukui Kitamachi, Koganei, Tokyo, 203-0023, Japan, tel/fax +81-42-379-7415, t-oshio@mb8.seikyou.ne.jp.

## 1. Introduction

Population is aging rapidly in almost every industrialized country. This demographic trend has placed strong pressures on the financial viability of the social security (hereafter referred to as SS) system, since its benefits to retired generations depend heavily on working and future generations. In most countries, the total fertility rate (TFR) remains well below the level that can sustain the current population size and there is a growing risk that the financial positions of the public pension will deteriorate and inequality between generations will continue to widen. Many economists propose to shift to a funded system from a pay-as-you-go one, or privatize the system along with introduction of mandatory individual accounts (e.g. Feldstein (1999)). Any SS reform should accompany a reduction of benefits and contributions and curtail intergenerational income transfer, which will become more difficult to sustain under an aging population[1].

Another way to strengthen the SS system against demographic pressures would be to provide households with financial childcare support, because children are expected to play a key role in financing SS benefits in the future. A sustained downtrend of fertility in many countries is attributable largely to the higher opportunity cost of childcare, presumably reflecting a rise in the labor force participation by highly educated women. Thus, in recent decades, policymakers in industrialized countries have been giving more importance to childcare support, including family and child allowance, maternity and childcare leave, and various tax incentives for childcare. Especially in Japan, which belongs to the group with the lowest fertility in OECD member countries and whose benefits to the "family and children" have a quite limited share of total social policy benefits, many argue for a shifting of

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[1] In this paper, we do not discuss other important roles of social security, such as to provide the elderly with the "national minimum" and to reduce uncertainty about future income.

benefits to childcare from income support for the elderly[2].

It is, however, uncertain whether and to what extent childcare support is effective in improving fertility.  Fertility depends heavily on various factors such as immigration, people's attitude toward marriage and family, and other social/cultural/religious backgrounds, and it takes at least a couple of decades for childcare support to have any impact.  If that is the case, it should be more reasonable and desirable to make SS less exposed to demographic pressures; that is, to lower a magnitude of income transfer between generations.

This paper aims to investigate how much SS reform and childcare support affects fertility and social welfare, based on a simple overlapping generations model with endogenous fertility.  There is extensive literature on endogenous fertility in the context with human capital and endogenous growth theory.  Among others, seminal works by Becker and Barro (1988) and Barro and Becker (1989) analyze how fertility is endogenously determined in small open and closed economies, respectively, assuming dynamic utility maximization by altruistic household.  Becker, Murphy, and Tamura (1990), which is another noted study on economic growth with endogenous fertility, showed a negative correlation between fertility and scale of human capital.  Nishimura and Zhang (1992), Kato (1999), and Oshio (2001) provide recent examples of studies on the relation between social security and fertility.  However, much remains to be analyzed about the effectiveness of SS reform and childcare support in terms of fertility, as well as social welfare.

Intuitively, childcare support appears to be a favorable policy option, since it is expected to be able to raise fertility and mitigate the distorting impact of SS on resource allocation under an aging population.  This second-best argument seems to make sense in

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[2] In Japan, the TFR was as low as 1.34 in 2001, and the share of benefits to the "Family and Children" of total SS benefits was 2.7% (compared to 9.0% in Germany (1996) and 10.5% in Sweden) according to ILO's "Cost of Social Security 1994-96: Nineteenth International Inquiry."

an open economy with no altruistic income transfer between generations, although the best policy option should be to reduce a size of the pay-as-you-go SS system. In a closed economy, however, the story becomes much more complicated because the policy impact on capital accumulation should be taken into account; childcare support might reduce the pace of per-capita capital accumulation in the long-run due to a greater number of descendants. Moreover, altruistic bequests, which are expected to at least partly offset public income transfer between generations, would likely make it more difficult to predict the direction of the policy impact. In addition, the well-known "double burden" problem that occurs during the transition to a funded SS system implies that it is important to analyze what the transition process to a new steady state looks like.

Section 2 sets up the basic model, starting with an open economy and no altruism, and examines a combination of SS and childcare support and its effectiveness. Then, it moves to a closed economy and takes a household's altruistic behavior into account, to investigate whether or not the second-best story continues to hold. Section 3 presents some simple simulations to compare the steady states before and after the policy changes. It also gives a rough picture of the transition process to the new steady state as well as the intergenerational distribution of the policy impact. Section 4 concludes.

## 2. Theoretical framework

### 2.1 The basic model in an open small economy

Let us consider a simple overlapping generations model of two periods (working and retirement periods) and two generations (working and retirement generations) in an open small economy. Also assume that the working generation consists of $m$ symmetric

households, whose utility functions and budget constraints are identical. Each household consumes $c_1$ and $c_2$, in each period, respectively, and it bears and cares for $n$ children in the working period. Since we consider a household in individual terms, the size of the population remains unchanged if $n$ is equal to unity, grows (shrinks) if $n$ is greater (less) than unity. So $n$-1 can be interpreted as the rate of population growth rate per generation, and $2n$ corresponds to TFR. We assume that the household's utility is determined solely by levels of consumption in each period and number of children, not by the children's "quality" such as the education with which they are endowed[3].

For simplicity, we assume that a representative household has a Cobb-Douglas utility function such as

$$u = a \ln n + b \ln c_1 + g \ln c_2, \quad a + b + g = 1, \quad 0 < a, b, g < 1.$$

(1)

At this point, we assume that the household cares only about its own utility and leaves no bequests. Denote $w$, $r$, and $z$ as wage, interest rate, and cost of childcare per child, and assume that the household takes these variables as exogenously given. Now let us consider two social policy measures. First, the government levies the SS tax, $p$, per household on the young generation, and gives SS benefit to the old generation which has retired. Under a pay-as-you-go system, total SS benefits are set to be equal to total SS tax revenues in each period. Second, the government gives the childcare subsidy, $s$, for each child, and finances it by levying the childcare tax on the working generation. All the SS benefit/ tax and childcare subsidy/tax are lump-sum.

The household determines consumption levels at each period and the number of children to maximize its utility, given the average number of children, $\bar{n}$, of the other

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[3] In this respect, see Becker and Tomes (1976) and Cigno (1991) for example.

households in the economy.  Then, the budget constraint of the household is given by

$$(z-s)n + c_1 + \frac{c_2}{1+r} = w - p + \frac{(m-1)\overline{n}+n}{(1+r)m}p - \frac{(m-1)\overline{n}+n}{m}s. \qquad (2)$$

Since the households live in an open economy, capital accumulation is neglected and both $w$ and $r$ are assumed to be constant.  The first term on the left hand side of (2) shows the net childcare cost, which is reduced by the childcare subsidy.  The third term on the right hand side is the present discount value of the SS benefit received after retirement, with the coefficient on $p$ – which is the same as that on $s$ – being the average number of children per household.  The last term shows the size of the lump-sum tax to finance the childcare subsidy.  The larger the number of children, the more SS benefit the household receives in the retirement period and the more SS benefit and the more childcare tax it pays in the working period.

If the number of households is very large, the budget constraints (2) can be condensed to

$$(z-s)n + c_1 + \frac{c_2}{1+r} = w - p + \left(\frac{p}{1+r} - s\right)\overline{n}, \qquad (3)$$

where the household assumes that the number of its own children does not affect the levels of SS benefit and childcare tax.  This condensed budget constraint also shows that the average number of children of other households affects each household's budget in opposite ways: more children of other  households increase  each  household's lifetime disposable income through the SS benefit on the one hand; and, they reduce it through the childcare tax on the other.  In this sense, children are "public goods" that have both positive and negative externalities.

The optimal number of children for each household, given the number of children of other households, is given by

$$n = \frac{a}{z-s}\left[w - p + \left(\frac{p}{1+r} - s\right)\overline{n}\right],$$

which maximizes utility (2) under the budget constraint (3).  Since households are assumed

to be symmetric, the average number of children per household in this economy is given by

a Nash equilibrium such that

$$n = \frac{a(w-p)}{z-(1-a)s - ap/(1+r)} \tag{4}$$

which is obtained assuming that $n = \overline{n}$ and that

$$z > (1-a)s + \frac{ap}{1+r} \tag{5}$$

to make sure that the number of children is positive.  This inequality (5) indicates the

condition that the *gross* cost of childcare, *z*, is greater than the weighted average of two

kinds of  social burden related to childcare per child; that is, the childcare subsidy, *s*, and

the present discount value of the SS benefit, *p*/(1+*r*).

