Sustainable Development in an Aging Economy*

by Tetsuo Ono†and Yasuo Maeda‡

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Abstract

In this paper, we analyze the effects of population aging on economic growth and the environment in a two-period overlapping generations model of growth, aging, and the environment. We show that aging may be beneficial to economic growth and the environment under perfect annuitisation, while possibly harmful under imperfect annuitisation. We also discuss the implications of our results for environmental policy in an aging economy.

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†College of Economics, Osaka Prefecture University, 1-1, Gakuen-cho, Sakai, Osaka 599-8531, Japan. E-mail: tono@eco.osakafu-u.ac.jp

‡Faculty of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. E-mail: maeda@econ.osaka-u.ac.jp

1 Introduction

In this paper, we consider the effects of population aging on economic growth and the environment in an overlapping generations model. It is often suggested that aging is harmful for economic growth and the environment since the old people dissave and decrease investment in capital and the environment. On the other hand, it is also suggested that aging is beneficial for economic growth and the environment since the young people invest more in capital and the environment in preparation for their longer lifetimes. Focusing on an aging population is therefore important for analyzing whether we can obtain a sustainable level of development in the future.

There are many studies that analyze economic growth or the environment in overlapping generations models.[1] To the best of our knowledge, there is none which examines the link between sustainable development and population aging. In this paper, we aim to consider this link.[2]

To pursue our goals, we utilize the overlapping generations model of growth and the environment by Pecchenino and Pollard (1997), and extend it by incorporating environmental externalities along the line of John and Pecchenino (1994) and John et al. (1995). A negative aspect of the effect of aging on growth and the environment is captured by a decrease in unintentional bequests. A longer life-span yields smaller unintentional bequests and thus lowers the level of the young agents' wealth. This implies a negative income effect on investment in capital

[1] John and Pecchenino (1994), John et al. (1995), and Jouvet et al. (1997) analyze optimal allocation of capital and the environment. Howarth and Norgaard (1990), Howarth (1991) and Mourmouras (1993) discuss intergenerational equity. Howarth and Norgaard (1992) and Howarth (1998) analyze climate change. Mourmouras (1991), Mäler (1994), Guruswamy Babu et al. (1997), and Krautkraemer and Batina (1999) focus on intergenerational allocation of environmental resources. Ollikainen (1998) and Amacher et al. (1999) consider timber bequests problems.

[2] John et al. (1995) develop a model with environmental externalities and population growth. They show that a higher population growth rate lowers environmental quality per capita but could raise the aggregate quality of the environment. In contrast to them, we focus on aging while keeping population growth rate constant, and achieve different results and implications from theirs.

and the environment. A positive aspect of aging is captured by more investment in capital and the environment in preparation for longer life expectancy.

We first show that an annuity market plays an important role in determining the effects of aging on economic growth and the environment. In the case of a perfect annuity, agents annuitise all their wealth and bequeath nothing to their heirs. A perfect annuitisation eliminates the negative income effect of aging through unintentional bequests. Hence, aging is beneficial to economic growth and the environment under a certain condition. In the case of an imperfect annuity where there are some unintentional bequests, aging leads to a decrease in unintentional bequests, which implies a negative income effect on growth and the environment. Since there are two competing effects of aging under imperfect annuity, greater longevity leads to either higher or lower levels of capital and environmental quality.

Second, we show that higher annuitisation rate yields lower levels of capital and environmental quality. An increase in annuitisation rate leads to the fall of unintentional bequests, thereby implying a negative income effect on capital and the environment. The zero annuitisation rate is the best way to obtain higher capital and environmental quality given the longevity of agents. We finally consider the effects of aging in the case of zero annuitisation rate, and show that aging may be beneficial to growth and the environment under a certain condition.

The organization of this paper is as follows. In Section 2, we develop the model. In Section 3, we show the sufficient conditions for the existence, uniqueness, and stability of the equilibrium. In Section 4, we analyze the effects of aging on economic growth and the environment. Section 5 contains our conclusion.

2 The Model

Consider a two-period overlapping generations model in which a continuum of agents is born. The population of newly born agents is normalized to one.[3] Each agent lives for two periods, youth and old age, at maximum, but may die at the end of youth. Let $p \in (0, 1)$ be the probability that the agent lives for two periods. In other words, p indicates the longevity of agents. The lifetime utility of an agent born in period t can be written as $(\ln c_t^1 + \ln E_t) + p(\ln c_{t+1}^2 + \ln E_{t+1})$ where c_t^1 is the consumption by an agent of generation t during youth, c_{t+1}^2 is the consumption by an agent of generation t during old age, E_t is an index for the quality of the environment in period t, and E_{t+1} is an index for the quality of the environment in period $t + 1$.[4,5] Following the literature of population aging, an increase in p implies aging in the model (see, for example, Ehrlich and Lui (1991) and Pecchenino and Pollard (1997)).

There is one private good and one public good, namely, the environment. Each agent is endowed with one unit of labor in youth and nothing in old age. The agent inelastically supplies the labor to the firms (defined below) and obtains wages in youth.

