

# **Characterizations of Consequentialism and Non-consequentialism\***

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## Abstract

By allowing for the possibility that individuals recognize the *intrinsic value* of choice along with the *instrumental value* thereof, we suppose that individuals express extended preference orderings of the following type: Choosing an *alternative*  $x$  from an *opportunity set*  $A$  is better than choosing an alternative  $y$  from an opportunity set  $B$ . Within this framework, we identify a *consequentialist* and a *non-consequentialist*, who show contrasting attitudes toward alternatives *vis-à-vis* opportunities. This paper characterizes these attitudes in terms of some axioms, whereas the companion paper explores the implications of these concepts in the context of social choice theory *à la* Arrow.

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# 1 Introduction

It is undeniable that most, if not all, welfare economists are *welfaristic* in their conviction in the sense that they regard an economic policy or economic system to be satisfactory if and only if it is warranted to generate consequences which score high in the measuring rod of social welfare.<sup>1</sup> It is equally undeniable, however, that there do exist people who care not only about welfaristic features of the consequences, but also about non-welfaristic features of the consequences, or even non-consequential features of the decision-making procedure through which these consequences are brought about. Even those welfare economists with strong welfaristic conviction should be ready to take the judgements of people with non-welfaristic convictions into account in order not to be paternalistic in their welfare analysis. The purpose of this paper is to develop several analytical frameworks which enable us to examine the choice behaviour of non-welfaristic people. More specifically, we develop several frameworks which can accommodate situations where an individual expresses his preferences of the following type: it is better for me that an alternative  $x$  is realized from the opportunity set  $A$  than another alternative  $y$  being realized from the opportunity set  $B$ .<sup>2</sup> Note, in particular, that he is expressing his intrinsic valuation of the opportunity for choice if he prefers choosing  $x$  from the opportunity set  $A$ , where  $x \in A$ , rather than choosing  $x$  from the singleton set  $\{x\}$ . Using this analytical framework, we can put forward a concise definition of consequentialism and non-consequentialism, and we can also characterize these concepts in terms of a few simple axioms.

The structure of this paper is as follows. In Section 2, we present the basic notation and definitions. Section 3 discusses the basic axioms which are assumed throughout this paper and identifies a simple implication thereof. In Section 4, we define the concepts of an *extreme consequentialist* and a *strong consequentialist*, and characterize them axiomatically. We then turn in Section 5 to the concepts of an *extreme non-consequentialist* and a *strong non-consequentialist* and their axiomatic characterizations. Section 6 concludes this paper with some remarks.

## 2 Basic Notations and Definitions

Let  $X$ , where  $3 \leq |X|$ , be the set of all mutually exclusive and jointly exhaustive *social states*. The elements of  $X$  will be denoted by  $x, y, z, \dots$ .  $K$  denotes the set of

all finite non-empty subsets of  $X$ . The elements in  $K$  will be denoted by  $A, B, C, \dots$ , and they are called *opportunity sets*. Let  $X \times K$  be the Cartesian product of  $X$  and  $K$ . Elements of  $X \times K$  will be denoted by  $(x, A), (y, B), (z, C), \dots$ , and they are called *extended alternatives*. Let  $\Omega \subseteq X \times K$  be such that for all  $(x, A) \in \Omega, x \in A$  holds. For all  $(x, A) \in \Omega$ , the intended interpretation is the following: the alternative  $x$  is chosen from the opportunity set  $A$ .

Let  $\succeq$  be a reflexive, complete and transitive binary relation over  $\Omega$ . The asymmetric and symmetric parts of  $\succeq$  will be denoted by  $\succ$  and  $\sim$ , respectively. For any  $(x, A), (y, B) \in \Omega$ ,  $(x, A) \succeq (y, B)$  is interpreted as “choosing  $x$  from  $A$  is at least as good as choosing  $y$  from  $B$ .” Thus, in the extended framework, it is possible to give an expression to the intrinsic value of opportunity set in addition to the instrumental value thereof. Indeed, the decision-maker recognizes the intrinsic value of the opportunity of choice if there exists an extended alternative  $(x, A) \in \Omega$  such that  $(x, A) \succ (x, \{x\})$ .