The optimal levels of consumption are also given by

$$c_1 = \frac{b(w-p)(z-s)}{z-(1-a)s - ap/(1+r)}, \tag{6-1}$$

$$c_2 = \frac{g(w-p)(z-s)(1+r)}{z-(1-a)s - ap/(1+r)} \tag{6-2}$$

in the working and retirement periods, respectively.  Plugging (4) and (6) into (1) yields the

household's indirect utility function, denoted as *v*, which is given by

$$v = \ln(w-p) + (1-a)\ln(z-s) + g\ln(1+r) - \ln[z-(1-a)s - ap/(1+r)] + const., \tag{7}$$

with *w*, *r*, *p*, and *s* being exogenously given.


## 2.2 The second-best combination of social security and childcare subsidy

In this section we consider the role of SS reform, childcare subsidy, and their combination for social welfare in an open economy. Differentiating (7) with respect to $p$ and $s$, together with (4), we obtain

$$\frac{\partial v}{\partial p} = \frac{n-1-r}{(1+r)(w-p)},$$

(8-1)

$$\frac{\partial v}{\partial s} = \frac{\boldsymbol{a}[p/(1+r)-s]}{(z-s)[z-(1-\boldsymbol{a})s-\boldsymbol{a}p/(1+r)]}.$$

(8-2)

Here, two things should be noted. First, we reconfirm from (8-1) the well-known threshold for the efficiency of a pay-as-you-go SS system: if the population growth rate, *n-1*, is higher (lower) than the interest rate, *r*, then SS is efficient (inefficient). In addition, since *n* is endogenously solved by (4), we get

$$n-1-r = \frac{\boldsymbol{a}w-(1+r)z+(1-\boldsymbol{a})(1+r)s}{z-(1-\boldsymbol{a})s-\boldsymbol{a}p/(1+r)}.$$

This implies that as far as

$$(1+r)z > \boldsymbol{a}w$$

(9)

and there is no or limited childcare subsidy, the population growth rate is lower than the interest rate and thus SS reduces the household's utility. At the same time, however, we cannot rule out the case that the childcare subsidy more than offsets the negative impact of SS on social welfare.

Second, we recognize from (8-2) that a childcare subsidy cannot be justified unless there is a pay-as-you-go SS system (or other systems that accompany income transfer to the elderly from the young). If there is no SS, children cannot be "public goods" that have positive externalities in this model, thus the childcare subsidy that aims to increase the number of children is not needed or rather should be avoided. If there is any SS, the

household's utility is maximized when

$$s = p/(1+r). \tag{10}$$

This complementary relationship between SS and childcare subsidy can be explained in another way. That is, we can show that the *privately* optimal number of children is smaller than its *socially* optimal level in a SS-rich economy, whereas it is larger in a childcare-support-rich economy. The socially optimal number of children, denoted as $n^*$, given the SS tax, $p$, is obtained to maximize (1) under the social budget constraint:

$$\left( z - \frac{p}{1+r} \right) n + c_1 + \frac{c_2}{1+r} = w - p,$$

where we assume that $\bar{n} = n$ in (3), and take into account the relationship that the childcare subsidy and tax are to be completely balanced in the government budget. A simple calculation yields the socially optimal number of children, $n^*$, which is given by

$$n^* = \frac{a(w-p)}{z - p/(1+r)}, \tag{11}$$

assuming that $z > p/(1+r)$. Comparing this with the privately optimal number of children given by (4) leads to

$$n < n^* \ \ if \ \ s < p/(1+r), \ \ n > n^* \ \ if \ \ s > p/(1+r).$$

These inequalities tell us that the number of children tends to be smaller than socially optimal if SS is richer than childcare support, whereas it tends to be larger than socially optimal if childcare support is richer than SS. Under a pay-as-you-go SS scheme, the socially and privately optimal number of children are identical if and only if (10) holds.

It is widely known that a pay-as-you-go SS scheme is not neutral regarding the household's utility maximization. However, its distorting effect can be offset by another distorting policy, that is, the childcare subsidy. This is a typical second-best situation, but at the same time the second-best policy combination cannot be the best by definition. We

discuss this more in detail in the next subsection.


## 2.3 The limited effectiveness of the policy combination


Discussions in the previous subsection suggest that the government can subdue the distorting effect of SS on social welfare by introducing a childcare subsidy. However, this does not mean that SS, even if accompanied by a childcare subsidy, can be justified in terms of social welfare. In an aging society, where the population growth rate is below the interest rate, the best solution should be to curtail the pay-as-you-go SS scheme. This can be confirmed as follows.

Assuming that the childcare subsidy is set so that (10) holds, the impact of an increase in the SS tax on a household's utility is given by

$$\left.\frac{dv}{dp}\right|_{s=p/(1+r)} = -\frac{1}{w-p} + \frac{a}{(1+r)z-p} = \frac{(1-a)p - [(1+r)z - aw]}{(w-p)[(1+r)z-p]}.$$

Then if (5) and (9) hold, we can show the relationship such that

$$\left.\frac{dv}{dp}\right|_{s=p/(1+r)} < 0 \;\; if \;\; p < \tilde{p}, \;\; \left.\frac{dv}{dp}\right|_{s=p/(1+r)} \geq 0 \;\; if \;\; p \geq \tilde{p},$$

where

$$\tilde{p} = \frac{(1+r)z - aw}{1-a}$$

which is positive as far as (9) holds. Thus, SS reduces social welfare as far as the level of SS benefit is below a certain level of $\tilde{p}$, even with the second-best policy combination.

Now, consider the impact on fertility. Assuming that $w > (1+r)z$, we know from (4) that a higher SS tax raises the number of children since

$$\frac{dn}{dp}\bigg|_{s=p/(1+r)} = \frac{w-(1+r)z}{(1+r)[z-p/(1+r)]^2} > 0$$

and also that

$$n < 1+r \Leftrightarrow p < \tilde{p}, \ n \geq 1+r \Leftrightarrow p \geq \tilde{p}.$$

Thus the impacts of the SS tax on the household's utility and number of children are illustrated in the left part of Figure 1. The second-best policy combination cannot revise the conventional wisdom that SS reduces social welfare as far as the population growth rate is below the interest rate, even if the population growth rate is endogenously determined. Also, it is ironic that the level of utility is *minimized* when the population growth rate reaches the interest rate, that is, when the so-called "golden rule" holds.

If we assume that $w \leq (1+r)z$, then a higher SS tax reduces both and the household's utility and number of children, as illustrated in the right part of Figure 1. In this case, the population growth rate cannot exceed the interest rate, and the gap between the two widens as the government raises the SS tax (and correspondingly, the childcare subsidy).

Thus, in both cases, we confirm that the best solution under an aging population is to reduce the size of a pay-as-you-go SS system by lowering the levels of SS tax and benefit. The childcare subsidy fails to completely offset the welfare-reducing effect of SS, even if it can raise the number of children. Thus, under an aging population the best policy combination is no SS and no childcare subsidy.

2.4 A closed economy with capital accumulation

Let us move to a closed economy, where non-human capital accumulates through household savings, and affects the wage and interest rate. We continue to assume that

each household takes wage, interest rate, and average number of children of other households as given and maximizes its utility with no bequests. Then, the privately optimal number of children and the levels of consumption in each period remain the same as in an open economy. In addition, the population growth rate is again below the interest rate, if there is no or limited childcare support and (9) holds.

Now, let us consider capital accumulation explicitly. In the steady state and Nash equilibrium (with $\bar{n} = n$), per-capita capital stock, $k$, is given by

$$k = \frac{w - p - ns - (z-s)n - c_1}{n} = \frac{w - p - c_1}{n} - z = \frac{g(z-s)}{a} - \frac{p}{1+r} \tag{12}$$

using (4) and (6) for rearrangement and assuming that capital stock is completely depreciated within one generation.

Firms act competitively, hiring labor to the point where the marginal product of labor is equal to the wage, and renting capital to the point where the marginal product of capital is equal to the interest rate. Also assuming that the production function is constant to scale with respect to capital and labor, we know

$$w = f(k) - kf'(k), \quad r = f'(k), \quad f'(k) > 0, \quad f''(k) < 0. \tag{13}$$

Then, it is easily shown that from (12) and (13) the introduction of either SS or childcare subsidy tends to slow the rate of capital accumulation, since

$$\frac{dk}{dp} = -\frac{1}{1+r}\left[1 - \frac{pr'}{(1+r)^2}\right]^{-1} < 0, \quad \frac{dk}{ds} = -\frac{g}{a}\left[1 - \frac{pr'}{(1+r)^2}\right]^{-1} < 0, \tag{14}$$

where $r' = dr/dk < 0$. A higher SS tax and a higher childcare subsidy both reduce the household's savings and in turn reduce the rate of capital accumulation. This suggests that any combination of the two policies cannot offset the negative impact of each policy on capital accumulation; rather it may amplify that of each policy. Hence, the effectiveness of the policy combination appears to be much more limited here in a closed economy than in

an open economy.

