# On the Persistence of Low Birthrate in Japan

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**Abstract**

We present two approaches to explain persistent low fertility in Japan. First we show that quality of consumption is an important determinant of fertility and labor supply. Taking this observation into account and using a general equilibrium model with vertical quality differentiation and heterogeneous labor, we show how low fertility may persist. This occurs because product quality and skilled labor supply adjust, never realizing the change in labor productivity necessary to reverse declining fertility. The second approach shows how network effect of child rearing implies that cost of having children increases as number of children decrease and thus cost of children based on past data always underestimate the current cost.

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# 1 Introduction

We present two new approaches to understanding persistence of low fertility. In the first part, we present a model of consumer choice where children and consumption experience require both goods and time. We demonstrate how change in marginal utility of consumption and change in wages generate different relationship between fertility and labor participation, i.e., possible source of the difference between cross section and time series. Then we embed a simplified version of this consumer into a general equilibrium model with heterogenous labor and vertically differentiated products. Through comparative statics, we analyze the cause and implications of low birthrate in the long run. We show that the feedback mechanism of the economy may not reverse the declining birthrate, contradicting an implication of the Easterlin Hypothesis cohort effect. This is because the labor market structure and product market adjusts to change in birthrate and thus the cohort effect never materializes.

The approach is in the spirit to papers in growth and trade that take into account the reaction of the economy in the long run (Acemoglu (1998), Flam and Helpman (1987), Thoenig and Verdier (2003)). Acemoglu (1998) showed that while in the short run, labor input is reduced in response to scarcity of skilled labor and high wages, skilled labor supply increase in response triggers technological change that makes skilled labor even more productive, raising skilled labor wage in the long run. Our analysis suggests that a similar long term adjustment of the economy will prevent a natural feedback mechanism from working. That is, smaller population will increase marginal product of labor in the short run but consumption pattern will change in the long run reducing such an advantage.

In the second part, we construct a simple model to analyze the network effect of raising children. Networks among parents seem to play an important role in determining the cost of child rearing for individual families. For instance, the existence of such network would facilitate helping each other and exchanging information. We adopt a static partial equilibrium model, and show that the network effect may magnify the decline of "birthrate", i.e., the number of children in each household. Thus once a society experiences low fertility, cost of having children increases making having children even less attractive. This also means that using past data to calculate cost of child rearing will under estimate. Public policy based on such calculation will not be effective.

There are many theoretical studies on fertility and population. Among them, our approach is most closely related to Becker and Barro (1988). They develop a model of fertility choices, in which the opportunity costs of child-rearing plays a crucial role in determining the optimal choice of fertility. We build on their work, but we incorporate the network effect of raising children into the model. As a result, we can analyze the fertility choice of each household in the presence of the network-effect of child-rearing.

# 2 Re-examination of female labor participation - birthrate relationship

Time series among many OECD countries show negative relationship between female labor participation and TFR (Figure 1) , while cross country in 2005 (average of years 1985-1996 as well as year 2000, Sleebos (2003), d'Addio and d'Ercole (2005), Da Rocha and Fuster (2006)) show a positive relationship. In Japan, although time series relationship has been negative for 1980 - 2000 (Figure 1) , cross section among prefectures show positive relationship in 1987 and 2002 (Figure 2). Obviously conditions that differ across regions in Japan are different from difference between two points in time. We also note that countries with high per capita GDP have low birthrates (Figure 3), suggesting low fertility may be correlated with high consumption. In this section we introduce a consumer optimization model to capture differences in income difference and quality of consumption.

We assume that a utility of a household depends on number of children, $n$, consumption of a good $x$. Both child rearing and consumption of a good requires time. Number of children is determined by amount of good $x_c$, and time devoted, $\ell_c$,

$$n = f(x_c, \ell_c), \quad f_x > 0, f_\ell > 0.$$

Subscripts on functions denote partial derivatives. The utility of consumer is actually determined by amount of $z$, which is consumption experience that depends on amount of the good, $x$, and time devoted, $\ell$,

$$z = g(x, \ell), \quad g_x > 0, g_\ell > 0.$$

Utility function is,

$$u(n, z), u_n > 0, u_z > 0.$$

Budget constraint depends on price of good and wage, and labor endowment, $\bar{\ell}$,

$$px + px_c + w\ell + w\ell_c = w\bar{\ell}.$$

Figure 4 demonstrates the optimization problem. The opportunity set is defined as,

$$\left\{(z, n) | n = f(x_c, \ell_c), \quad z = g(x, \ell), \quad p(x + x_c) + w(\ell + \ell_c) = w\bar{\ell}\right\}.$$

The frontier is downward sloping (see Appendix). It reflects the budget constraint as well as the technologies, $g$ and $f$.

We further index consumption ( consumption experience) by quality, $Q$. Utility function is

$$u(Qz, n)$$

where $z$ measures quantity of consumption. First-order condition for utility maximization are,

$$\frac{f_x}{f_\ell} = \frac{g_x}{g_\ell} = \frac{p}{w}, \tag{1}$$

$$\frac{u_n}{u_z} = Q\frac{g_x}{f_x}. \tag{2}$$

Equation (1) implies less labor intensive consumption and child rearing method will be used when wage increase. The time series of female wage has been rising in Japan would lead to less labor intensive methods which means greater labor participation. Equation (2) implies better quality of consumption leads to more consumption and less children.

Higher wage but not significantly higher quality means positive relationship. However with the same higher relative wage and higher quality consumption means negative relationship between labor participation and fertility. Availability of consumption goods, such as entertainment and restaurants, is much greater in larger cities. This means higher $Q$, meaning less children and more consumption in cities.[1]

## 2.1 General Equilibrium with high quality product and heterogenous labor

In this section we analyze a general equilibrium model in which consumers have a utility function that reflect the previous analysis, although somewhat simplified. Consumers differ by two attributes, their preference and quality of labor. Consumers choose either to consumer high quality product or standard (low quality) product. Child bearing choice differ according to which product they choose, as well as if they are skilled or not. Skilled workers produce the high quality product and the labor supply level determine the level of quality.

### Consumers

We simplify the consumer's problem so that she chooses between consumption ($x$) and childbearing ($n$). Her preference is represented by the following utility function which also depends on the quality of the good consumed, $Q$,

$$U_\rho(n,x) = (Qx^\rho + n^\rho)^{\frac{1}{\rho}}, \quad 0 < \rho < 1. \tag{3}$$

Consumers preference, $\rho$, is distributed uniformly over [0,1]. Consumption good is either the standard (low quality) $Q = 1$ or high quality $Q > 1$.

[1]For instance, there are 191 Tokyo restaurants listed in the Michelin restaurant guide, compared to 64 in Paris and 42 in New York (Robinson (2007)). Same hours spent at a Tokyo restaurant yields higher $Qz$ on the average compared to other locations in Japan.