### 3 Basic Axioms and Their Implication

In this section, we introduce two basic axioms for the ordering  $\succeq$ , and examine the implication of combining them together.

**Independence (IND):** For all  $(x, A), (y, B) \in \Omega$ , and all  $z \in X - A \cup B$ ,  $(x, A) \succeq (y, B) \Leftrightarrow (x, A \cup \{z\}) \succeq (y, B \cup \{z\})$ .

**Simple Indifference (SI):** For all  $x \in X$ , and all  $y, z \in X - \{x\}$ ,  $(x, \{x, y\}) \sim (x, \{x, z\})$ .

(IND) corresponds to the standard independence axiom used in the literature (see, for example, Pattanaik and Xu [8]). Its requirement is simple: for all opportunity sets  $A$  and  $B$ , if an alternative  $z$  is not in both  $A$  and  $B$ , then the extended preference ranking over  $(x, A \cup \{z\})$  and  $(y, B \cup \{z\})$  corresponds to that over  $(x, A)$  and  $(y, B)$ , independently of the nature of the added alternative  $z$ . (SI) requires that choosing  $x$  from “simple” cases, each involving two alternatives, is regarded as indifferent to each other.

The following result summarizes the implication of the above two axioms.

**Theorem 3.1.** If  $\succeq$  satisfies (IND) and (SI), then for all  $(x, A), (x, B) \in \Omega$ ,  $|A| = |B| \Rightarrow (x, A) \sim (x, B)$ .

**Proof.** Let  $\succeq$  satisfy (IND) and (SI). Let  $(x, A), (x, B) \in \Omega$  be such that  $|A| = |B|$ .

First, consider the case where  $A \cap B = \{x\}$ . Let  $A = \{x, a_1, \dots, a_m\}$  and  $B = \{x, b_1, \dots, b_m\}$ . Since opportunity sets are finite,  $m < +\infty$  holds. From (SI), clearly,  $(x, \{x, a_i\}) \sim (x, \{x, b_j\})$  for all  $i, j = 1, \dots, m$ . Two simple applications of (IND) lead us to have  $(x, \{x, a_1, a_2\}) \sim (x, \{x, a_1, b_1\})$  and  $(x, \{x, a_1, b_1\}) \sim (x, \{x, b_1, b_2\})$ . By the transitivity of  $\succeq$ ,  $(x, \{x, a_1, a_2\}) \sim (x, \{x, b_1, b_2\})$  follows easily. By using similar arguments as above, from (IND) and transitivity of  $\succeq$ , we can obtain  $(x, A) \sim (x, B)$ .

Next, consider the case that  $A \cap B = \{x\} \cup C$  where  $C$  is non-empty. From the above, noting that  $|A - C| = |B - C|$ , we must have  $(x, (A - C) \cup \{x\}) \sim (x, (B - C) \cup \{x\})$ . Since opportunity sets are finite,  $(x, A) \sim (x, B)$  can then be obtained from (IND). ■

## 4 Consequentialism

In this section, we define and characterize two versions of consequentialism: *extreme consequentialism* and *strong consequentialism*. First, we define the extreme consequentialism and strong consequentialism, respectively, as follows.

**Definition 4.1.**  $\succeq$  is said to be *extremely consequential* if, for all  $(x, A), (x, B) \in \Omega$ ,  $(x, A) \sim (x, B)$ .

**Definition 4.2.**  $\succeq$  is said to be *strongly consequential* if, for all  $(x, A), (y, B) \in \Omega$ ,  $(x, \{x\}) \sim (y, \{y\})$  implies  $[(x, A) \succeq (y, B) \Leftrightarrow |A| \geq |B|]$ , and  $(x, \{x\}) \succ (y, \{y\})$  implies  $(x, A) \succ (y, B)$ .