The environment is assumed to be a public good that is reduced by aggregate consumption but that can be improved by maintenance investment. We express this mechanism as the formula:

$$E_{t+1} = E_t + b(E^n - E_t) - \beta(c_t^1 + pc_t^2) + \gamma m_t,$$

where $E^n > 0$ is the natural environmental quality in absence of human intervention, $b \in [0, 1]$ measures the speed of the autonomous change in environmental quality, $\beta > 0$ is a parameter of consumption externalities, $c_t^1 + pc_t^2$ is an aggregate consumption in period t, $\gamma > 0$ is a

[3] We make abstraction from a free rider problem within a generation in order to focus our attention on intergenerational externalities.

[4] The superscript "1" means young and "2" means old. The subscript "t" means period t.

[5] This specification of the utility function has the advantage of rendering our current analytic objects tractable. In Section 4, we will evaluate how the specification affects our results.

parameter that represents the technology for environmental maintenance, and m_t is an aggregate maintenance investment in period t.[6] Note that $\beta p c_t^2$ is an intergenerational externality from generation $t-1$ to generation t. This simple formulation is based on John and Pecchenino (1994) and John et al. (1995).[7]

Following Pecchenino and Pollard (1997), we consider a situation in which actuarially fair contracts are unavailable on the private market.[8] The government overcomes this market failure by establishing a market in actuarially fair annuity contracts. The government controls access to this market so that each agent can only buy an annuity up to $a \in (0, 1]$ percent of his savings. Since agents without bequests motives would prefer to annuitise all their wealth, they annuitise their wealth up to this limit.[9] Unannuitised wealth is transferred to their heirs as unintentional bequests if the agent dies. Under this framework, an agent born in period t divides wage w_t and unintentional bequests B_t into savings s_t, consumption c_t^1, and maintenance investment m_t when young and consumes $(R_{t+1} + \mu_{t+1})s_t$ when old, where R_{t+1} is the gross return rate on savings and μ_{t+1} is the excess returns on annuities.

An agent in generation t takes as given the wage, w_t, bequests, B_t, the return on savings, R_{t+1}, the excess return on annuities, μ_{t+1}, environmental quality in period t, E_t, the aggregate

[6] Since the number of young agents is one in each period, a per capita consumption of young agents in period t, c_t^1, is equal to their aggregate consumption. On the other hand, aggregate consumption of old agents in period t is pc_t^2 since the number of old agents is p.

[7] Our formulation of the environmental equation differs from John and Pecchenino (1994) and John et al. (1995) on the following point. They define E as an index that can take on positive and negative values. A value of zero is the quality of the environment in absence of human intervention. On the other hand, we assume that the index takes on only positive values and $E^n > 0$ is the natural environmental quality in absence of human intervention.

[8] For example, in Japan, there are few means to make reverse mortgage contracts. Wealth such as land or residence is often bequeathed to heirs unintentionally. This indicates the difficulty of actuarially fair contracts on the private market.

[9] In addition to this voluntary plan, Pecchenino and Pollard (1997) consider another plan: the mandatory plan. Under this plan, each agent must place a part of his fixed amount of income in an annuity. In this paper, we adopt the voluntary plan and make clear the difference between a perfect annuity ($a = 1$) case and an imperfect annuity ($a < 1$) case.

consumption of old agents in generation $t-1$, pc_t^2, and the maximum percentage of savings that can be annuitised, a. The problem of an agent in generation t is:[10]

$$\max_{\{c_t^1, m_t, s_t\}} \ln c_t^1 + \ln E_t + p(\ln c_{t+1}^2 + \ln E_{t+1})$$

subject to

$$s_t + c_t^1 + m_t = w_t + B_t, \tag{1}$$

$$c_{t+1}^2 = (R_{t+1} + \mu_{t+1})s_t, \tag{2}$$

$$E_{t+1} = (1-b)E_t + bE^n - \beta(c_t^1 + pc_t^2) + \gamma m_t, \tag{3}$$

$$m_t \geq 0, c_t^1 \geq 0, s_t \geq 0.$$

Throughout this paper, we focus on the case of positive maintenance, $m > 0$.[11] The first-order conditions of the utility maximization are (1) - (3) and

$$1/c_t^1 = (\beta + \gamma)p/E_{t+1}, \tag{4}$$

$$(R_{t+1} + \mu_{t+1})/c_{t+1}^2 = \gamma/E_{t+1}. \tag{5}$$

Taking c_t^2, E_t, w_t, R_{t+1}, and μ_{t+1} as given, these five equations characterize the allocation $\{c_t^1, c_{t+1}^2, E_{t+1}, m_t, s_t\}$ which is the outcome of the utility maximization. (4) states that an individual chooses consumption when young to equate the marginal rate of substitution between consumption in youth and environmental quality in old age to the marginal rate of transformation, $\beta+\gamma$. At the utility maximum, a decrease in utility due to falling consumption during youth

[10] We can interpret the problem alternatively to mean that a short-lived government chooses an environmental tax, m_t, to maximize utility of generation t.