Consumer's labor endowment is $\bar{\ell}$ and wage is $w$ which is also the opportunity cost of children. Denoting price of the good by $p$, consumer chooses consumption and number of children to maximize (3) with respect to the budget constraint,

$$px + wn = w\bar{\ell}.$$

Each consumer's consumption and number of children given quality $Q$ is determined by the utility maximization given the budget constraint,

$$x^*_\sigma(p, w; Q) = \frac{Q^\sigma \bar{\ell}}{(\frac{p}{w})^\sigma \left(Q^\sigma (\frac{p}{w})^{1-\sigma} + 1\right)}, \quad n^*_\sigma(p, w; Q) = \frac{\bar{\ell}}{Q^\sigma (\frac{p}{w})^{1-\sigma} + 1}, \tag{4}$$
$$\text{where } \sigma \equiv \frac{1}{1-\rho} > 1.$$

Consumption is increasing and number of children is decreasing in quality, as in the previous section. The indirect utility is,

$$v_\sigma(p, w; Q) = \bar{\ell} \left( Q^\sigma (\frac{w}{p})^{\sigma-1} + 1 \right)^{\frac{1}{\sigma-1}}.$$

The consumer must choose which quality to consume. If her marginal utility from more consumption is relatively large, she devotes less resources to children and has fewer children. If the quality is low and not as beneficial, she derives utility by having many children. She compares the utility levels from consuming each quality and buys whichever yields higher utility. We denote the prices of the goods with different qualities by $p_H$ and $p_L$. Consumer will buy the high quality good when

$$v_\sigma(p_H, w; Q) > v_\sigma(p_L, w; 1).$$

This condition is equivalent to,

$$\sigma < \hat{\sigma} \equiv \frac{\ln \frac{p_H}{p_L}}{\ln \frac{p_H}{p_L} - \ln Q}. \tag{5}$$

Since $\sigma > 1$, there will be no demand for the low quality good if $\ln \frac{p_H}{p_L} < \ln Q$. This occurs if low quality product is more expensive ( $p_L \geq p_H$) since $Q > 1$ and $p_H > p_L$ but the price premium for the high quality is small relative to difference in quality. It does not depend on the level of income.

Consumer's labor supply is the hours not devoted to raising children,

$$\ell_\sigma(p, w; Q) = \bar{\ell} - n^*_\sigma(p, w; Q) = \frac{Q^\sigma}{Q^\sigma + (\frac{p}{w})^{\sigma-1}}. \tag{6}$$

**Markets**

The labor each consumer supplies is either skilled ($s$) or unskilled ($u$). There are total of $N$ consumers, and $\theta \in (0, 1)$ of the consumers are skilled. Labor endowment, $\bar{\ell}$, is the same for both types. We denote wages for skilled and unskilled by $w_s$ and $w_u$. Production technology is constant returns to scale in labor: one unit of skilled labor produces one unit of high quality product and one unit of unskilled labor produces one unit of the standard product. Furthermore we assume both products are supplied competitively. Thus we have $p_H = w_s$ and $p_L = w_u$.

One skilled worker's demand for high quality product is , denoting relative wage by $\xi = \frac{w_s}{w_u} > 1$ and using (4),

$$x^H_s(\xi) = x^*_\sigma(w_s, w_s; Q) = \frac{Q^\sigma \bar{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma} = \frac{\ln \xi}{\ln \xi - \ln Q},$$

and demand for low quality is,

$$x^L_s(\xi) = x^*_\sigma(w_u, w_s; Q) = \frac{\bar{\ell}}{\xi^{-\sigma}(\xi^{\sigma-1} + 1)}, \quad \sigma > \hat{\sigma}.$$

There will be positive demand for the low quality only if $\xi > 1$ since $\xi = \frac{p_H}{p_L}$. We make the following observation

**Claim 1.** *High skilled consumers consume more of both quality, $x^H_s(\xi) > x^H_u(\xi)$ and $x^L_s(\xi) > x^L_u(\xi)$.*

Total demands from all the skilled workers for high quality product and low quality product are ,

$$X_s^H(\xi) = \theta N \int_1^{\hat{\sigma}} x_s^H(\xi) d\sigma, \quad X_s^L(\xi) = \theta N \int_{\hat{\sigma}} x_s^L d\sigma.$$

Similarly for unskilled workers, we have the individual demands for high quality good,

$$x_u^H(\xi) = x_\sigma^*(w_s, w_u; Q) = \frac{Q^\sigma \bar{\ell}}{\xi^\sigma \left(Q^\sigma \xi^{1-\sigma} + 1\right)}, \quad \sigma < \hat{\sigma} = \frac{\ln \xi}{\ln \xi - \ln Q},$$

and demand for low quality good,

$$x_u^L(\xi) = x_\sigma^*(w_u, w_u; Q) = \frac{\bar{\ell}}{2}, \quad \sigma > \hat{\sigma}.$$

Total demands for each quality from all unskilled workers are,

$$X_u^H(\xi) = \int_1^{\hat{\sigma}} x_u^H(\xi) d\sigma, \quad X_u^L(\xi) = \int_{\hat{\sigma}} x_u^L(\xi) d\sigma.$$

Since production of one unit of good requires one unit of labor, demand for skilled and unskilled labor, $L_s^D$ and $L_u^D$ are,

$$L_s^D(\xi) = \theta N X_s^H(\xi) + (1-\theta) N X_u^H(\xi), \tag{7}$$

$$L_u^D(\xi) = \theta N X_s^L(\xi) + (1-\theta) N X_u^L(\xi). \tag{8}$$

Labor supply is constructed in a similar manner from individual supplies. Individual labor supply as function of relative wage is , using (6) ,

$$\ell_s^H(\xi) = \ell_\sigma^*(w_s, w_s; Q) = \frac{Q^\sigma \bar{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma},$$

$$\ell_s^L(\xi) = \ell_\sigma^*(w_u, w_s; 1) = \frac{\bar{\ell}}{\xi^{1-\sigma} + 1}, \quad \sigma > \hat{\sigma}$$

$$\ell_u^H(\xi) = \ell_\sigma^*(w_s, w_u; Q) = \frac{Q^\sigma \bar{\ell}}{Q^\sigma + \xi^{\sigma-1}}, \quad \sigma < \hat{\sigma},$$

$$\ell_u^L(\xi) = \ell_\sigma^*(w_u, w_u; 1) = \frac{\bar{\ell}}{2}, \quad \sigma > \hat{\sigma}.$$

Aggregation yields the total labor supply of each type,

$$L_s^S = N\bar{\ell}\int_1^{\hat{\sigma}} \left\{\theta\frac{Q^\sigma}{Q^\sigma+1} + (1-\theta)\frac{Q^\sigma}{Q^\sigma+\xi^{\sigma-1}}\right\} d\sigma, \tag{9}$$

$$L_u^S = N\bar{\ell}\int_{\hat{\sigma}}^{\infty} \left\{\theta\frac{Q^\sigma}{Q^\sigma+\xi^{1-\sigma}} + (1-\theta)\frac{1}{2}\right\} d\sigma. \tag{10}$$

It is easy to show, from (5), that $\hat{\sigma}$ is decreasing in $\xi$ that $L_s^D$ and $L_u^S$ is decreasing in $\xi = \frac{w_s}{w_u}$ and $L_s^S$ and $L_u^D$ are increasing in $\xi$. Equilibrium relative wage for a given quality level, $\xi^*(Q)$, is determined by the skilled labor market clearing condition,

$$L_s^D(\xi) = L_s^S(\xi).$$

The unskilled labor market has cleared by Walrus Law.