Thus, according to the extreme consequentialism, two choice situations  $(x, A)$  and  $(x, B)$  are judged exclusively on their consequences  $x$  and  $y$ , and the opportunity sets  $A$  and  $B$  from which these alternatives are chosen are irrelevant. On the other hand, the strong consequentialism stipulates that opportunity sets do not matter when the individual has a strict preference over  $(x, \{x\})$  and  $(y, \{y\})$ . Only when the individual is indifferent between  $(x, \{x\})$  and  $(y, \{y\})$  do opportunities matter.

To characterize the extreme consequentialism and strong consequentialism, the following axioms will prove useful.

**Local Indifference (LI):** For all  $x \in X$ , there exists  $(x, A) \in \Omega - \{(x, \{x\})\}$  such that  $(x, \{x\}) \sim (x, A)$ .

**Local Strict Monotonicity (LSM):** For all  $x \in X$ , there exists  $(x, A) \in \Omega - \{(x, \{x\})\}$  such that  $(x, A) \succ (x, \{x\})$ .

(LI) is a minimal and local requirement of extreme consequentialism: there exists an opportunity set  $A \in K$ , which is distinct from  $\{x\}$ , such that choosing an alternative  $x$  from  $A$  is regarded as indifferent to choosing  $x$  from the singleton set  $\{x\}$ . (LSM) requires that there exists an opportunity set  $A$  such that, choosing  $x$  from the opportunity set  $A$  is strictly better than choosing  $x$  from the singleton set  $\{x\}$ . In other words, the individual values opportunities per se at least in this very limited sense.

**Theorem 4.1.**  $\succeq$  satisfies (IND), (SI) and (LI) if and only if it is extremely consequential.

**Proof.** If  $\succeq$  is extremely consequential, then it clearly satisfies (IND), (SI) and (LI). Therefore, we have only to prove that, if  $\succeq$  satisfies (IND), (SI) and (LI), then, for all  $(x, A), (x, B) \in \Omega$ ,  $(x, A) \sim (x, B)$  holds.

Let  $\succeq$  satisfy (IND), (SI) and (LI). First, note that from Theorem 3.1 we have the following:

(4.1) For all  $(x, A), (x, B) \in \Omega$ ,  $|A| = |B| \Rightarrow (x, A) \sim (x, B)$ .

Hence, we have only to show that

(4.2) For all  $(x, A), (x, B) \in \Omega$ ,  $|A| > |B| \Rightarrow (x, A) \sim (x, B)$ .

To begin with, we show that

(4.3) For all  $x \in X$  and all  $y \in X - \{x\}$ ,  $(x, \{x, y\}) \sim (x, \{x\})$ .

Let  $x \in X$ . Suppose for some  $a \in X - \{x\}$ ,  $(x, \{x, a\}) \succ (x, \{x\})$ . Given (SI), by the transitivity of  $\succeq$ , we have

(4.4)  $(x, \{x, y\}) \succ (x, \{x\})$  for all  $y \in X - \{x\}$ .

Then, by (IND),

(4.5) For all  $z \in X - \{x, y\}$ ,  $(x, \{x, y, z\}) \succ (x, \{x, z\})$ .

Using the similar argument as for (4.4) and (4.5), we can show that

(4.6) For all  $(x, A) \in \Omega$ , all  $m = 4, \dots, |A| = m \Rightarrow (x, A) \succ (x, \{x\})$ .

(4.6), together with (4.4) and (4.5), is in contradiction with (LI). Therefore, we cannot have  $(x, \{x, a\}) \succ (x, \{x\})$  for some  $a \in X - \{x\}$ . Similarly, if  $(x, \{x\}) \succ (x, \{x, b\})$  for some  $b \in X$ , we can show that  $(x, \{x\}) \succ (x, A)$  holds for all  $(x, A) \in \Omega$  with  $A \neq \{x\}$ , another contradiction with (LI). Hence, by the completeness of  $\succeq$ , (4.3) holds.

From (4.3), noting the finiteness of opportunity sets and by the repeated use of (IND), (4.1) and the transitivity of  $\succeq$ , (4.2) obtains. ■

Before turning to the full characterization of the strong consequentialism, we note the following result which will prove useful in establishing the remainder of our results in this paper.