[11] Suppose that an initial quality of the environment E_1 is sufficiently high and the initial capital stock k_1 is sufficiently low. Then, successive generations beginning with generation 1 will choose not to invest in the environment, since they do not find maintenance worthwhile. Hence, capital accumulation occurs, whereas the quality of the environment decreases continuously in the future because of both the accumulation of consumption externalities and the lack of maintenance investment. At some period t, agents born in this period will find it worthwhile to invest in the environment. The equilibrium path of generations with positive maintenance may display capital accumulation and improvement in the quality of the environment into the future. Thus, we focus on the equilibrium sequence with positive maintenance. See John and Pecchenino (1994) for a detailed analysis of the equilibrium path including zero maintenance.

is equal to an increase in utility due to the sum of an increase in maintenance effort, γ, and a decrease in a consumption externality, β. (5) states that an individual chooses savings to equate the marginal rate of substitution between consumption in old age and environmental quality in old age to the marginal rate of transformation, $\gamma/(R_{t+1} + \mu_{t+1})$. At the utility maximum, a decrease in utility due to falling consumption during old age, $(R_{t+1} + \mu_{t+1})$, is equal to an increase in utility due to an increase in maintenance effort, γ.

If an agent dies at the end of his youth, his unannuitised wealth is distributed to his heirs as the unintentional bequest, B_t :

$$B_t = (1-a)(1-p)R_t s_{t-1}, \tag{6}$$

where $1-a$ is the share of wealth not annuitised, $1-p$ is the percentage of agents who die at the end of their youth period, and $R_t s_{t-1}$ is the gross return from savings. On the other hand, annuitised wealth of those who die, $a(1-p)R_t s_{t-1}$, is distributed to the agents who are alive in old age. Thus, $p\mu_t s_{t-1} = a(1-p)R_t s_{t-1}$, or

$$\mu_t = a(1-p)R_t/p. \tag{7}$$

The structure of production is based on Pecchenino and Pollard (1997). The firms are perfectly competitive profit maximizers that produce output using the production function $Y_t = A_t {K_t}^{\alpha} {L_t}^{1-\alpha}$, $\alpha \in (0,1)$. K_t is the capital stock in period t, L_t is employment in period t, and $A_t > 0$ is a productivity scalar. We assume full depreciation of capital. The production function can be written in the intensive form $y_t = A_t {k_t}^{\alpha}$ where y_t is the output-labor ratio and k_t is the capital-labor ratio. Assume, because of external effects of aggregate capital on productivity as discussed in Romer (1986), that $A_t = A{K_t}^{\eta}$ where $A > 0$ and $\eta > 0$ are constant parameters. Therefore, the aggregate capital stock, K_t, enters the technology as a constant from the perspective of current producers.[12]

Taking as given wages (w_t), rental rates (ρ_t), and the productivity scalar (A_t), each firm chooses labor (L_t) and capital (K_t) to maximize its profit. Thus, the first-order conditions are

$$w_t = A_t(1-\alpha){k_t}^{\alpha} = A(1-\alpha){k_t}^{\alpha+\eta}, \tag{8}$$

$$\rho_t = \alpha A_t {k_t}^{\alpha-1} = \alpha A {k_t}^{\alpha+\eta-1}, \tag{9}$$

where (8) and (9) state that the firm hires labor and capital until the marginal products equal the factor prices. The second equalities in (8) and (9) hold since $L_t = 1$ and $A_t = AK_t^{\eta} = Ak_t^{\eta}$. Because of the assumptions of a constant returns production technology and inelastic labor supply, (8) and (9) also define factor markets clearing.

The economy starts at $t = 1$. In this period, there are both young agents in generation 1 and initial old agents in generation 0. Each agent in generation 0 is endowed with k_1 units of capital, earns the return $R_1 k_1$, and consumes it. We assume that the measure of the initial old agents is p. The utility of an agent in generation 0 is $\ln c_1^2 + \ln E_1$ where E_1 is given.

3 Laissez-faire Equilibrium

In this section, we characterize a laissez-faire equilibrium. A *laissez-faire equilibrium with positive maintenance* is a sequence $\{\bar{c}_t^1, \bar{c}_t^2, \overline{E}_t, \overline{m}_t, \bar{s}_t, \bar{k}_t, \overline{w}_t, \bar{\rho}_t, \overline{R}_t, \bar{\mu}_t, \overline{B}_t\}_{t=1}^{\infty}$ with the initial condition $\{k_1, E_1\}$ such that, in each period $t = 1, 2, ...$: (i) agents maximize utility; (ii) firms maximize profits; and (iii) markets clear.

[12] An alternative way of introducing an externality is to assume that $A_t = AK_{t-1}^{\eta}$, as in John and Pecchenino (1994): the capital stock of the previous period affects the current output. This form makes the analysis quite complicated, so we adopt the form, $A_t = AK_t^{\eta}$.