### Comparative statics

We first see how the equilibrium labor supply and relative wage change with quality.

**Claim 2.** *(i) $L_s^S$, $L_u^S$ and $L_s^D$ are increasing and $L_u^D$ are decreasing in $Q$.*

*(ii) Equilibrium relative wages and level of skilled labor are increasing in quality. That is, $\partial\xi^*(Q)/\partial Q > 0$ and $\partial L_s^*(Q)/\partial Q > 0$.*

(See Figures 5 and 6. Proof is in the Appendix.) Higher quality makes consumption attractive for skilled workers and also increase proportion of all workers that consume the high quality product. Thus both demand and supply of skilled labor is increasing in quality. The same effect increases the supply of unskilled workers and reduces demand for low quality good. The latter effect implies demand for unskilled workers decreases when quality improves.

Skilled labor supply is increasing in population, $\partial L_s^S/\partial N > 0$, from (9) and demand is also increasing in population, $\partial L_s^D/\partial N > 0$, from (7). (See proof of Claim 2 in the Appendix.) This implies

**Claim 3.** *Both equilibrium skilled and unskilled labor will increase when population increases, $\partial L_s^*/\partial N > 0$ and $\partial L_u^*/\partial N > 0$.*

Again, using the proof of Claim 2 in the Appendix, both demand and supply of skilled labor is also increasing in proportion of skilled consumers, $\partial L_s^S/\partial\theta > 0$, from (9) and $\partial L_s^D/\partial\theta > 0$, from (7).

**Claim 4.** *Equilibrium skilled labor and equilibrium relative wage are increasing in the proportion of skilled consumers,* $\partial L_s^*/\partial\theta > 0$ *and* $\partial\xi^*/\partial\theta > 0$.

**Birthrate**

Individual number of children are,

$$
\begin{aligned}
n_s^H(\xi) = n_\sigma^*(w_s, w_s; Q) &= \frac{\bar{\ell}}{Q^\sigma + 1}, \quad \sigma < \hat{\sigma}, \\
n_s^L(\xi) = n_\sigma^*(w_u, w_s; 1) &= \frac{\bar{\ell}}{\xi^{\sigma-1} + 1}, \quad \sigma > \hat{\sigma} \\
n_u^H(\xi) = n_\sigma^*(w_s, w_u; Q) &= \frac{\bar{\ell}}{Q^\sigma \xi^{1-\sigma} + 1}, \quad \sigma < \hat{\sigma}, \\
n_u^L(\xi) = n_\sigma^*(w_u, w_u; 1) &= \frac{\bar{\ell}}{2}, \quad \sigma > \hat{\sigma}.
\end{aligned}
$$

It is clear that for given wage level, those that consume high quality good devoted even more resources for consumption and thus reduce number of children when quality improves. Since the equilibrium relative wage in increasing in quality, we can say the following,

**Claim 5.** *(i) Skilled consumers have less children . That is ,* $n_s^H < n_u^H$ *for* $\sigma < \hat{\sigma}$ *and* $n_s^L < n_u^L$ *for* $\sigma > \hat{\sigma}$.

*(ii) Skilled consumers have less children when quality of product improves. That is,* $dn_s^H/dQ < 0$ *for* $\sigma < \hat{\sigma}$ *and* $dn_s^L/dQ < 0$ *for* $\sigma > \hat{\sigma}$.

*(iii) Unskilled consumers that consume low quality product have the same number of children when quality improves. That is,* $dn_u^L/dQ = 0$ *for* $\sigma > \hat{\sigma}$.

Although there is an income effect, the substitution effect dominates and skilled workers that consumer low quality reduce number of children. For unskilled consumers that bought high quality good, improvement makes consumption more attractive (reduce children) but their relative wage becomes lower and the substitution effect works in the opposite direction. The total effect is not clear.

**Endogenous quality**

Assume that level of quality is increasing in the size of the skilled labor. That is, $Q = Q_T(L_s)$ is an increasing function of $Q$. Subscript $T$ refers to "technology" which is what this relationship reflects. We will denote the inverse relationship between the market equilibrium supply of skilled labor and quality of $L_s^*(Q)$ by $Q = Q_M(L_s)$, which is an increasing function from Claim 2. The equilibrium level of labor $L_s^*$ and equilibrium level of quality, $Q^* = Q_M(L_s^*) = Q_T(L_s^*)$, is the intersection of the two curves.

When marginal increase in quality from labor is very large, then the equilibrium is unstable. Graphically, this would mean slope of $Q_T$ is steeper than $Q_M$ $(Q_T' > Q_M')$ . This is the case around equilibrium $E_1$ in Figure 7. A perturbation away from $E_1$ results in either spiral increase in quality and skilled labor supply or decrease of quality and skilled labor supply. When technology is mature so that marginal quality improvement is very small, then equilibrium is stable $(Q_T' < Q_M')$ . This is equilibrium $E_2$ in Figure 7. There may be multiple equilibria, some stable and others unstable. A slight perturbation from low quality with small skilled labor force will start a spiral of labor and quality improvement until $E_2$ is reached.

Now using Claim 3, we analyze the effect of declining population. The claim implies that the $Q_M(L_s)$ function will shift upward in the $L_s - Q$ space (Figure 8).

**Claim 6.** *(i) If the technology is in its infancy, then equilibrium quality and skilled labor supply increase when population declines. That is ,*

$$Q_T' > Q_M' \quad \Rightarrow \quad \frac{\partial Q^*}{\partial N} < 0, \quad \frac{\partial L_s^*}{\partial N} < 0.$$

*(ii) If the technology is mature, then equilibrium quality and skilled labor*

*supply decrease when the population decreases. That is ,*

$$Q'_T < Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial N} > 0, \quad \frac{\partial L_s^*}{\partial N} > 0.$$

When the technology is mature, then declining population results in "contraction" of the economy. That is, quality and supply of skilled labor are reduced. Claim 5 suggests that lower quality will increase the birthrate. Recall that all but unskilled consumers that consumed high quality product will increase birthrate when quality improves. This situation is consistent with a cohort effect.

The situation is different when the technology still has not exhausted increasing marginal returns. The new equilibrium results in more skilled labor and higher quality. Products are more polarized, skilled labor has higher relative wages and work more. Utility is derived from more consumption and there is less children. The cohort effect does not hold because the economy adjusts to the lower level of population according to the available technology.