**Lemma 4.1.** If  $\succeq$  satisfies (IND), (SI) and (LSM), then, for all  $(x, A), (x, B) \in \Omega$ ,  $|A| \geq |B| \Leftrightarrow (x, A) \succeq (x, B)$ .

**Proof.** Let  $\succeq$  satisfy (IND), (SI) and (LSM). Note that, from Theorem 3.1, we have the following:

(4.7) For all  $(x, A), (x, B) \in \Omega$ ,  $|A| = |B| \Rightarrow (x, A) \sim (x, B)$ .

Therefore, we have only to show that

(4.8) For all  $(x, A), (x, B) \in \Omega$ ,  $|A| > |B| \Rightarrow (x, A) \succ (x, B)$ .

We first note that, by following a similar argument as in the proof of Theorem 4.1, the following can be established:

(4.9) For all  $x \in X$ , all  $y \in X$ ,  $x \neq y \Rightarrow (x, \{x, y\}) \succ (x, \{x\})$ .

Now, from (4.9), by the repeated use of (IND), we can derive the following:

(4.10) For all  $(x, A) \in \Omega - \{(x, X)\}$ , and  $y \in X - A$ ,  $(x, A \cup \{y\}) \succ (x, A)$ .

Then, given the finiteness of opportunity sets, (4.7) and (4.10), (4.8) follows from

the transitivity of  $\succeq$ . ■

To characterize the strong consequentialism, we need an additional condition which requires that, for all  $(x, A), (y, B) \in \Omega$  and all  $z \in X$ , if the individual ranks  $(x, A)$  higher than  $(y, B)$ , then adding  $z$  to  $B$  while maintaining  $y$  being chosen from  $B \cup \{z\}$  will not affect the individual's ranking:  $(x, A)$  is still ranked higher than  $(y, B \cup \{z\})$ . Formally:

**Robustness (ROB):** For all  $(x, A), (y, B) \in \Omega$  and all  $z \in X$ , if  $(x, \{x\}) \succ (y, \{y\})$  and  $(x, A) \succ (y, B)$ , then  $(x, A) \succ (y, B \cup \{z\})$ .

We are now ready to put forward the following full characterization of strong consequentialism.

**Theorem 4.2.**  $\succeq$  satisfies (IND), (SI), (LSM) and (ROB) if and only if it is strongly consequential.

**Proof.** If  $\succeq$  is strongly consequential, then it clearly satisfies (IND), (SI), (LSM) and (ROB). Therefore, we have only to prove that, if  $\succeq$  satisfies (IND), (SI), (LSM) and (ROB), then, for all  $(x, A), (y, B) \in \Omega$ ,  $(x, \{x\}) \sim (y, \{y\})$  implies  $[(x, A) \succeq (y, B) \Leftrightarrow |A| \geq |B|]$ , and  $(x, \{x\}) \succ (y, \{y\})$  implies  $(x, A) \succ (y, B)$ .

Let  $\succeq$  satisfy (IND), (SI), (LSM) and (ROB). Note that, from Lemma 4.1, we have the following:

(4.11) For all  $(x, A), (y, B) \in \Omega$ ,  $|A| \geq |B| \Leftrightarrow (x, A) \succeq (y, B)$ .

Now, for all  $x, y \in X$ , consider  $(x, \{x\})$  and  $(y, \{y\})$ . If  $(x, \{x\}) \sim (y, \{y\})$ , then, since  $X$  contains at least three alternatives, by (IND), for all  $z \in X - \{x, y\}$ , we must have  $(x, \{x, z\}) \sim (y, \{y, z\})$ . From (4.11) and by the transitivity of  $\succeq$ , we then have  $(x, \{x, y\}) \sim (y, \{x, y\})$ . Then, by (IND), we have  $(x, \{x, y, z\}) \sim (y, \{x, y, z\})$ . Since the opportunity sets are finite, by repeated application of (4.11), the transitivity of  $\succeq$  and (IND), we then obtain

(4.12) For all  $(x, A), (y, B) \in \Omega$ , if  $(x, \{x\}) \sim (y, \{y\})$ , then  $|A| \geq |B| \Leftrightarrow (x, A) \succeq (y, B)$ .