Then, let us consider the effects of SS and the childcare subsidy on social welfare, explicitly considering their impact on capital accumulation. Our focus is on whether and to what extent the childcare subsidy can mitigate the SS's distorting impact on social welfare. Differentiating the household's indirect utility function, (7), with respect to *p and s,* respectively, yields

$$\frac{dv}{dp} = \frac{\partial v}{\partial p} + \frac{dv}{dk}\frac{dk}{dp} = \left[ -\frac{1}{w-p} + \frac{a/(1+r)}{z-(1-a)s-ap/(1+r)} \right] + \Omega\frac{dk}{dp}, \qquad (15\text{-}1)$$

$$\frac{dv}{ds} = \frac{\partial v}{\partial s} + \frac{dv}{dk}\frac{dk}{ds} = \left[ -\frac{1-a}{z-s} + \frac{1-a}{z-(1-a)s-ap/(1+r)} \right] + \Omega\frac{dk}{ds}, \qquad (15\text{-}2)$$

where

$$\Omega \equiv \frac{dv}{dk} = \frac{w'}{w-p} + \frac{gr'}{1+r} - \frac{apr'/(1+r)}{z-(1-a)s-ap/(1+r)},$$

and $w' = dw/dk > 0$. The first term of the right hand side in (15-1) and (15-2) captures the direct impact of the SS tax and childcare subsidy, respectively, on the household's utility, whereas the second term represents their indirect impact on the household's utility through capital accumulation. $\Omega$ gives the impact of a change in capital stock on the household's utility through wage and interest rate. With no SS or childcare subsidy, $\Omega$ is shown to be positive: that is; the higher per-capita capital raises the household's utility. We cannot generally tell the sign of $\Omega$, which depends on parameters. We can show, however, that the less the childcare subsidy, the higher becomes the case that $\Omega$ is positive and that the childcare subsidy reduces utility through capital accumulation.

It is difficult to algebraically solve the optimal combination of the SS tax and the childcare subsidy from (15), but it is interesting to assess the combination of $s = p/(1+r)$, which is the second-best solution in an open economy, here in a closed economy in terms of

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efficiency. First, focus on (15-2) given *p* and consider whether or not the combination of $s = p/(1+r)$ remains optimal. The first term of the right hand side of (15-2) is zero when $s = p/(1+r)$ and positive (negative) when *s* is smaller (larger) than $p/(1+r)$, as already discussed for an open economy. And, a tedious but simple calculation yields

$$\Omega\big|_{s=p/(1+r)} > 0 \ \ if \ \ p < \tilde{p}, \ \ \Omega\big|_{s=p/(1+r)} \leq 0 \ \ if \ \ p \geq \tilde{p},$$

assuming that the production function is constant to scale and firms act competitively (and thus $w' + kr' = 0$). Considering that $p < (\geq) \tilde{p}$ corresponds to $n < (\geq) 1 + r$ in a closed economy as in an open economy, and assuming that $n < 1 + r$ under an aging population, we get

$$\frac{dv}{ds}\bigg|_{s=p/(1+r)} = \Omega\frac{dk}{ds} < 0.$$

Thus, the combination of $s = p/(1+r)$ is no longer the second-best solution in a closed economy. Given the level of the SS tax, *p*, the government should set the level of childcare subsidy lower than $p/(1+r)$, since the childcare subsidy adds to the SS's negative impact on capital accumulation.

Furthermore, we cannot *a priori* rule out the possibility that curtailing the childcare subsidy is favorable. The negative impact of the childcare subsidy on capital accumulation might dominate its total impact on the household's utility, even if its direct impact on the household's utility–which is reflected in the second term of the right hand side of (14-1)–is positive. Section 3 examines this possibility, along with the impact on the number of children, based on simple simulations.

## 2.5 Bequest motive and fertility in an open economy

Let us introduce altruistic bequests into the model and analyze how they affect the

previous conclusions, assuming that households care about their future generations as well as themselves. The well-known Barro's neutrality argues that individuals offset the change in public income transfers–such as net contributions of SS–by an offsetting change in altruistic bequests, making the net transfers between generations unaffected. Does this argument still hold in this model with endogenous fertility? That is, are SS and childcare subsidy neutral to the household's utility? And, is the combination of the SS tax and childcare subsidy, $s = p/(1+r)$, desirable as in the case of no bequest motive?

First, let us consider the case of an open economy, where the wage and the interest rate are exogenously given and capital accumulation can be neglected. And, assume that the household cares about its children's welfare by weighting the children's utility in its own utility function, and denote the utility function of the altruistic household as *U*. To make the calculations simple, set up *U* as

$$U = a \ln n + b \ln c_1 + g \ln c_2 + \frac{n^e}{1+d} U_{+1},$$
$$a + b + g = 1, \quad 0 < a, b, g < 1, \quad 0 < e, d,$$

where $U_{+1}$ is the per-capita utility of the children. *e* and *d* are parameters that show the degree of altruism in different ways: *e* is the elasticity of altruism toward children with respect to the number of children, and *d* is the rate to discount the children's utility. The number of children affects the household's utility in two ways: directly (the first term) and indirectly by multiplying the child's utility by $n^e$ (the fourth term). Although this seems to be partly redundant, we use this model to make the results easy to compare with those in the model of no altruism.

To make the system dynamically stable, we have to assume that $n^e < 1+d$ hereafter. In addition, assume that the household receives bequests from its parents and leaves bequests to its children in the working period, so the budget constraint in each period is expressed as

$$c_1 = w + (1+r_-)b_- - (z-s+b)n - p - s\bar{n} - x, \tag{16-1}$$

$$c_2 = (1+r)x + p\bar{n}, \tag{16-2}$$

where $b$ is per-capita bequests left to the children, $b_-$ is bequests left by the parents, and $x$ is savings. Bequests, like savings, bear the interest rate, $r$, and $r_-$ denotes the interest rate on the bequests received from the parent generation. Each household maximizes its utility, $U$, under the above budget constraints, taking wage, interest rate, and number of children of other households to be as assumed in the previous subsections.

Differentiating $U$ with respect to $x$, $b$, and $n$ gives

$$\frac{\partial U}{\partial x} = -\frac{b}{c_1} + \frac{g(1+r)}{c_2} = 0, \tag{17-1}$$

$$\frac{\partial U}{\partial b} = -\frac{bn}{c_1} + \frac{n^e}{1+d}\frac{b(1+r)}{c_{1,+1}} = 0, \tag{17-2}$$

$$\frac{\partial U}{\partial n} = \frac{a}{n} - \frac{b(z-s+b)}{c_1} + \frac{en^{e-1}}{1+d}U_{+1} = 0, \tag{17-3}$$

where $c_{1,+1}$ is the children's consumption in the working period. Let us concentrate on a Nash equilibrium in the steady state, where $\bar{n} = n$, $b_{-1}=b$, $r_{-1}=r$, $c_1=c_{1,+1}$, and $c_1=c_{2,+1}$. Then, it follows from (17-2) that

$$n = \left(\frac{1+r}{1+d}\right)^{\frac{1}{1-e}}, \tag{18}$$

which means that in an open economy the number of children depends solely on the (world's) interest rate and the degree of altruism against children[4]. If there is perfect arbitrage between human capital and non-human capital, the interest rate can be interpreted as a rate of return on human capital. Then, it makes sense that the higher interest rate

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[4] This result has already been obtained by Becker and Barro (1988)'s analysis on endogenous fertility.

encourages the household to increase the number of children.  The net cost of childcare, which reflects SS and childcare support, is neutral to fertility due to the household's altruistic income transfer.  Moreover, if (18) holds, the assumption that $n^e < 1 + d$ that makes the system dynamically stable is transformed to the inequality $n < 1 + r$.

However, the fact that neither SS nor childcare subsidy affects the number of children does not necessarily mean that they are neutral to the household's utility.  This can be confirmed as follows.  In a Nash equilibrium in the steady state, the budget constraints of the household, (16-1) and (16-2), are aggregated with some rearrangement into

$$c_1 + \frac{c_2}{1+r} + zn = w + (1+r-n)\left(b - \frac{p}{1+r}\right).$$
(16)'

This budget constraint (16)', together with (17-1), (17-3), and (18), yields the first-order solutions for $c_1$, $c_2$, and $b$ to maximize economic welfare, given the number of children, which is independently determined by (16), as well as SS benefit and the childcare subsidy.