Now we consider the effect of more skilled workers, using Claim 4. The claim implies that the $Q_M(L_s)$ function will shift downward in the $L_s - Q$ space (Figure 9). Immediately we have the following,

**Claim 7.** *(i) If the technology is in its infancy, then equilibrium quality and skilled labor supply decrease when the proportion of skilled workers increase. That is ,*

$$Q'_T > Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial \theta} < 0, \quad \frac{\partial L_s^*}{\partial \theta} < 0.$$

*(ii) If the technology is mature, then equilibrium quality and skilled labor supply increase when the proportion of skilled workers increase. That is ,*

$$Q'_T < Q'_M \quad \Rightarrow \quad \frac{\partial Q^*}{\partial \theta} > 0, \quad \frac{\partial L_s^*}{\partial \theta} > 0.$$

Equilibrium quality will decrease (increase) when technology is in its infancy (maturity). When proportion of skilled consumers increase, each skilled worker needs to supply less labor to maintain the same quality. When marginal quality from labor is very large, quality must be lower to accommodate it. Lower quality (and lower wage) likely to imply higher birthrate. Thus when technology is sufficiently productive, the increasing skilled workers will increase the birthrate. On the other hand when the marginal product of labor is low, then higher labor implies higher quality. This may reduce the birthrate.

Claims 6 and 7 suggest that increasing the proportion of skilled labor can be effective in reversing decline in birthrate whenever the cohort effect may not hod. This was the case when marginal return from increasing skilled labor is large. On the other hand, when the technology is mature, Esterlin Hypothesis is likely to hold and the same policy will prevent the feedback mechanism that otherwise will function.

# 3 Network Effect of Childbearing

Let us consider a simple model of fertility choice. There are $N$ identical households in an economy. They enjoy raising children as well as consuming a good. Let $n$ denote the number of children and $c$ denote the consumption of a good in each household. Preferences are represented by a Cobb-Douglas utility function,

$$u(n, c) = n^{\theta} c^{1-\theta},$$

where $\theta \in (0, 1)$. Each household has one unit of time and allocates its time to earning wages and rasing children. We assume that each child costs $\beta$ in time, and thus $n\beta$ is the total time cost of raising children. In addition to its time, each household is endowed with $k$ units of capital. Let $w$ denote the wage rate and $r$ denote the (gross) rental rate of capital. The budget constraint for each household is

$$c + w\beta n = w + rk. \tag{11}$$

Note that the price of the consumption good equals one since it is nemeraire. Each household maximizes the utility function subject to the budget constraint. Let $u_n$ and $u_c$ denote the partial derivatives of $u$ with respect to $n$

and $c$ respectively. The first order condition for this problem is

$$\frac{u_n}{u_c} = \frac{\theta c}{(1-\theta)n} = w\beta. \tag{12}$$

The higher wage rate leads to the larger opportunity cost of raising children. Thus, as the wage rate increases, the number of children per consumption good, $n/c$, declines in each household. The choice of fertility also depends on the time cost of rasing children $\beta$. An increase in $\beta$ implies that it is more time-consuming to raise children. Thus, a larger $\beta$ leads to smaller number of children per consumption good, $n/c$, in each household.

**The Network Effect**

We take the view that there are networks among parents having children, and the networks can play an important role in determining the cost of rasing children. For instance, in a city with a large population of families, parents may help each other when their children are sick but they cannot be absent from work. Parents can also exchange information about raising children such as the quality of a day-care center or a pediatrician. This kind of network would facilitate raising children for each household by lowering the cost of raising children. Also, the network effect would be stronger as the population size of families becomes larger.

There also may be other reasons why cost of raising children decreases with number of children. When there are many children, there will be more doctors, day-care, and other service providers for children. Figure 10 shows such a decline in Japan.[2] Transportation cost would be lower as distribution of service providers become more dense. One can also argue search costs would decline but this effect may be indistinguishable from the aforementioned network effect.

We now introduce the network effect of raising children into the model.

[2]We do not claim less service providers explains less children. In fact, the causality is in the other direction. We only claim that there was in fact a decline of child related service providers in Japan and this would have raised cost of child rearing over the year. We still can observe from Figure 10 that distance to a service provider would have increased.

For this purpose, let $\beta = \beta(Nn)$ and $\beta'(Nn) < 0$ *i.e.* the cost of raising children negatively depends on the total number of children in the economy. We also assume that each household does not recognize the network effect, that is, it is a kind of a positive externality.

Using the budget constraint (11) and first order condition (12), we can derive the demand for children,

$$n(\beta) = \frac{\theta(w + rk)}{w\beta}. \tag{13}$$

If $\beta$ is constant, the demand for children is determined for the given values of $w$ and $r$. However, $\beta$ is not constant in the presence of the network effect since it depends on the total number of children. For the simplification of the analysis, let us consider a specific function of $\beta$,

$$\beta(Nn) = \frac{\overline{\beta}}{(Nn)^{\alpha}}, \tag{14}$$

where $\alpha \in (0, 1)$ and $\overline{\beta} > 0$. Clearly, $\beta$ is decreasing in the total number of children. It can be shown that $\beta$ is convex in $n$ since $\frac{d^2\beta}{dn^2} > 0$. For the given wage and rental rate, the number of children and the cost of raising children are determined by the two equations (13) and (14). Let $n_e$ and $\beta_e$ denote the equilibrium values of $n$ and $\beta$. Then, we can derive

$$\begin{aligned} n_e &= \left[\frac{\theta\,(w + rk)}{w\overline{\beta}}\right]^{\frac{1}{1-\alpha}} N^{\frac{\alpha}{1-\alpha}}, \\ \beta_e &= \overline{\beta}^{\frac{1}{1-\alpha}} \left[\frac{\theta\,(w + rk)}{w}\right]^{-\frac{\alpha}{1-\alpha}} N^{-\frac{\alpha}{1-\alpha}}. \end{aligned}$$

Figure 11 shows the determination of equilibrium values of $n$ and $\beta$. At the equilibrium point $(n_e, \beta_e)$, the curve of the demand for children is steeper than that of the cost of raising children. This case is guaranteed by the assumption that $\alpha$ is smaller than 1 in the cost function (14). Under this assumption, it can be show that the equilibrium is stable. To confirm this point, let us consider the following two scenarios.

## 3.1 The Adjustment Process of Birthrate

Suppose that the cost of child rearing is given by $a > \beta_e$ in Figure 12. For this value of the cost, the number of children chosen by each household is $n(a)$. However, when the number of children per household is $n(a)$, the actual cost of child rearing is $\beta(n(a))$ due to the network externality. Since $\beta(n(a))$ is smaller than $a$, each household would increase the the number of children. Again, $\beta$ declines due to the increase in $n$, and so on. This process continues until the equilibrium is reached. If the cost of raising children is higher than the equilibrium value, the number of children per household would increase in the adjustment process.

In contrast, suppose that the cost of child rearing is given by $b < \beta_e$ in Figure 12. For each household, it is optimal to choose $n(b)$. For this number of children, the network effect is too weak to keep the cost of child rearing as low as $b$. As a result, the cost rises up to $\beta(n(b))$, under which each household chooses the smaller number of children. Then, the decline in $n$ raises $\beta$ further more, and this process continues until the equilibrium is reached. If the cost of raising children is smaller than the equilibrium value, each household would reduce the number of children during the adjustment process.