If, on the other hand,  $(x, \{x\}) \succ (y, \{y\})$ , then, for all  $z \in X$ , by (ROB),  $(x, \{x\}) \succ (y, \{y, z\})$ . Since all opportunity sets are finite, by repeated use of (ROB),

we then have  $(x, \{x\}) \succ (y, A)$  for all  $(y, A) \in \Omega$ . Therefore, from (4.11) and by the transitivity of  $\succeq$ , we obtain

(4.13) For all  $(x, A), (y, B) \in \Omega$ , if  $(x, \{x\}) \succ (y, \{y\})$ , then  $(x, A) \succ (y, B)$ .

(4.13), together with (4.11) and (4.12), completes the proof. ■

In Suzumura and Xu [18], we have exemplified that the axioms (IND), (SI) and (LI) are independent, and also that the axioms (IND), (SI), (LSM) and (ROB) are independent. Thus, our characterization theorems, viz. Theorem 4.1 for extreme consequentialism and Theorem 4.2 for strong consequentialism, do not contain any redundancy.

## 5 Non-consequentialism

In this section, we define and characterize two versions of non-consequentialism: *extreme non-consequentialism* and *strong non-consequentialism*. Their definitions are given below.

**Definition 5.1.**  $\succeq$  is said to be extremely non-consequential if, for all  $(x, A), (y, B) \in \Omega$ ,  $(x, A) \succeq (y, B) \Leftrightarrow |A| \geq |B|$ .

**Definition 5.2.**  $\succeq$  is said to be strongly non-consequential if, for all  $(x, A), (y, B) \in \Omega$ ,  $|A| > |B| \Rightarrow (x, A) \succ (y, B)$ , and  $|A| = |B| \Rightarrow [(x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow (x, A) \succeq (y, B)]$ .

According to the extreme non-consequentialism, consequences do not matter at all, and what is valued is the richness of opportunity involved in the choice situation. Thus, two extended alternatives,  $(x, A)$  and  $(y, B)$ , are ranked exclusively according to the cardinality of  $A$  and  $B$  and the consequences do not have any influence at all. In its complete neglect of consequences, extreme non-consequentialism is indeed extreme, but it captures the sense in which people may claim: “Give me liberty, or give me death.” On the other hand, strong non-consequentialism pays attention to consequences if and only if two opportunity sets contain the same number of alternatives.

To give characterizations of the extreme non-consequentialism and strong conse-

quentialism, the following axioms will be used.

**Indifference of No-choice Situations (INS):** For all  $x, y \in X$ ,  $(x, \{x\}) \sim (y, \{y\})$ ;

**Simple Preference for Opportunities (SPO):** For all distinct  $x, y \in X$ ,  $(x, \{x, y\}) \succ (y, \{y\})$ .

(INS) is simple and easy to interpret. It says that in facing two choice situations in which each alternative is restricted to choices from singleton sets, the individual is indifferent between them. (INS) thus conveys the message that in these simple cases, the individual feels that there is no real freedom of choice in each choice situation and is ready to express his indifference among these cases *regardless of the nature of the alternatives*. In a sense, it is the lack of freedom of choice that “forces” the individual to be indifferent among these situations. This idea is similar to an axiom proposed by Pattanaik and Xu [8] for ranking opportunity sets in terms of freedom of choice, which stipulates that all singleton sets offer the same amount of freedom of choice. On the other hand, (SPO) stipulates that it is always better for the individual to choose an alternative from the set containing two elements (one of which is the chosen alternative) than to choose an alternative from the singleton set. (SPO) thus displays the individual’s desire for having some genuine opportunities of choice.

**Theorem 5.1.**  $\succeq$  satisfies (IND), (SI), (LSM) and (INS) if and only if it is extremely non-consequential.

**Proof.** If  $\succeq$  is extremely non-consequential, then it clearly satisfies (IND), (SI), (LSM) and (INS). Therefore, we have only to prove that, if  $\succeq$  satisfies (IND), (SI), (LSM) and (INS), then, for all  $(x, A), (y, B) \in \Omega$ ,  $|A| \geq |B| \Leftrightarrow (x, A) \succeq (y, B)$ .