3.1 Characterization of the Equilibrium

The first-order conditions of the utility maximization are (1) - (5), and the first-order conditions of profit maximization are (8) and (9). A market clearing condition for capital is $s_t L_t = K_{t+1}$, which says that the total savings by young agents in generation t, $s_t L_t$, must equal their own addition to the future stock of capital, K_{t+1}. Since $L_t = 1$ for all t, this condition is rewritten as

$$s_t = k_{t+1}. \tag{10}$$

Since the market for capital is competitive, we have an arbitrage condition of the form:

$$R_{t+1} = \rho_{t+1}. \tag{11}$$

Given $\{k_1, E_1\}$, the sequence $\{\overline{c}_t^1, \overline{c}_t^2, \overline{E}_t, \overline{m}_t, \overline{s}_t, \overline{k}_t, \overline{w}_t, \overline{\rho}_t, \overline{R}_t, \overline{\mu}_t, \overline{B}_t\}_{t=1}^{\infty}$ is characterized by (1) - (11). Summarizing these equations, we have

$$\begin{aligned}
\overline{E}_{t+1} &= \gamma \overline{k}_{t+1}, && (12)\\
\overline{E}_{t+1} &= (1-b)\overline{E}_t + bE^n - \beta\left\{\frac{1}{(\beta+\gamma)p}\overline{E}_{t+1} + p\left(1+\frac{a(1-p)}{p}\right)\alpha A\overline{k}_t^{\alpha+\eta}\right\} && (13)\\
&\quad +\gamma\left\{(1-\alpha)A\overline{k}_t^{\alpha+\eta} + (1-a)(1-p)\alpha A\overline{k}_t^{\alpha+\eta} - \overline{k}_{t+1} - \frac{1}{(\beta+\gamma)p}\overline{E}_{t+1}\right\}
\end{aligned}$$

which characterize the equilibrium path $\{\overline{k}_t, \overline{E}_t\}$ given the initial conditions $\{k_1, E_1\}$. (12) is a rewrite of (5), and (13) is a rewrite of (3).

Substituting (12) into (13) leads to

$$\begin{aligned}
\gamma\overline{k}_{t+1} &= (1-b)\gamma\overline{k}_t + bE^n - \beta\left\{\frac{\gamma}{(\beta+\gamma)p}\overline{k}_{t+1} + p\left(1+\frac{a(1-p)}{p}\right)\alpha A\overline{k}_t^{\alpha+\eta}\right\} && (14)\\
&\quad +\gamma\left\{(1-\alpha)A\overline{k}_t^{\alpha+\eta} + (1-a)(1-p)\alpha A\overline{k}_t^{\alpha+\eta} - \overline{k}_{t+1} - \frac{\gamma}{(\beta+\gamma)p}\overline{k}_{t+1}\right\},
\end{aligned}$$

or

$$\overline{k}_{t+1} = G(k_t) \equiv \frac{(1-b)}{(2+1/p)}\overline{k}_t + \frac{bE^n}{\gamma(2+1/p)} + \frac{\xi}{\gamma(2+1/p)}\overline{k}_t^{\alpha+\eta} \tag{15}$$

where

$$\xi \equiv -\beta p(1+a(1-p)/p)\alpha A + \gamma A\{(1-\alpha)+(1-a)(1-p)\alpha\}.$$

We assume $\xi > 0$ in the following analysis.[13] (15) characterizes the laissez-faire equilibrium path with positive maintenance $\{\overline{k}_t\}_{t=1}^{\infty}$ with the initial condition $\overline{k}_1 > 0$.

A *steady state equilibrium* is a sequence $\{\overline{k}_t\}$ with $\overline{k}_t = \overline{k}_{t+1} = \bar{k}$. Since $\overline{E}_{t+1} = \gamma\overline{k}_{t+1}$ holds, the quality of the environment also remains constant along the steady state equilibrium path. A nontrivial steady state equilibrium is a sequence $\{\overline{k}_t\}$ with $\overline{k}_t = \overline{k} > 0$.

3.2 Existence, Uniqueness, and Stability of the Equilibrium

In this subsection, we analyze the existence, uniqueness, and stability of the steady state equilibrium. In preparation for the analysis, we define

$$E^* \equiv \frac{(\alpha+\eta-1)}{b\xi^{1/(\alpha+\eta-1)}}\left(\frac{\gamma(1+1/p+b)}{\alpha+\eta}\right)^{(\alpha+\eta)/(\alpha+\eta-1)}.$$

Then, we obtain the following results.

Proposition 1: (i) *Suppose that $\alpha+\eta<1$. There exists a unique and stable nontrivial steady state equilibrium.*

(ii) *Suppose that $\alpha+\eta=1$. There exists a unique and stable nontrivial steady state equilibrium if $\{\gamma(1-b)+\xi\}/\gamma(2+1/p) < 1$. There exists no nontrivial steady state equilibrium if $\{\gamma(1-b)+\xi\}/\gamma(2+1/p) \geq 1$.*

(iii) *Suppose that $\alpha+\eta>1$. There exist two nontrivial steady state equilibria if $E^n < E^*$; the one with lower k is stable and the other with higher k is unstable. There exists a unique*

[13] As we will see below, the equilibrium path may display monotone convergence to the steady state if $\xi > 0$. On the other hand, it may display cyclical behavior if $\xi \leq 0$. Since our analysis is restricted to the stable steady state, we assume $\xi > 0$.

and unstable nontrivial steady state equilibrium if $E^n = E^*$. There exists no nontrivial steady state equilibrium if $E^n > E^*$.