## The Effect of the Wage Rate

Let us examine the impact of a wage increase on the number of children in each household. If the wage rate increases, then the opportunity cost of child rearing would rise. Thus, each household would choose to have fewer children. This is confirmed by $\frac{dn_e}{dw} < 0$. The point is that the network externality can magnify the impact of the wage increase. In Figure 13, the curve of the demand for children shifts down due to an increase in the wage rate. If there were no network effect, the cost of child rearing would be constant, and thus the decline in the number of children would be smaller than that in the presence of the network effect. This implies that the network effect will magnify a decline in "birthrate".

## The Effect of the Number of Households

Let us turn to a change in the number of households. In Figure 14, a fall in $N$ shifts the curve of the cost of child rearing upward, and thus the number of children per household would decline. The reason is straightforward. The decline in the number of households weakens the network effect of child rearing, and increasing the cost of raising children. It is worth noting that this effect does not appear in the absence of the network effect. This result also implies that a decline in the number of households reduces more than proportionally the total number of children in the economy.

## Socially Optimal Number of Children

We determine the relationship between the equilibrium and socially optimal number of children. Since all households are identical, social welfare is,

$$W(n, c) = Nu(n, c).$$

The resource constraint is simply $N$ times 11) with $\beta$ replaced by the function $\beta(Nn)$. The social planner takes the externality into account. The first order condition is,

$$\frac{u_n}{u_c} = \frac{N\theta c}{N(1-\theta)n} = w\beta(Nn) + w\beta'(Nn)N. \tag{15}$$

Since $\beta'(Nn) < 0$, comparison with (12) shows that the positive externality actually makes the cost of an extra child smaller. Using the explicit formulation (14), we get the relationship between $\beta$ and $n$,

$$n = \frac{\theta(w + rk)}{w\beta} + \alpha N(1-\theta). \tag{16}$$

The second term utilizes

$$\beta'(Nn) = -\alpha\bar{\beta}N^{-\alpha}n^{-\alpha-1} = -\alpha\frac{\beta(Nn)}{n}.$$

Unlike (13), this is an implicit demand function of $n$. It does show the relationship between $n$ and the value of $\beta$. This is depicted in dotted lines in Figure 11. We can see that the socially optimal number of children $n^*$ is more than $n_e$ and the corresponding cost will be lower, $\beta^* < \beta_e$.

## Optimal Subsidy

In this section, we analyze the government's optimal subsidy. Suppose that the government provides a subsidy $s$ per child and levies a lump-sum tax $t$ on income. Then, the budget constraint for each household is

$$c + (w\beta - s)n = w + rk - t.$$

Under this budget constraint, each household maximizes the utility. It can be shown that the first order condition is

$$\frac{\theta c}{(1-\theta)n} = w\beta - s.$$

The subsidy stimulates the demand for children by reducing the cost of child-rearing. By using the first order condition with the budget constraint, we can obtain the following condition:

$$n\beta = \frac{\theta(w + rk - t)}{w} + \frac{sn}{w}. \tag{17}$$

Under the subsidy, the demand for children must satisfy this condition. The balanced budget condition for the government implies that

$$Nsn = Nt.$$

With this condition, we can rearrange (17) as

$$n\beta = \frac{\theta(w + rk - t)}{w} + \frac{(1-\theta)sn}{w}. \tag{18}$$

In the previous section, we derived the optimal condition (16). We can rewrite (16) as follows:

$$n^*\beta^* = \frac{\theta(w+rk)}{w} + \alpha N(1-\theta)\beta^*. \tag{19}$$

If the government chooses the subsidy optimally, then $(n^*, \beta^*)$ must satisfy (18) under the optimal subsidy $s^*$. Thus, by using (18) and (19), we have

$$\frac{\theta(w+rk)}{w} + \frac{(1-\theta)s^*n^*}{w} = n^*\beta^* = \frac{\theta(w+rk)}{w} + \alpha N(1-\theta)\beta^*.$$

By solving for $s^*$, we can derive the optimal subsidy,

$$s^* = \frac{w\alpha N\beta^*}{n^*} = \frac{\alpha w\overline{\beta}N^{1-\alpha}}{n^{*1+\alpha}},$$

where the second equality is obtained by (15). We can show that the optimal subsidy is positively related to the wage rate. In Figure 11, an increase in the wage rate shifts the dotted line upward, and thus the optimal number of children falls. Then, we can easily see that an increase in wage rate raises the optimal subsidy. The intuition is straightforward. A rise in the wage rate increases the opportunity costs for child-rearing. Thus, the government must provide the larger subsidy to stimulate birthrate.

# 4 Concluding Remarks

We have employed comparative statics of a general equilibrium framework to understand the long term (stationary equilibrium) effect of declining population on the economy, including labor supply, cost of child rearing and birthrate. We first incorporated vertically differentiated goods in the general equilibrium model based on the observation of time series and cross sectional date of birthrate - female labor participation relationship. We focused on network effect to explain change in cost of child rearing.

Our analysis in the fist part suggests that if the technology is productive enough, the economy will adjust to smaller population and the cohort effect does not reverse the trend of declining population. We also showed that increasing the proportion of skilled consumers (potential workers) can increase birthrate and reverse the trend precisely when the cohort effect does not hold. We note that the same relationship between population size and proportion of skilled consumers means that changing the proportion can prevent the natural feedback mechanism from functioning when it would have functioned.

The two situations are characterized by if the technology has high marginal return from skilled labor (infant) or if this has been exhausted (mature). The economy will correct itself when it is mature, where we also observed the equilibrium to be stable. Therefore, another possible policy is to let the technology mature quickly.

Besides extending the model to a dynamic framework, analysis of an economy such as Japan requires understanding the effect of international trade. Assuming Japan will export high quality products, trade should reduce the substitution effect of high quality product while maintaining or increasing the income effect. This suggests trade by itself could correct the bias towards consumption and less children. On the other hand, existing trade literature (Flam and Helpman (1987), Theonig and Verier (2003)) suggest that trade will lead to greater specialization, particularly in a dynamic framework. This is left for future research.

In the second half, we developed a simple model to examine the network-effect of child rearing. If the network effect exists, the cost of raising children decreases with the total number of children. Then, the equilibrium number of children is too small as compared to the socially optimal value, and the network effect can magnify the decline of birthrate. The government can provide the optimal subsidy to stimulate birthrate. In a high-wage country, the opportunity cost of child-rearing is large for each household. Thus, the size of the optimal subsidy would increase with income of the economy.

In a general equilibrium setting, the wage rate is determined endogenously. Then, a decline in the size of population leads to the higher wage rate by reducing the labor supply. The rise in the wage rate leads to the higher opportunity costs for raising children, and thus the demand for children declines as well. In the presence of the network effect, a decrease in population can magnify the reduction in birthrate since the cost of child-rearing increases with a decline in the size of population.

Our approach here is also static. It may be possible to extend the model to a dynamic setting. Immigration is another aspect of globalization. It is interesting to examine the impact of immigration on birthrate. These are tasks for our future research. Case of open economy is explored in Yomogida and Aoki (2005).