To begin with, from Lemma 4.1, we have the following

(5.1) For all  $(x, A), (y, B) \in \Omega$ ,  $(x, A) \succeq (y, B) \Leftrightarrow |A| \geq |B|$ .

Now, for all  $x, y \in X$ , by (INS),  $(x, \{x\}) \sim (y, \{y\})$ . For all  $z \in X - \{x, y\}$ , by (IND),  $(x, \{x, z\}) \sim (y, \{y, z\})$ . It follows from (5.1) that  $(x, \{x, y\}) \sim (y, \{x, y\})$ , where use is made of the transitivity of  $\succeq$ . By the repeated use of (5.1), (IND) and the transitivity of  $\succeq$  and noting that all opportunity sets are finite, we can show that

(5.2) For all  $(x, A), (y, B) \in \Omega$ ,  $(x, A) \succeq (y, B) \Leftrightarrow |A| \geq |B|$ .

(5.2) completes the proof of Theorem 5.1. ■

Before presenting the characterization theorem for the strong non-consequentialism, we note the following result which will prove useful in establishing the characterization theorem.

**Lemma 5.1.** If  $\succeq$  satisfies (IND), (SI) and (SPO), then it also satisfies (LSM).

**Proof.** Let  $\succeq$  satisfy (IND), (SI) and (SPO). Let  $x \in X$ . For all  $y \in X - \{x\}$ , by (SPO),  $(x, \{x, y\}) \succ (y, \{y\})$ . Then, (IND) implies  $(x, \{x, y, z\}) \succ (y, \{y, z\})$  for all  $z \in X - \{x, y\}$ . By (SI),  $(y, \{y, z\}) \sim (y, \{x, y\})$ . The transitivity of  $\succeq$  now implies  $(x, \{x, y, z\}) \succ (y, \{x, y\})$ . By (SPO),  $(y, \{x, y\}) \succ (x, \{x\})$ . Then,  $(x, \{x, y, z\}) \succ (x, \{x\})$  follows from the transitivity of  $\succeq$ . That is, (LSM) holds. ■

**Theorem 5.2.**  $\succeq$  satisfies (IND), (SI) and (SPO) if and only if it is strongly non-consequential.

**Proof.** If  $\succeq$  is strongly non-consequential, then it satisfies (IND), (SI) and (SPO). Therefore, we have only to prove that, if  $\succeq$  satisfies (IND), (SI) and (SPO), then, for all  $(x, A), (y, B) \in \Omega$ ,  $|A| > |B| \Rightarrow (x, A) \succ (y, B)$  and  $|A| = |B| \Rightarrow [(x, \{x\}) \succeq (y, \{y\}) \Leftrightarrow (x, A) \succeq (y, B)]$ .

From Lemma 5.1 and Lemma 4.1, we have (5.1). For all distinct  $x, y \in X$ , by (SPO),  $(x, \{x, y\}) \succ (y, \{y\})$ . Then, from (5.1) and the transitivity of  $\succeq$ , for all  $z \in X - \{x\}$ ,  $(x, \{x, z\}) \succ (y, \{y\})$ . By (IND), from  $(x, \{x, y\}) \succ (y, \{y\})$ , for all  $z \in X - \{x, y\}$ ,  $(x, \{x, y, z\}) \succ (y, \{y, z\})$ . Noting (5.1) and the transitivity of  $\succeq$ , we then have:

(5.3) For all  $(x, A), (y, B) \in \Omega$ , if  $|A| = |B| + 1$  and  $|B| \leq 2$ , then  $(x, A) \succ (y, B)$ .

From (5.3), by the repeated use of (IND), (5.1) and the transitivity of  $\succeq$ , coupled with the finiteness of opportunity sets, we can obtain the following:

(5.4) For all  $(x, A), (y, B) \in \Omega$ , if  $|A| = |B| + 1$ , then  $(x, A) \succ (y, B)$ .