Proof: (i) When $\alpha+\eta<1$ holds, the function $G(\cdot)$ is strictly increasing and strictly concave with $G(0) = bE^n/\gamma(2+1/p) > 0$, $\lim_{k\to\infty} G(k) = \infty$, $\lim_{k\to 0} G'(k) = \infty$, and $\lim_{k\to\infty} G'(k) = (1-b)/(2+1/p) < 1$. In the $k_t - k_{t+1}$ space where the horizontal axis is k_t and the vertical axis is k_{t+1}, the graph of $G(\cdot)$ cuts the 45^o line once from above since $\lim_{k\to\infty} G'(k) < 1$ (see Figure 3.1).

(ii) When $\alpha+\eta = 1$, (15) is

$$\overline{k}_{t+1} = \frac{\gamma(1-b)+\xi}{\gamma(2+1/p)}\overline{k}_t + \frac{bE^n}{\gamma(2+1/p)}.$$

The graph of $G(\cdot)$ cuts the 45^o line once from above if the slope of $G(\cdot)$ is less than one, i.e., $\{\gamma(1-b)+\xi\}/\gamma(2+1/p) < 1$ (see Figure 3.2). On the other hand, the graph of $G(\cdot)$ never cuts the 45^o line if the slope of $G(\cdot)$ is equal to or greater than one, i.e., $\{\gamma(1-b)+\xi\}/\gamma(2+1/p) \geq 1$.

(iii) When $\alpha+\eta > 1$, the function $G(\cdot)$ is strictly increasing and strictly convex as shown in Figure 3.3. Thus the number of steady state equilibria is two, one, or none.

Let us define $\hat{k}$ satisfying $dG(\hat{k})/dk = 1$. Then, $\hat{k}$ is

$$\hat{k} = \{\gamma(1+1/p+b)/(\alpha+\eta)\xi\}^{1/(\alpha+\eta-1)}.$$

If $G(\hat{k}) < \hat{k}$, there exist two nontrivial steady state equilibria. If $G(\hat{k}) = \hat{k}$, there exists one nontrivial steady state equilibrium. If $G(\hat{k}) > \hat{k}$, there exists no nontrivial steady state equilibrium. The relation $G(\hat{k}) \lesseqqgtr \hat{k}$ is reduced to $bE^n \lesseqqgtr (\alpha+\eta-1)\xi\{\gamma(1+1/p+b)/(\alpha+\eta)\xi\}^{(\alpha+\eta)/(\alpha+\eta-1)}$. **Q.E.D.**

In case of $\alpha+\eta<1$, for any initial condition of $k_1 > 0$, the economy displays a monotone convergence to the unique nontrivial steady state equilibrium.

In case of $\alpha + \eta = 1$, for any initial condition of $k_1 > 0$, the economy displays a monotone convergence to the nontrivial unique steady state equilibrium if $\{\gamma(1-b)+\xi\}/\gamma(2+1/p) < 1$. This sufficient condition is rewritten as

$$\begin{aligned} 0 \quad > \quad & -\gamma b - \frac{\beta\gamma}{(\beta+\gamma)p} - \beta p \left(1 + \frac{a(1-p)}{p}\right) \alpha A \\ & + \gamma(1-\alpha)A + \gamma(1-a)(1-p)\alpha A - \gamma - \frac{\gamma^2}{(\beta+\gamma)p}. \end{aligned} \tag{16}$$

To interpret this condition, we rearrange (14) and set $\alpha + \eta = 1$:

$$\begin{aligned} \gamma\overline{k}_{t+1} - \gamma\overline{k}_t \quad > \quad & -\gamma b\overline{k}_t + bE^n - \beta\left\{\frac{\gamma}{(\beta+\gamma)p}\overline{k}_{t+1}\right\} - \beta p\left(1 + \frac{a(1-p)}{p}\right)\alpha A\overline{k}_t \\ & + \gamma(1-\alpha)A\overline{k}_t + \gamma(1-a)(1-p)\alpha A\overline{k}_t - \gamma\overline{k}_{t+1} - \frac{\gamma^2}{(\beta+\gamma)p}\overline{k}_{t+1}. \end{aligned} \tag{17}$$