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# Appendix

### Optimization of $u(Qx, n)$

Denoting the Lagrange multiplier by $\lambda$, fist-order conditions are,

$$u_n f_x = \lambda p, \quad u_n f_\ell = \lambda w, \quad u_n g_x = \lambda p, \quad u_n g_\ell = \lambda w,$$

and the budget constraint. This implies

$$\frac{f_x}{f_\ell} = \frac{g_x}{g_\ell} = \frac{p}{w}.$$

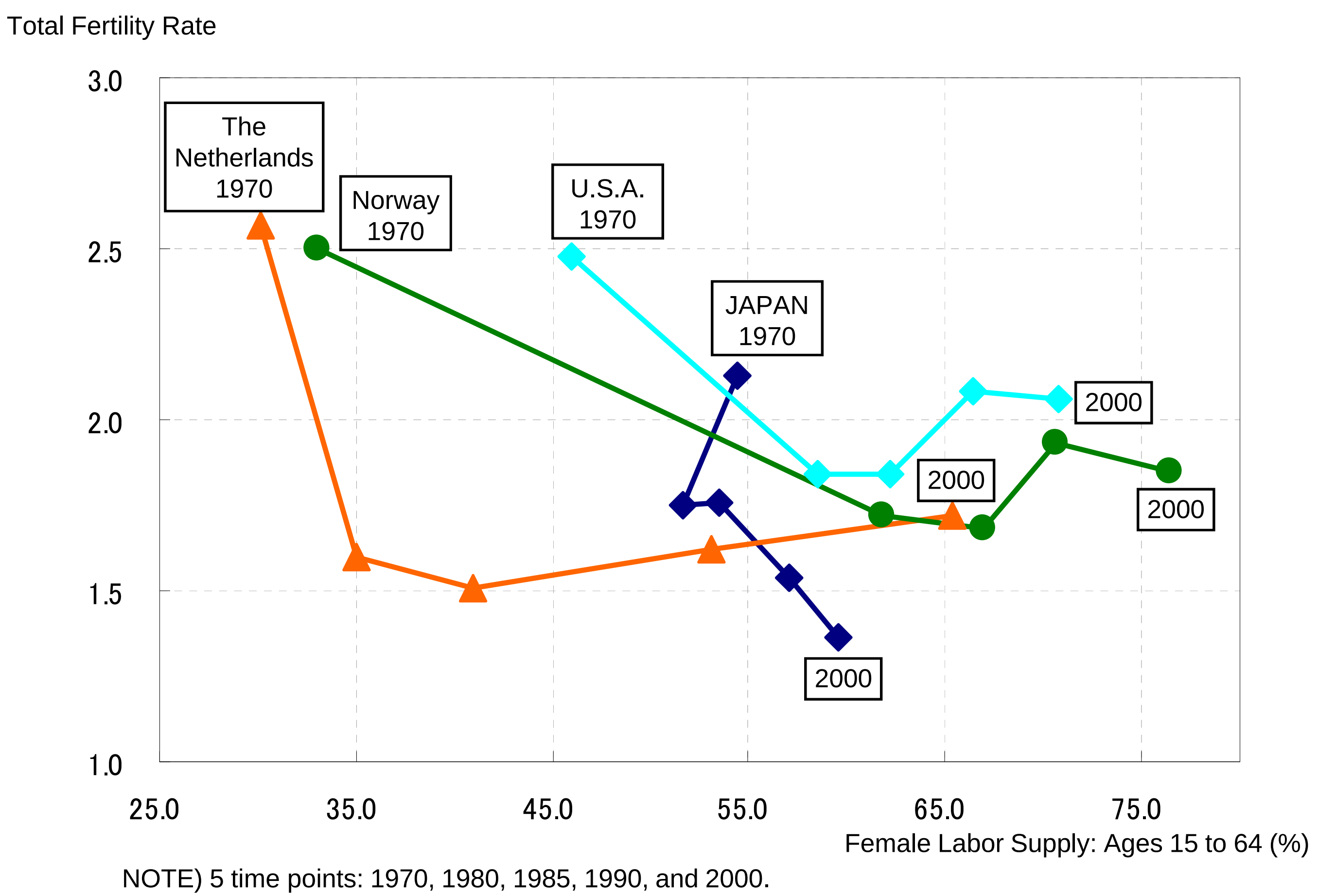
When $w$ increases, $\ell_c$ and $\ell$ decrease while $x$ and $x_c$ increase.

### Proof of Claim 2

The demand and supply functions, (7),(8), (9), and (10), can be rewritten as,

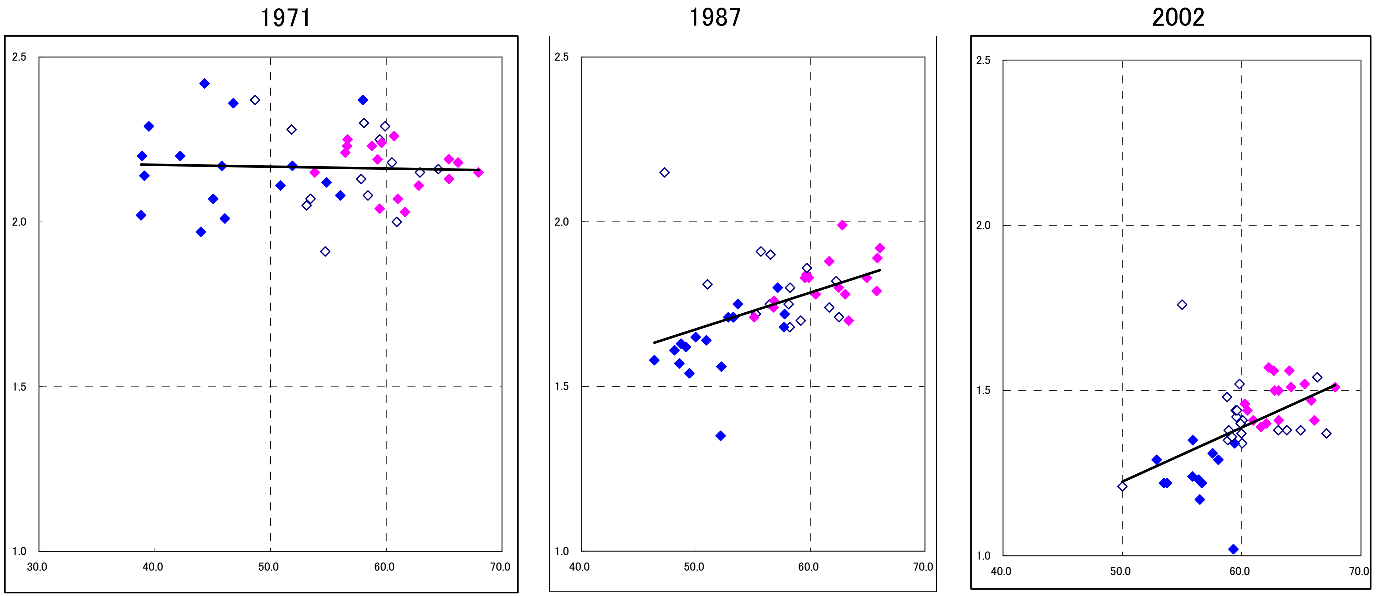
$$\begin{aligned} L_s^S &= \theta N\bar{\ell}\int_1^\infty \frac{Q^\sigma}{Q^\sigma + \xi^{1-\sigma}} d\sigma + \theta N\bar{\ell}\int_{\hat{\sigma}}^\infty \left\{ \frac{Q^\sigma}{Q^\sigma + \xi^{1-\sigma}} - \frac{Q^\sigma}{Q^\sigma + 1} \right\} d\sigma \\ L_s^D &= \theta N\bar{\ell}\int_1^{\hat{\sigma}} \frac{Q^\sigma}{Q^\sigma + 1} d\sigma + (1-\theta)N\bar{\ell}\int_1^{\hat{\sigma}} \frac{Q^\sigma}{Q^\sigma \xi + \xi^\sigma} d\sigma \\ L_u^S &= (1-\theta)N\bar{\ell}\int_1^\infty \left\{ \frac{Q^\sigma \xi^{1-\sigma}}{Q^\sigma \xi^{1-\sigma} + 1} - \frac{1}{2} \right\} d\sigma + (1-\theta)N\bar{\ell}\int_1^\infty \frac{1}{2} d\sigma, \\ L_u^D &= (1-\theta)N\bar{\ell}\int_{\hat{\sigma}}^\infty \frac{1}{2} d\sigma + \theta N\bar{\ell}\int_{\hat{\sigma}}^\infty 1\xi^{-1} + \xi^{-\sigma} d\sigma. \end{aligned}$$