From (5.4), by the transitivity of  $\succeq$  and (5.1), we have

(5.5) For all  $(x, A), (y, B) \in \Omega$ , if  $|A| > |B|$ , then  $(x, A) \succ (y, B)$ .

Consider now  $(x, \{x\})$  and  $(y, \{y\})$ . If  $(x, \{x\}) \sim (y, \{y\})$ , a similar argument as in the proof of Theorem 5.1 enables us to assert that

- (5.6) For all  $(x, A), (y, B) \in \Omega$ , if  $(x, \{x\}) \sim (y, \{y\})$  and  $|A| = |B|$ , then  $(x, A) \sim (y, B)$ .

If, on the other hand,  $(x, \{x\}) \succ (y, \{y\})$ , a similar argument as in the proof of Theorem 4.2 leads us to assert that

- (5.7) For all  $(x, A), (y, B) \in \Omega$ , if  $(x, \{x\}) \succ (y, \{y\})$  and  $|A| = |B|$ , then  $(x, A) \succ (y, B)$ .

(5.7), together with (5.5) and (5.6), completes the proof. ■

In Suzumura and Xu [18], we have exemplified that the axioms (IND), (SI), (LSM) and (INS) are independent, and also that the axioms (IND), (SI) and (SPO) are independent. Thus, our characterization theorems, viz., Theorem 5.1 for extreme non-consequentialism and Theorem 5.2 for strong non-consequentialism, do not contain any redundancy.

## 6 Concluding Remarks

Using the analytical framework of extended preference orderings, where individuals express preferences over the pairs of alternatives and opportunity sets from which alternatives are chosen, we developed in this paper a simple analysis of consequentialism and non-consequentialism. We have identified two types of consequentialism, extreme consequentialism and strong consequentialism, and two types of non-consequentialism, extreme non-consequentialism and strong non-consequentialism. Although these identified types are rather extreme, they are meant to illustrate the kind of analysis in which we may talk about similarity and dissimilarity of individual attitude toward alternatives and opportunities. Such an analysis is presented in our companion paper, Suzumura and Xu [19], where we examined how and to what extent Arrow's general impossibility theorem hinges on his basic assumption of welfarist-consequentialism, on the one hand, and whether or not Arrow's suggestion that "the possibility of social welfare judgements rests upon a similarity of attitudes toward social alternative (Arrow [1, p.69])" can be sustained within the wider con-

ceptual framework than Arrow's own.

It should be noted that, although individual attitudes toward alternatives and opportunities reflected in the extreme and strong consequentialism, and the extreme and strong non-consequentialism are quite diverse, they all satisfy (IND) and (SI). More remarkably, the strong consequentialism, the extreme non-consequentialism and the strong non-consequentialism have more in common: they all satisfy not only (IND) and (SI), but also (LSM). With our axiomatic characterizations of the extreme and strong consequentialism, and the extreme and strong non-consequentialism, we hope that the contrast and similarity of these concepts have received some clarifications. These characterization theorems are summarized in Diagram 1, where  $(A) \oplus (B)$  indicates the logical combination of the two axioms  $A$  and  $B$ .

It is interesting to note that, despite their diverse attitudes toward opportunities and alternatives, both the extreme and strong consequentialism, and the extreme and strong non-consequentialism all satisfy the following property:

**Monotonicity (MON):** For all  $(x, A), (x, B) \in \Omega$ ,  $B \subseteq A \Rightarrow (x, A) \succeq (x, B)$ .

According to (MON), the individual is not averse to richer opportunities, that is, choosing an alternative  $x$  from the opportunity set  $A$  is at least as good as choosing the same  $x$  from the opportunity set  $B$  which is a subset of  $A$ .<sup>3</sup> Clearly, (MON) and (LI), and (MON) and (LSM) are independent.

It is hoped that our attempt in this paper will be suggestive enough to motivate further exploration of the analytical framework of extended preference orderings. To orient some possible directions to be explored, two final remarks may be in order.