This budget constraint (16)' suggests that a change in the SS tax is neutral to the household's decisions about the number of children and consumption only if $\Delta b = (1+r)\Delta p$. (17-3) tells, however, that if $\Delta b = (1+r)\Delta p$, a change in the SS tax affects the household's behavior unless $\Delta s = \Delta p/(1+r)$.  Second, (17-3) suggests that a change in the childcare subsidy is neutral to the household's decisions about the number of children and consumption only if $\Delta b = \Delta s$.  (16)' tells, however, that if $\Delta b = \Delta s$, a change in the childcare subsidy affects the household's budget constraint unless $\Delta s = \Delta p/(1+r)$.

Therefore, neither SS nor childcare subsidy is neutral to the household's behavior, unless the government sets the policy combination of $s = p/(1+r)$.  If $s = p/(1+r)$, then the household's bequest function looks like

$$b = \bar{b} + \frac{p}{1+r} = \bar{b} + s$$
(19)

where $\bar{b}$ is a component that is not related to SS and childcare subsidy. Any change in those policies is offset by an opposite change in bequests, and does not affect the levels of consumption and utility. Note, however, that this does not mean the optimality of this policy combination.

The main difference from the case of no altruism is that we cannot rule out the case in which a higher SS tax *raises* social welfare, even if the population growth rate is below the interest rate. Also, the impact of the childcare subsidy is mixed. These features can be confirmed as follows. First, in the steady state of the model, the level of utility is the same for all generations and it is given by

$$U = U_{+1} = \frac{1}{1 - n^e/(1+d)} \left( a \ln n + b \ln c_1 + g \ln c_2 \right)$$

$$= \frac{1+r}{1+r-n} \left[ a \ln n + (1-a) \ln c_1 + g \ln \frac{g(1+r)}{b} \right],$$

(20)

using (17-1) and (18). Second, plugging (20) and the budget constraint (16)' into (17-3) yields

$$\frac{b}{c_1} \left[ \frac{(1+r)z - w}{1+r-n} + \left( \frac{p}{1+r} - s \right) \right] - \frac{a(1+r) - n}{n(1+r-n)}$$

$$= \frac{e}{1+r-n} \left[ a \ln n + (1-a) \ln c_1 + g \ln \frac{g(1+r)}{b} \right].$$

(21)

The left hand side of this equation represents a net marginal decrease in the household's own utility caused by an increase in the number of children, while the right hand side represents a marginal increase in the children's utility discounted by the household. Taking the special case of $e = 0$, differentiating (21) with respect to $c_1$, $p$, and $s$ yields

$$\frac{dc_1}{dp} = \frac{bn(1+r-n)}{(1+r)[a(1+r)-n]} = \frac{bd}{(1+r)[a(1+d)-1]}, \quad \frac{dc_1}{ds} = -\frac{(1+r)dc_1}{dp}.$$

(22)

Hence, if the population growth rate is low enough to meet the condition of $n < a(1+r)$--

which is equivalent to the condition of $a(1+d)>1$ in the case of $e=0$ --then a higher SS tax would *raise* the level of consumption and accordingly that of overall social welfare. The number of children is not affected by any policy change, and capital accumulation is neglected here in an open economy.

This conclusion--even if depending on parameters such as $a$ and $d$ --is the opposite of the usual conclusion of conventional life-cycle models, which a pay-as-you-go SS system reduces welfare if $n<1+r$. Also, the latter part of (22) implies that an increase in the childcare subsidy, albeit likely to mitigate the negative impact of SS on social welfare, could *lower* the level of the household's consumption and utility if the number of children is so small that $n<a(1+r)$.

The intuitive reason for these paradoxical results is that a higher SS tax raises the net cost of childcare, which in turn endows future generations with a higher level of consumption and utility, whereas the childcare subsidy reduces the net cost of childcare, which in turn reduces consumption and social welfare utility of future generations. It should be noted, however, that if the initial number of children is not so small ($n>a(1+r)$), then the negative income effect of a higher SS benefit or a lower childcare subsidy on the household's income dominates the overall impact on consumption and utility. We also notice from (21) that the SS tax and childcare subsidy, if combined so that $s=p/(1+r)$, are neutral to social welfare.

## 2.6 Bequest motive and fertility in a closed economy

Finally, let us move to a closed economy and take capital accumulation into account. The budget constraints (16) and the first-order conditions (17) remain the same as in the case of an open economy. We now consider capital accumulation, where per-capita capital

stock is determined by savings and bequests such that

$$k = \frac{w + (1+r)b - p - ns - (z - s + b)n - c_1}{n} + b = \frac{w + (1+r)b - p - nz - c_1}{n}$$

in the steady state. It is impossible to algebraically capture the overall impact of SS and childcare subsidy, as well as to foretell the direction of a change. A higher SS tax itself is likely to subdue savings under a pay-as-you-go SS system. At the same time, however, it may encourage the household to raise altruistic bequests, because parents get worried about heavier burdens levied on their children. If the latter effect is larger than the former, capital accumulation could be stimulated. And if that is the case, interest rate declines and number of children falls (see (18)), thus per-capita capital stock increases further. On the other hand, a higher childcare subsidy is expected to directly encourage the household to rear children and lower the pace of capital accumulation. In any case, however, we cannot know the overall impact of each policy on the household's utility.

Finally, does the conclusion that the combination of $s = p/(1+r)$ makes both SS and the childcare subsidy neutral to social welfare still hold? Assuming that the government arranges policy so that $s = p/(1+r)$ and that the household sets up its bequests as shown in (19), per-capita capital stock is given by

$$k = \frac{w + (1+r)\overline{b} - zn - c_1}{n}. \tag{22}$$

Thus, both SS and childcare subsidy disappear from all of the budget constraints (16), the first-order conditions (17), and capital accumulation (23). The two policies, if combined as $s = p/(1+r)$, do not affect social welfare at all, which again is the same conclusion as that drawn from the model of an open economy.

## 3. Simulations

### 3.1 Assumptions

In this section, we illustrate the impact of SS, childcare subsidy, and their combination on social welfare and number of children based on simple simulations.  We compare the policy effects for two models (a model with no altruism and a model with altruism) under two economies (an open economy and a closed economy), starting with the initial situation where a pay-as-you-go SS system has already been incorporated and that total fertility rate is relatively low.  Discussions in Section 2 imply that the policy results depend much on the setup of the model and the economy: the impact of introducing the childcare subsidy is expected to be favorable in a model with no altruism in an open economy, while its direction is not determined in other cases.

We establish the initial condition by providing parameters with tentative but seemingly plausible values, as summarized in Table 1.  For the utility function, the same weight (1/3) is put on the number of children and each level of consumption in the working and retired periods.  The production function is given by $y = k^q$, with the share of capital income, $q$, being equal to 1/3.  The lump-sum SS tax is set at 20% of the initial wage, and the childcare subsidy is not introduced initially.  For models with altruism, $e$, one of two parameters of altruism, is set to be 0.003.  We find after several trials that a high value of $e$ tends to make the simulations unstable and unlikely to converge.

Then, the gross cost of childcare, z, is left fixed.  We first set the initial value of $n$ as 0.8, reflecting the fact that the average TFR is close to 1.6 among major industrialized nations.  Then we run simulations and search for the value of $z$ that makes $n$ 0.8; we find that $z$ is about 35.8% of the wage.  We assume that $z$ is proportional to the wage (unlike the theoretical analysis in Section 2), since it seems reasonable to think that the opportunity

cost of childcare depends greatly on wage. In the case of models with altruism, we further need to fix $d$ , another parameter of altruism. We use the same childcare cost/wage ratio (z/w=0.358) as in models with no altruism, and search for the value of $d$ that makes $n$ 0.8. We find that $d$ is 1.827.

The initial values for endogenous variables, which are to be solved by simulations based on the above-mentioned parameters, are reported in Tables 2 and 3 (see below).

### 3.2 Steady state comparisons

Let us first make steady state comparisons, comparing the long-run impact of policy changes with the initial state. In each model, we consider three policy options: (1) the "CC subsidy," which introduces the childcare subsidy, equivalent to 50% of the SS tax, that is, 10% of the initial wage; (2) the "SS cut," which reduces the SS tax (as well as the SS benefit) to its half size, that is, 10% of the initial wage, again; and (3) the "Mix," which combines (1) and (2). The "Mix" case keeps the total contributions to SS and childcare subsidy unchanged, and aims to make the overall system more childcare-friendly. The "CC Subsidy" case aims to assess the second-bestness of childcare support.