The claim follows from noting that $\hat{\sigma}$ is decreasing in $\xi$ and increasing in $Q$, and that $Q^\sigma \xi^{1-\sigma} > 1$ for $\sigma < \hat{\sigma}$.




# Figure 1: FTR and female labor supply 1970,80,85,90,2000

(Council for Gender Equality, Special Committee on the Declining Birthrate and Gender-Equal Participation, 2006a)

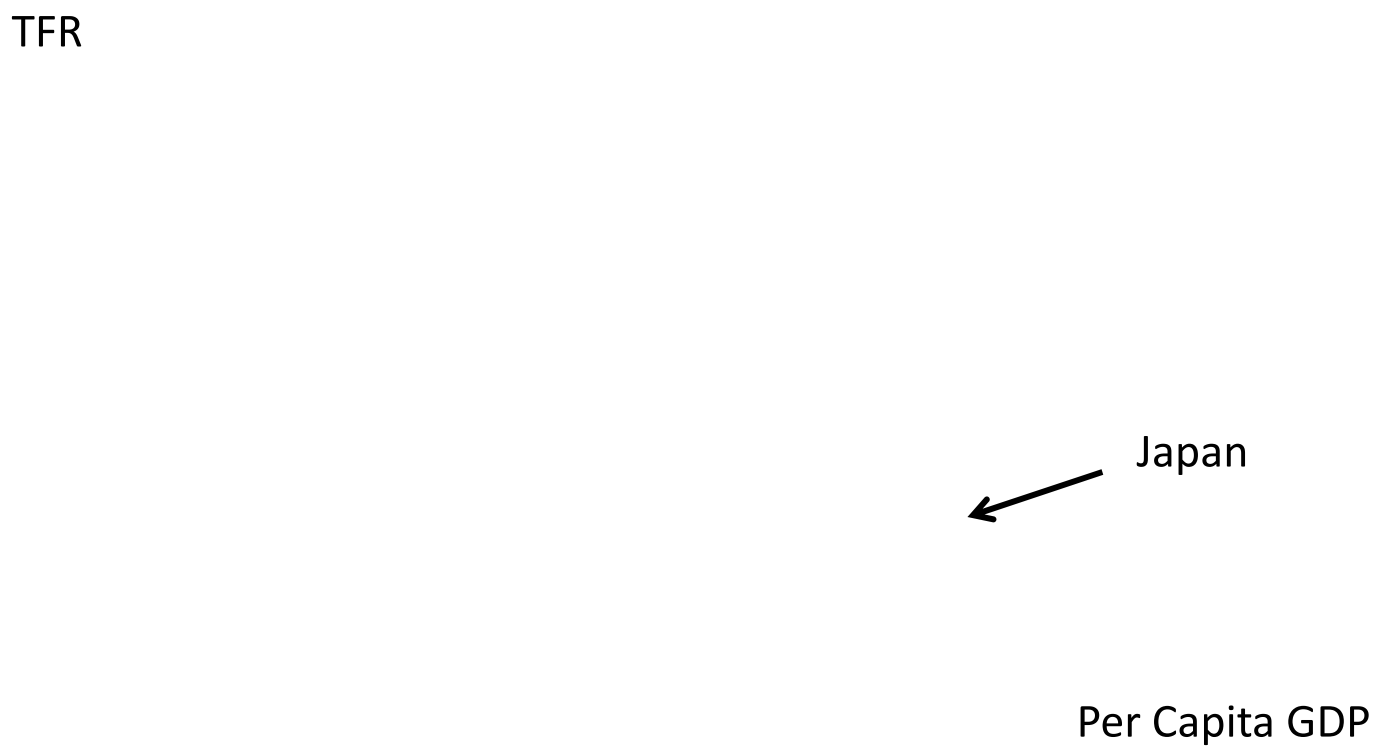



NOTE) Pink points are TYPE1(low declining rate in TFR and high level of TFR and female labor supply). Blue points are TYPE7(high declining rate in TFR and low level in TFR and female labor supply).

Sources) Ministry of Internal Affairs and Communications “Employment Status Survey," National Institute of Population and Social Security Research “Indicators of Fertility by Prefecture in 1970-1985,“ and Health, Labor and Welfare Ministry “Population Survey Report.”

Figure 2: FTR and female labor participation ratio by prefecture in 1971, 1987, 2002

(Council for Gender Equality, Special Committee on the Decling Birthrate and Gender-Equal Participation, 2006b)




Source) United Nations Population Fund “State of World Population 2004,” OECD “National Accounts of OECD Countries Main Aggregation Volume 1 2005,” and IMF “World Economic Outlook Databases 2003.”