First, the two basic axioms, viz., (IND) and (SI), which are commonly invoked in our axiomatic characterizations of consequentialism and non-consequentialism, are not in fact beyond any dispute. Indeed, it is fairly common that an added alternative may have “epistemic value” in that it tells us something important about the nature of the choice situation. Sen [15, p. 753] provides us with a telling example: “[I]f invited to tea ( $t$ ) by an acquaintance you might accept the invitation rather than going home ( $O$ ), that is, pick  $t$  from the choice over  $\{t, O\}$ , and yet turn the invitation down if the acquaintance, whom you do not know very well, offers you a *wider* menu of either having tea with him, or some heroin and cocaine ( $h$ ), that is, you may pick  $O$ , rejecting  $t$ , from the larger set  $\{t, h, O\}$ . The expansion of the menu offered by this acquaintance may tell you something about the kind of person he is, and this could

affect your decision even to have tea with him.” This means a clear violation of (IND) when  $A = B$ . (SI) is also not immune to possible exceptions. Take, for example, the case where  $X$  denotes the set of alternative measures of transportation for moving from city  $A$  to city  $B$ . If  $x$  and  $y$  stand, respectively, for exactly the same car except for the serial number, you may feel indifferent between choosing  $x$  and  $y$ , so that you may feel fine even if you must choose  $x$  rather than  $y$  from the opportunity set  $\{x, y\}$ . However, when the choice is between  $x$  and  $z$  where  $z$  is a comfortable train connecting  $A$  and  $B$ , you may feel very unhappy if you are forced to choose  $x$  in the presence of  $z$ . Thus, you may express a preference for  $(x, \{x, y\})$  against  $(x, \{x, z\})$ , which is a clear violation of (SI).

Second, our axiomatizations of consequentialism and non-consequentialism were concerned with rather extreme cases where unequivocal priority is given to consequences (resp. opportunities) not only in the case of extreme consequentialism (resp. extreme non-consequentialism) but also in the case of strong consequentialism (resp. strong non-consequentialism). It goes without saying that further research should be pursued so that active interactions between consequential considerations and procedural considerations are allowed to play essential role.

Diagram 1: Characterization Theorems

$$(IND) \oplus (SI) \left\{ \begin{array}{l} \oplus (LI) = \text{extreme consequentialism} \\ \\ \oplus (LSM) \left\{ \begin{array}{l} \oplus (ROB) = \text{strong consequentialism} \\ \oplus (INS) = \text{extreme non-consequentialism} \\ \oplus (SPO) = \text{strong non-consequentialism} \end{array} \right. \end{array} \right.$$

IND: Independence

SI: Simple Indifference

LI: Local Indifference

LSM: Local Strict Monotonicity

ROB: Robustness

INS: Indifference of No-choice Situations

SPO: Simple Preference for Opportunities

**Note:**  $(A) \oplus (B)$  indicates the logical combination of the two axioms  $A$  and  $B$ .

## Endnotes

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<sup>1</sup> For a general observation on the concept and content of welfarism, see, among others, Sen [10,11]

<sup>2</sup> Much attention has been focussed on the opportunity set evaluation, beginning with Sen [12,13]. See, among many others, Bossert, Pattanaik and Xu [2], Pattanaik and Xu [8,9], and Sen [14,15]. To the best of our knowledge, Gravel [4,5] remains the unique precursor in analysing the extended preference ordering on  $X \times K$ , where  $X$  is the set of social states and  $K$  is the set of opportunity sets. However, his approach is quite different from ours in that he assumes that an individual has two preference orderings, one for ordering alternatives in  $X$  and another for ordering the choice situations in  $X \times K$ . His analysis is focused on the possibility of conflict between these two orderings, and it has nothing to do with the consequentialism and non-consequentialism. Capitalizing on Arrow's [1, pp. 89-91] insightful observation, Pattanaik and Suzumura [6,7] and Suzumura [16,17] developed a conceptual framework for the analysis of non-consequential features of the decision-making procedures through which consequences are brought about.

<sup>3</sup> Note that we are neglecting decision-making cost and other factors which may make a larger opportunity set a liability rather than a credit. In this context, see Dworkin [3].

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