Tables 2 and 3 summarize the simulation results, with the former for models with no altruism and the latter for those with altruism. To begin with, let us look at the results for the most basic model (no altruism/small open) in the top part of Table 2. The childcare subsidy raises the household's utility, because it increases the number of children and mitigates the negative impact of the existing SS, in line with the argument presented in 2.2. However, the improvement of utility is relatively limited, reflecting reduced consumption levels. Actually, the optimal level of the childcare subsidy, which corresponds to the second-best combination of $s = p/(1+r)$, is calculated to be 36.3% of the SS benefit (7.26% of the

wage), given that the interest rate is 1.755, which is equivalent to 3.4% at a thirty-year composite annual rate. A reduction in the SS tax becomes more efficient in improving welfare, while its impact on fertility is not so strong as in the case of introducing the childcare subsidy. This is also consistent with the argument in 2.3. The mix of the two policies is more efficient in raising fertility, but its impact on utility is less than in the case of reducing the SS tax.

A closed economy with capital accumulation presents a quite different picture (see the bottom part of Figure 2). First, the childcare subsidy fails to improve the household's utility, although it raises fertility. As suggested from discussions in 2.4, the explanation lies with capital accumulation; per-capita capital stock drops sharply due to an increase in the population size, which in turn lowers wage income substantially. By contrast, curtailing SS is welfare-improving, mainly because it stimulates the pace of capital accumulation. In this "SS cut" case, fertility also improves due to an increase in wage income, which in turn helps to raise utility. In the "Mix" case, the impact on welfare lies between the two.

Next, let us move to Table 3, where altruistic bequests are incorporated. In an open economy, where wage and interest rate are exogenously given and capital accumulation is neglected, both introducing the childcare subsidy and reducing the SS tax are welfare-improving and neutral to fertility (see the top part). Both of these policies lead to a substantial increase in bequests, which finances higher levels of consumption for all generations in the steady state. The "Mix" case amplifies the results of the two policies. Discussions in 2.5 point to the possibility that both of "CC subsidy" and "SS cut" may reduce consumption and utility, but our simulation obtains the opposite results[5].

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[5] In our simulation, $e$ is very close to 0, and $n = 0.8,$ $a(1+r) = 1/3 * (1+1.263) = 0.754$. Hence, $n > a(1+r)$, which probably means that $dc_1/dp$<0 and $dc_1/ds$>0 from (22). If we start a simulation with a much lower number of children, we could have the opposite results as the substitution effect becomes

In a closed economy, models of altruism lead to completely the opposite results: introducing childcare subsidy and curtailing SS both hurt social welfare, although they succeed in raising fertility (see the bottom part). The mechanism behind this is such that an increase in the number of children reduces per-capita capital stock, which in turn lowers wage income and at the same time further raises fertility. It should be remembered that lower per-capital stock increases fertility due to a higher interest rate in the models with altruism. Another noteworthy result is that the household reduces bequests substantially in the "SS cut" case, as suggested in 2.6. This is probably because a lower SS tax, which means a lower SS benefit, reduces the burdens on future generations and discourages the household from leaving altruistic bequests, and further dampens the pace of capital accumulation.

## 3.3 The transition and policy impact by generation

Steady state comparisons do not complete the assessment of each policy option, since any policy reform affects each generation differently during the transition to a new steady state. A well-known example of this is the "double burden" problem in SS reform. A shift from a pay-as-you-go system to a funded one requires the current working generation to pay "double" SS burdens to finance the benefits for both the current pensioners and themselves. This means that this SS reform makes the future generation better off at the expense of the current generation, making it difficult to be politically accepted. By contrast, introduction of the childcare subsidy is not likely to affect each generation so differently. This is because childcare subsidy and tax, which are balanced within the same generation,

larger than the income effect.

cause *intra*generational rather than *inter*generational income redistribution, although the impact is expected spill over to subsequent generations through capital accumulation, bequests, and fertility.

Our model is a very simple overlapping generations model, which consists of only two generations and two periods, but it can illustrate the basic picture of the transition process, as well as the intergenerational distribution of the policy impact. In simulations, we assume for simplicity that the household has static expectations about wage and interest rate, and we concentrate on the simulation results in a closed economy. Policy options to be considered are the same as those shown in Tables 2 and 3. The government announces and implements policy changes at the time when generation 1 is in the working period and generation 0 is in the retired period. In the "CC subsidy" case, generations 1 and younger receive a subsidy equivalent to p/2 per child and finance it by themselves. In the "SS cut" case, generation 1 pays *p* in the working period (to finance the SS benefit for generation 0) and receives *p*/2 times the number of children per household in the retired period. Generations 2 and younger pay *p*/2 in the working period and receives *p*/2 times the number of children per household in their retired period. Generation 0, which retired at the time as the policy announcement, continue to receive the SS benefit, *p,* despite the policy change.

Figures 2 to 4 summarize the results of models with no altruism. We first find from Figure 2 that curtailing the SS makes generation 1 worse off, although it raises utility in the long run. This result corresponds to a so-called "double burden" situation, pointing to a difficulty in incorporating SS reform. By contrast, introduction of the childcare subsidy immediately reduces the utility of generation 0 to its steady state level due to a reduction of per-capita capital, as its tax and benefit are balanced within that generation. In the "Mix" case, the adverse impact is concentrated heavily on generation 1.

Figure 3 shows that the pace of improvement in fertility is more moderate during the

transition in the "SS cut" case than in the "CC subsidy" case.  This is because generation 1

needs to save more to offset a reduction in the SS benefit it will receive after retirement,

reducing income available for bearing children.  Figure 4 compares a change in per-capita

capital stock and confirms that curtailing SS gradually raises it, whereas introducing the

childcare subsidy lowers it immediately, in line with the results in Figure 3.

Turning to models with altruism, we find a more complicated picture of the transition

as illustrated in Figures 5 to 7.  The most remarkable difference from models with no

altruism is that the utility curve becomes bumpier in the "SS cut" case, with generation 1

being worse off, generations 2 and 3 being better off, and generation 4 and younger being

worse off.  A reduction of per-capita capital stock dominates social welfare in the long run,

but the transitory generations are more exposed to the short-run impact: generation 1 faces

a "double burden" problem and generation 2 enjoys a reduction in net SS contributions, both

in line with the results for an open economy.  Generation 3 seems to be able to enjoy the

spillover effect from its parent generation.  However, for generation 4 and beyond, a

reduction in per-capita income dominates the net impact on the household's utility, as

suggested in Figures 6 and 7.  Figure 7 suggests that generation 1 increases savings for

their own incomes after retirement, but that smaller SS discourages altruistic bequests,

which together with an increase in population lowers the pace of per-capita capital

accumulation.  In the "CC subsidy" case, the transition is more gradual than in the "SS cut"

case and it takes several generations to reach to a new steady state.

## 4. Concluding Remarks

Our analysis shows that the impact of SS reform and childcare support on fertility and

social welfare depends much on the setup of models and assumptions about the openness

of the economy and altruism. The main results are summarized as follows.

Assuming no altruism in an open economy where wage and interest rate are given exogenously, introduction of a childcare subsidy is a second-best solution that offsets the SS's distorting effect on resource allocation under an aging population. If we move to a closed economy and/or assume altruistic bequests, however, childcare support is not necessarily desirable. And, we cannot rule out the case that even curtailing SS, which is the best solution in an open economy with no altruism, fails to improve social welfare. Meanwhile, both introducing the childcare subsidy and reducing the SS tax are expected to raise fertility (except for the case of an open economy with altruism), but their impact on capital accumulation and the household's altruistic behavior make it difficult to foretell the direction of the policy impact. We also find that SS reform and childcare support affect the transition process to the new steady state differently. SS reform, which affects intergenerational income transfer, tends to make the directions of long-term and short-term policy impacts opposite. By contrast, childcare support does not, since the childcare subsidy is financed within the same generation.

These results are likely to hold for wider definitions of SS (including any policy that requires income transfer to the elderly from the young, such as health and nursing care for the elderly) and of childcare support (including any policy that helps rearing the future generations, such as financial aid for education). However, our analysis is based on a very simple overlapping generations model, and there remains much to be investigated. Most of all, it is interesting to know how childcare support, which aims just to raise fertility in our analysis, can improve labor productivity and at least partly offset the adverse effect of a larger population on per-capita income. Childcare support could be more welfare-improving than implied in this paper.