We can interpret the sufficient condition by comparing (16) and (17). The first term in the right-hand side of (16), $-\gamma b$, is the negative effect of the autonomous decrease in environmental quality. The second term, $-\beta\gamma/(\beta+\gamma)p$, is the negative consumption externality of young agents. The third term, $-\beta p(1 + a(1-p)/p)\alpha A$, is the negative consumption externality of old agents. The fourth term, $\gamma(1-\alpha)A$, is the positive income effect, which implies that the increase in wages enhances maintenance investment. The fifth term, $\gamma(1-a)(1-p)\alpha A$, is the positive income effect of unintentional bequests, which implies that the increase in bequests enhances maintenance activity. The sixth term, $-\gamma$, is the substitution effect of savings for maintenance investment; the decrease in one unit of maintenance investment caused by the increase in one unit of savings implies the decrease of γ units of the environmental quality. The seventh term, $-\gamma^2/(\beta+\gamma)p$, is the negative substitution effect of consumption in youth for environmental maintenance. If the sum of the positive effects is smaller than the sum of the negative effects, then the economy displays the convergence to the unique and nontrivial steady state equilibrium.

In case of $\alpha + \eta > 1$, there are two steady state equilibria if $E^n < E^*$: the one with lower per capita capital, k^L, and the other with higher per capita capital $k^H (> k^L)$. When $k_1 \in (0, k^H)$, the economy displays the monotone convergence to the steady state with k^L. When $k_1 = k^H$, the equilibrium path continues to stay at the steady state with $k = k^H$. When $k_1 > k^H$, the economy displays the divergence: capital and environmental quality continues to increase over time. If $E^n > E^*$, capital and environmental quality grow infinitely.

4 The Effects of Aging on Economic Growth and the Environment

In this section, we undertake comparative static analysis at the stable steady state equilibrium and then show the effects of aging on growth and the environment.

Proposition 2: (i) *If wealth is totally annuitised* ($a = 1$), *greater longevity leads to higher steady state levels of capital and environmental quality.*

(ii) *If wealth is not totally annuitised* ($a < 1$), *greater longevity leads to either higher or lower steady state levels of capital and environmental quality.*

Proof: Differentiating (15) with respect to k and p and evaluating at the steady state, we obtain:

$$(1 - G'(\overline{k}))d\overline{k} = \left\{ \frac{\gamma(1-b)\overline{k} + bE^n + \xi\overline{k}^{\alpha+\eta}}{\gamma p^2(2+1/p)^2} + \frac{-\alpha A(\beta+\gamma)(1-a)\overline{k}^{\alpha+\eta}}{\gamma(2+1/p)} \right\} dp. \qquad (18)$$

The inequality $1 - G'(\overline{k}) > 0$ holds at the stable steady state (see Figures 3.1-3.3). When $a = 1$, (18) is rewritten as

$$\frac{\partial\overline{k}}{\partial p} = \frac{\{\gamma(1-b)\overline{k} + bE^n + \xi\overline{k}^{\alpha+\eta}\}}{\{1 - G'(\overline{k})\}\gamma p^2(2+1/p)^2} > 0.$$

Since E is positively related to k ($E = \gamma k$), we have $\partial\overline{E}/\partial p > 0$. When $a < 1$, there is a negative effect of aging as shown in the second term on the right-hand side of (18).

Therefore, a higher p leads to either higher or lower capital and environmental quality.

Q.E.D.

If wealth is totally annuitised, aging is beneficial to growth and the environment. Greater longevity heightens the incentive to invest in saving and the environment, thereby increasing capital accumulation and improving environmental quality; there is only this positive effect. If wealth is not totally annuitised, aging reduces unintentional bequests. This is a negative income effect that counteracts the positive effect. Therefore, aging may be either harmful or beneficial to growth and the environment.

We should notice that, in our model, there is no substitution effect of aging since we assume a log-linear utility function. If we adopt the general form of a utility function, saving depends on the return $R_{t+1} + \mu_{t+1}$ so that aging has a substitution effect. If wealth is totally annuitised, greater longevity leads to the lower return to saving since the return is decreasing in $p : R_{t+1} + \mu_{t+1} = R_{t+1} + (1-p)R_{t+1}/p = R_{t+1}/p$. If wealth is not totally annuitised, greater longevity leads to the lower return to the annuity: $\mu_{t+1} = a(1-p)R_{t+1}/p$. Therefore, a decrease in return affects the saving if we adopt the general form of the utility function. In particular, under the gross substitutability assumption, greater longevity leads to lower rate of return and thus a smaller amount of saving (see Azariadis (1993), Chapter 13); there appears to be an additional negative substitution effect that counteracts the positive effect. Therefore, even if the wealth is totally annuitised ($a = 1$), aging may be harmful to growth and the environment.[14]

The results in Proposition 2 imply that the annuitisation is a key factor in evaluating the effects of aging. Therefore, we next consider the effects of an increase in annuitisation rate on growth and the environment.

[14] We would like to thank an anoymous referee for pointing this out.