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**Figure 1  Social security tax, utility, and fertility**
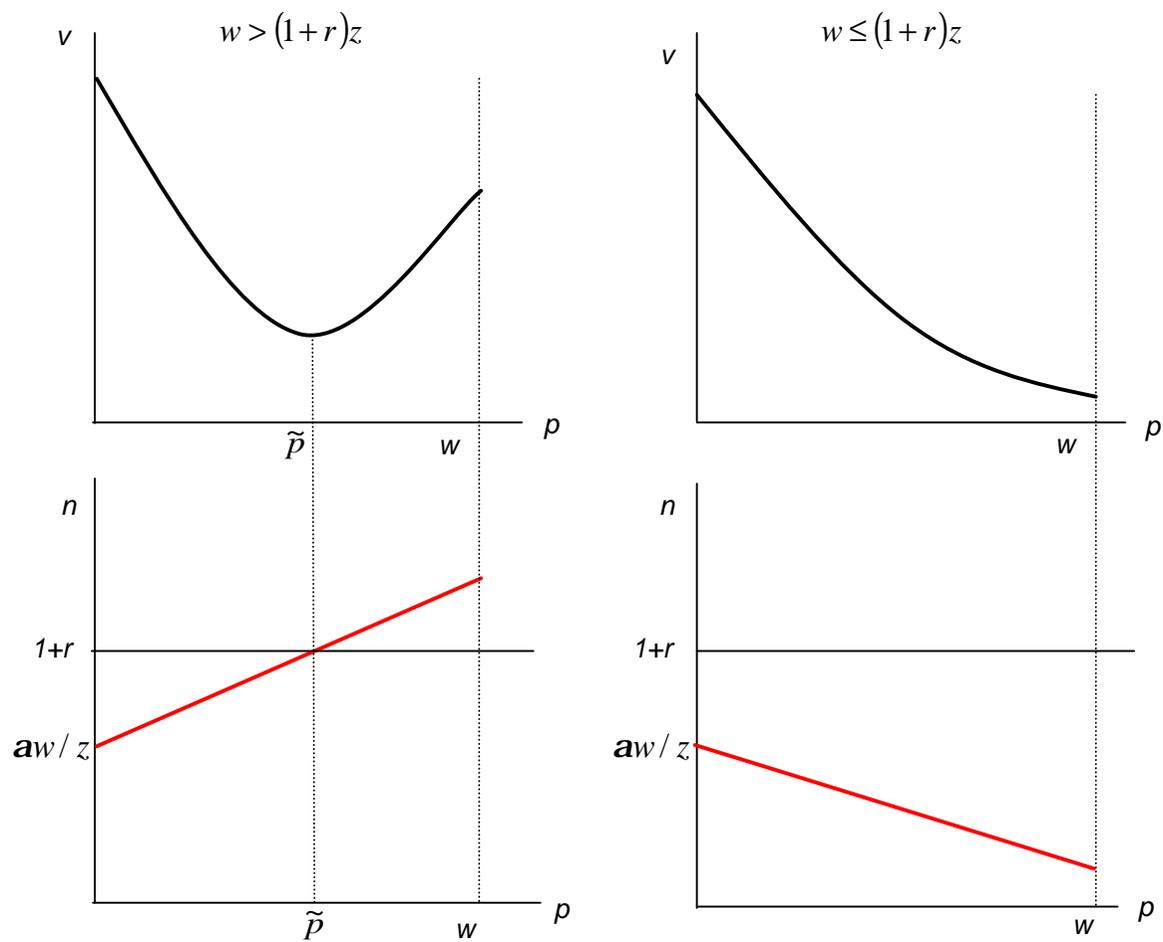


Note: The policy combination $s = p/(1+r)$ in an open economy with no altruism is assumed.

**Table 1: Assumptions on parameters and initial values**

|  | |
|---|---|
|  | 1/3 |
|  | 1/3 |
|  | 1/3 |
|  | 1/3 |
|  | 0.003 |
|  | 1.827 |
| *z/w* | 0.358 |
| *s* | 0 |
| *p/w* | 0.2 |
| *n (2n)* | 0.8  (1.6) |

Note:  The initial vaules of     and z/w are solved to make n equal to 0.8.

**Table 2:  Models of no altrusim:  steady state comparisons**

| Variables | $s$ | $p$ | $2n$ | $k$ | $w$ | $r$ | $c_1$ | $c_2$ | $u$ |
|---|---|---|---|---|---|---|---|---|---|
| Initial values | 0.000 | 0.058 | 1.600 | 0.083 | 0.291 | 1.755 | 0.083 | 0.229 | -1.395 |
| Open economy | | | | | | | | | |
| CC subsidy | 0.029 | 0.058 | 2.000 | - | 0.291 | 1.755 | 0.075 | 0.206 | -1.391 |
| SS cut | 0.000 | 0.029 | 1.737 | - | 0.291 | 1.755 | 0.090 | 0.249 | -1.313 |
| Mix | 0.029 | 0.029 | 2.152 | - | 0.291 | 1.755 | 0.081 | 0.222 | -1.317 |
| Closed economy | | | | | | | | | |
| CC subsidy | 0.029 | 0.058 | 1.978 | 0.035 | 0.217 | 3.143 | 0.048 | 0.199 | -1.554 |
| SS cut | 0.000 | 0.029 | 1.749 | 0.099 | 0.308 | 1.559 | 0.096 | 0.247 | -1.291 |
| Mix | 0.029 | 0.029 | 2.198 | 0.051 | 0.247 | 2.433 | 0.065 | 0.223 | -1.379 |

Note: "CC subsidy"introduces the childcare subsidy, which is equivalent to 50% of the SS tax (10% of initial w);
    "SS Cut" reduces the SS tax to its half size (10% of initial w); and "Mix" does both of them.
    In an open economy, per-capita capital stock is assumed to be fixed.

**Table 3:  Models of altrusim:  steady state comparisons**

| Variables | $s$ | $p$ | $2n$ | $k$ | $w$ | $r$ | $c_1$ | $c_2$ | $b$ | $U$ |
|---|---|---|---|---|---|---|---|---|---|---|
| Initial values | 0.000 | 0.068 | 1.600 | 0.136 | 0.342 | 1.263 | 0.116 | 0.262 | 0.021 | -1.918 |
| Open economy | | | | | | | | | | |
| CC subsidy | 0.034 | 0.068 | 1.600 | - | 0.342 | 1.263 | 0.407 | 0.922 | 0.420 | -0.620 |
| SS cut | 0.000 | 0.034 | 1.600 | - | 0.342 | 1.263 | 0.243 | 0.550 | 0.180 | -1.152 |
| Mix | 0.034 | 0.034 | 1.600 | - | 0.342 | 1.263 | 0.539 | 1.221 | 0.585 | -0.330 |
| Closed economy | | | | | | | | | | |
| CC subsidy | 0.034 | 0.068 | 1.996 | 0.078 | 0.285 | 1.821 | 0.085 | 0.240 | 0.017 | -2.006 |
| SS cut | 0.000 | 0.034 | 1.724 | 0.112 | 0.321 | 1.438 | 0.104 | 0.253 | 0.005 | -1.952 |
| Mix | 0.034 | 0.034 | 2.238 | 0.061 | 0.262 | 2.163 | 0.073 | 0.232 | 0.006 | -2.043 |

Note:  See the note of Table 2.

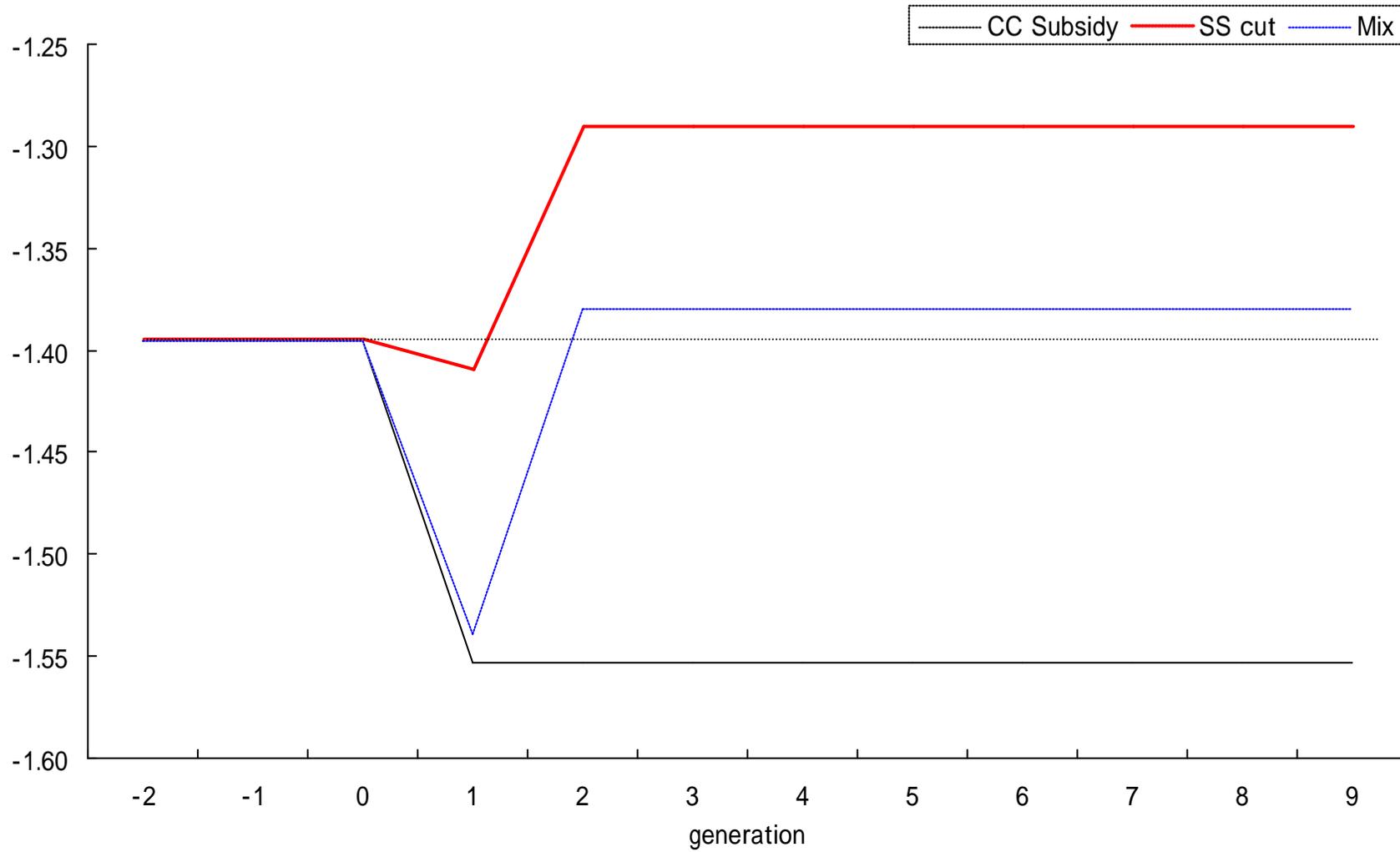Figure 2 Models of no altruism utility

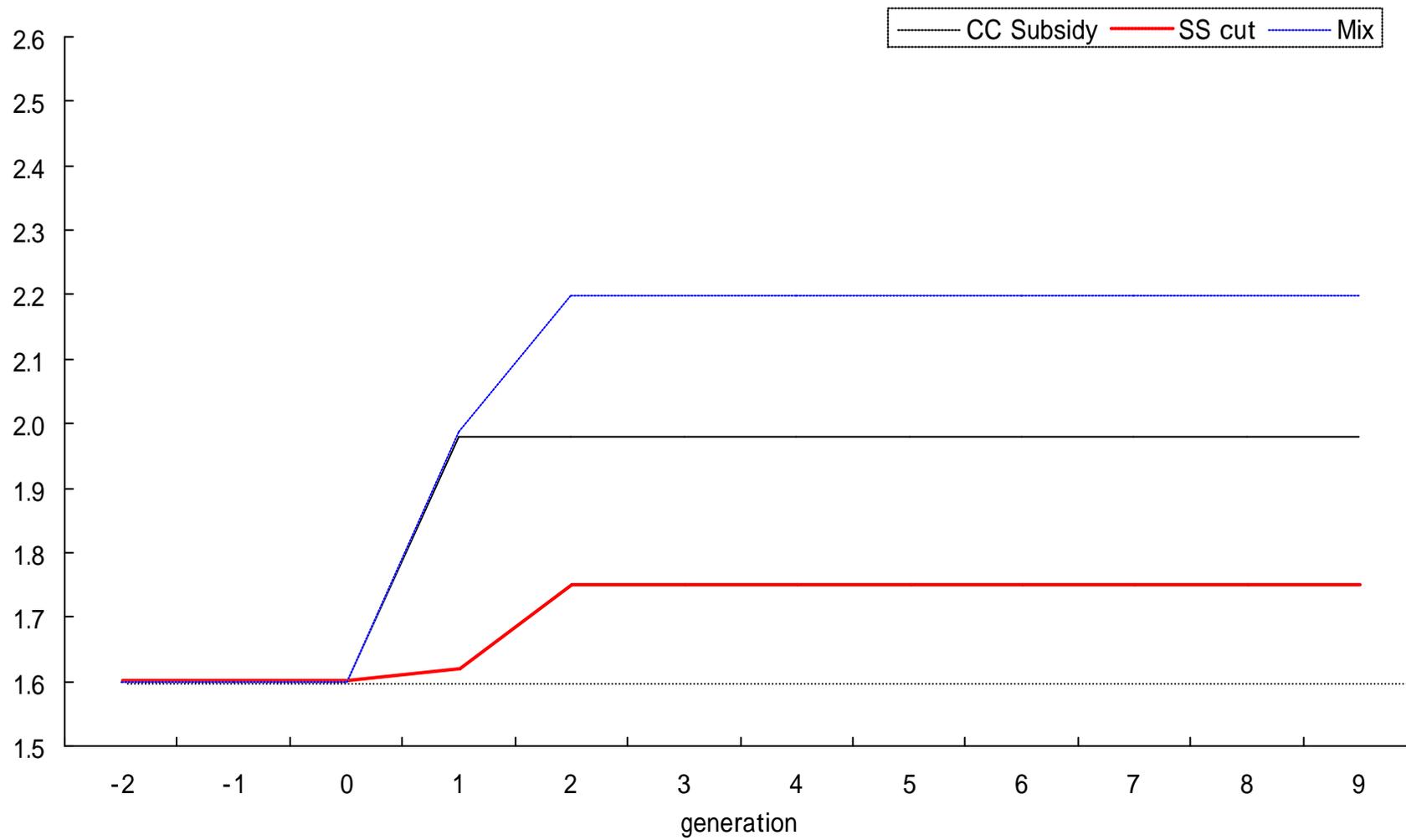**Figure 3 Models of no altruism fertility rate**

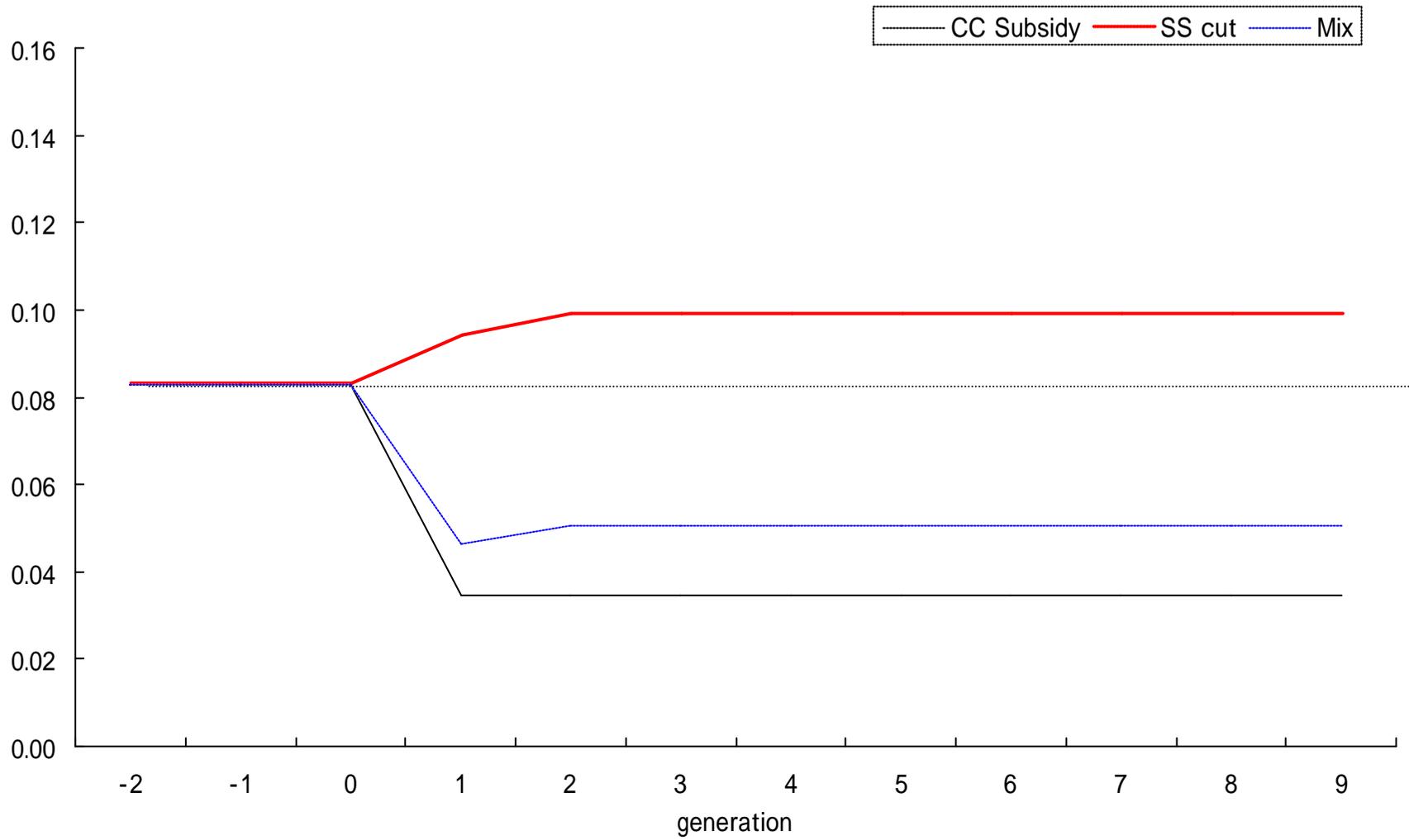# Figure 4  Models of no altruism per-capita capital stock