Proposition 3: *The higher degree of annuitisation leads to lower levels of capital and the environment.*

Proof: Differentiating (15) with respect to k and a and evaluating at the steady state, we obtain

$$\frac{\partial \overline{k}}{\partial a} = \frac{-(\beta+\gamma)(1-p)\alpha A\overline{k}^{\alpha+\eta}}{(1-G'(\overline{k}))\gamma(2+1/p)} < 0.$$

Since $\overline{E} = \gamma\overline{k}$ holds, we also have $\partial\overline{E}/\partial a < 0$. **Q.E.D.**

An increase in annuitisation rate leads to the fall of unintentional bequests, implying a negative income effect on the young. Agents respond to this effect by reducing saving, consumption in youth, and maintenance investment. A higher annuitisation rate leads to lower levels of capital and environmental quality. Therefore, setting $a = 0$ is the best way to obtain higher capital and environmental quality given the longevity p.

We finally consider the effects of aging when the wealth is not annuitised: $a = 0$.

Proposition 4: *Suppose that $a = 0$: wealth is not annuitised. Greater longevity leads to higher levels of capital and environmental quality if $\gamma - 2\alpha p(1+p)(\beta+\gamma) \geq 0$.*

Proof: When $a = 0$, (18) is rewritten as

$$\begin{aligned}&(1-G'(\overline{k}))d\overline{k} \\ = &\left\{\frac{\gamma(1-b)\overline{k} + bE^n + A[\gamma - 2\alpha p(1+p)(\beta+\gamma)]\overline{k}^{\alpha+\eta}}{\gamma p^2(2+1/p)^2}\right\} dp.\end{aligned}$$

Since the denominator in the right-hand side is positive and the first and the second terms of the numerator in the right-hand side are positive, $\partial\overline{k}/\partial p > 0$ if the third term of the numerator in the right-hand side, i.e., $\gamma - 2\alpha p(1+p)(\beta+\gamma) \geq 0$. **Q.E.D.**

When wealth is not annuitised, there are two competing effects of aging as shown in Proposition 2 (ii). First is a positive effect: greater longevity heightens the incentive to save and invest in the environment. Second is a negative effect: greater longevity decreases unintentional bequests. The sufficient condition implies that aging is beneficial to growth and the environment even if the wealth is not annuitised; the positive effect of aging overcomes the negative one if the original longevity (p) is low, the degree of the negative effect of consumption on the environment (β) is low, and the capital share (α) is low.[15]

Many industrialized countries are likely to violate the sufficient condition in Proposition 4 since the longevity in those countries is already high. The longer life expectancy toward the future would result in lower levels of capital and environmental quality. They must lower the degree of the negative effect of consumption (β) in order to satisfy the sufficient condition. Examples are the wide spread of a recycling scheme and the development of the new technology that controls engine emissions. These efforts lead the industrialized countries to achieve sustainable development in an aging economy.

5 Conclusion

In this paper we have developed an overlapping generations model with uncertain lifetimes and environmental externalities. In particular, we have considered a situation in which actuarially fair contracts are unavailable on the private market, and in which the government overcomes this market failure by establishing a market in actuarially fair annuity contracts. Under this framework, we have analyzed the effects of aging on growth and the environment.

Our main finding is that annuitisation plays an important role in determining the effects of aging on growth and the environment. In case of perfect annuitisation, the higher longevity

[15] The effect of γ is ambiguous since γ has two competing effects on the sufficient condition.

yields more accumulation of capital and the environment under a certain condition. On the other hand, in case of imperfect annuitisation, aging results in a decrease in unintentional bequests since agents who die at the end of youth transfer part of their wealth to their heirs as unintentional bequests. Aging has a negative income effect on economic growth and the environment which counteracts a positive effect of aging. In addition, when the annuitisation rate is zero, aging could be beneficial to growth and the environment if the original longevity is low and the degree of the negative effect of consumption is low; that is, setting annuitisation rate is zero is optimal in view of utility maximization under a certain condition. This result implies that many industrialized countries with high longevity must lower the degree of consumption externality and set annuitisation rate as zero in order to achieve sustainable development in an aging economy.