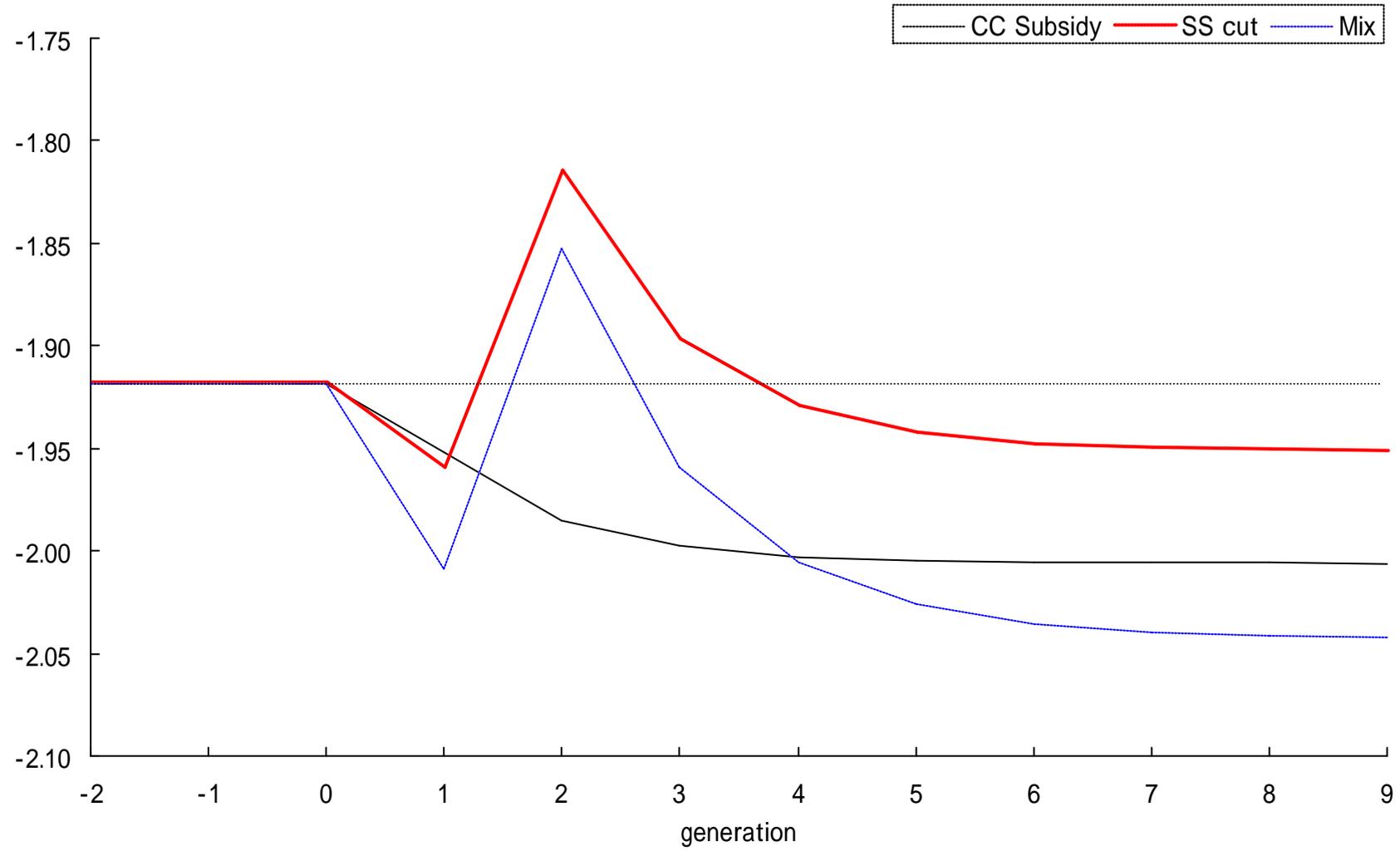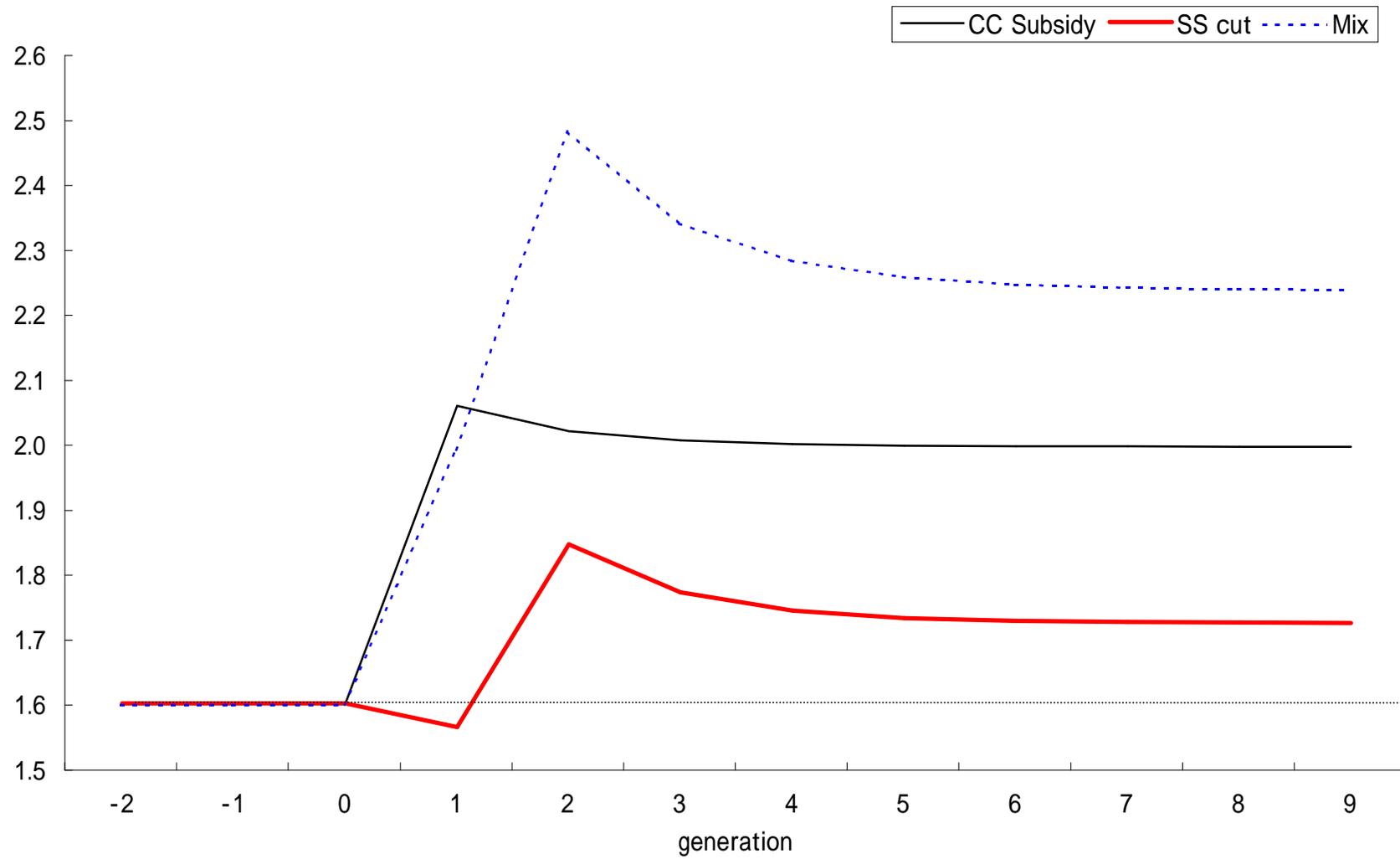**Figure 5 Models of altruism utility**

Figure 6 Models of altruism fertility rate

**Figure 7 Models of altruism per-capita capital stock and bequests**

Per-capita capital stock

Bequests

generation