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Figure 3.1

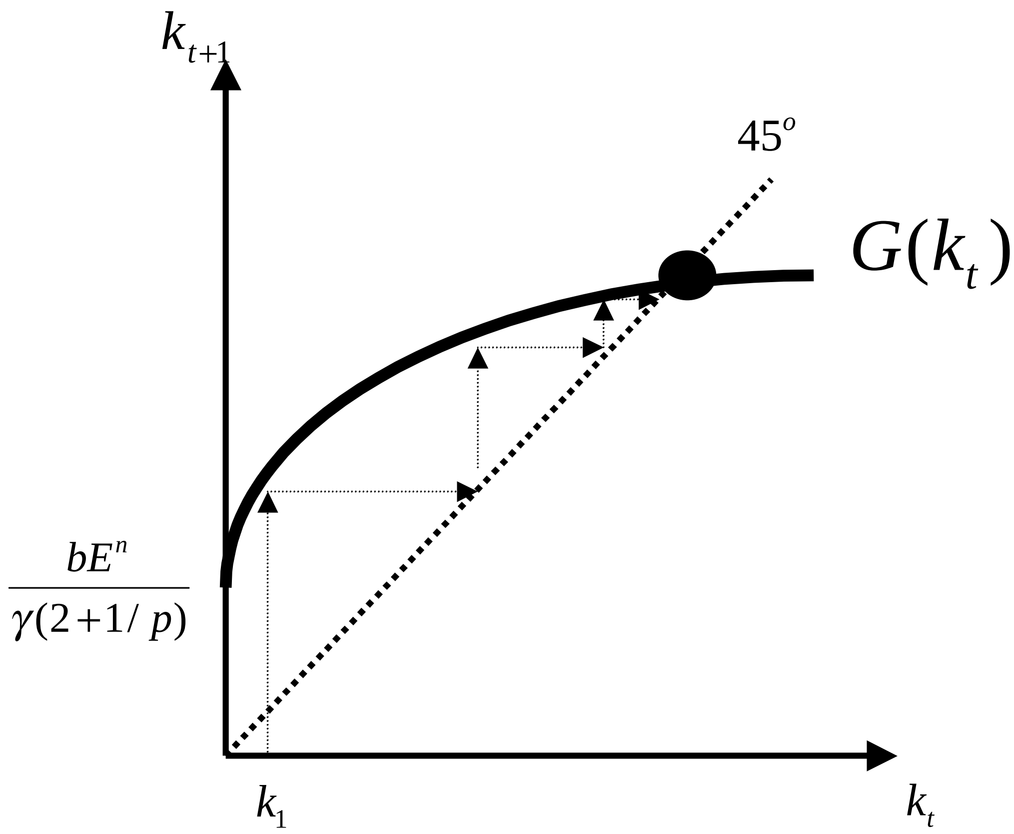

Figure 3.2

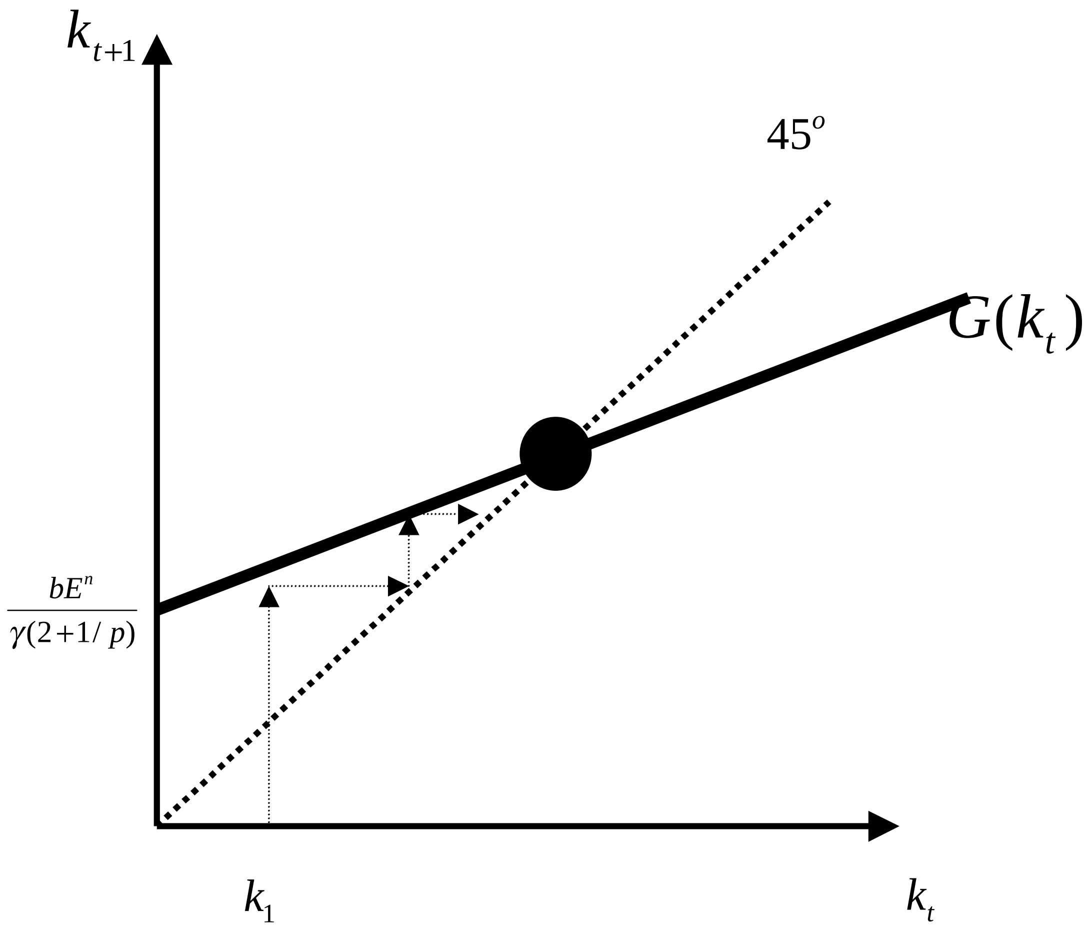

Figure 3.3