## Figure 3: TFR and Per Capita GDP

(Council for Gender Equality, Special Committee on the Declining Birthrate and Gender-Equal Participation, 2006a)

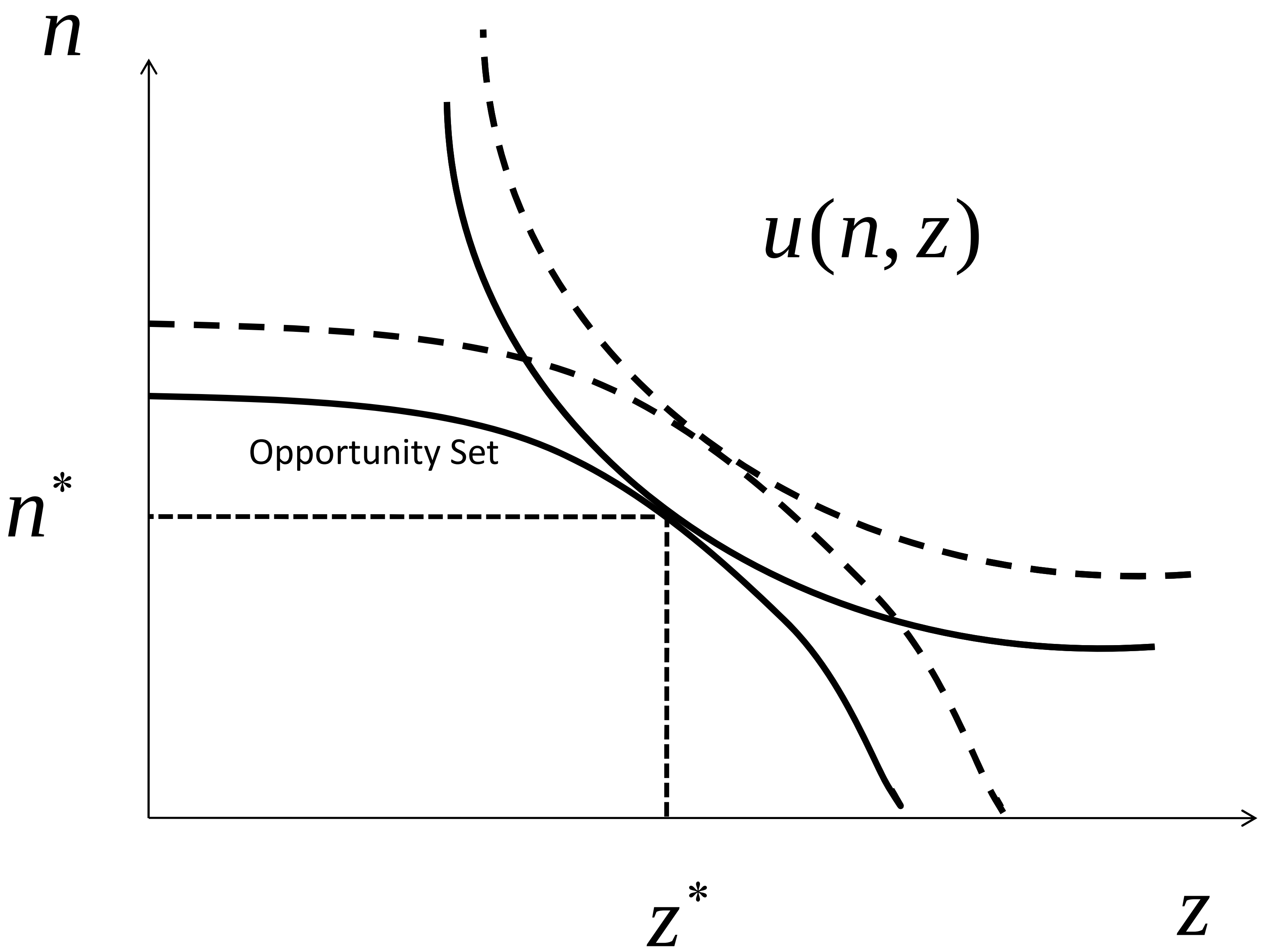



Figure 4: Optimization Problem

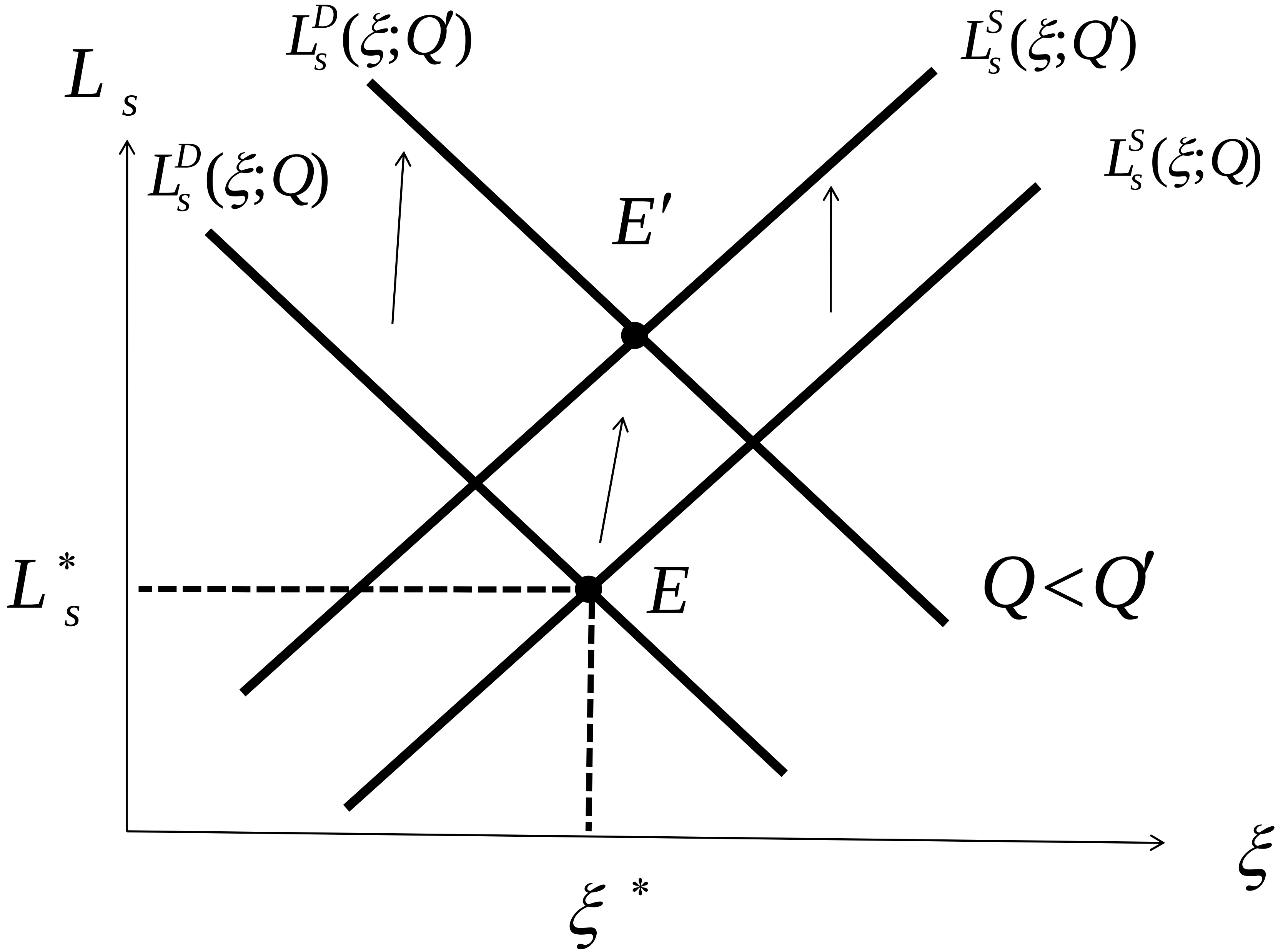



Figure 5: Skilled Labor Market

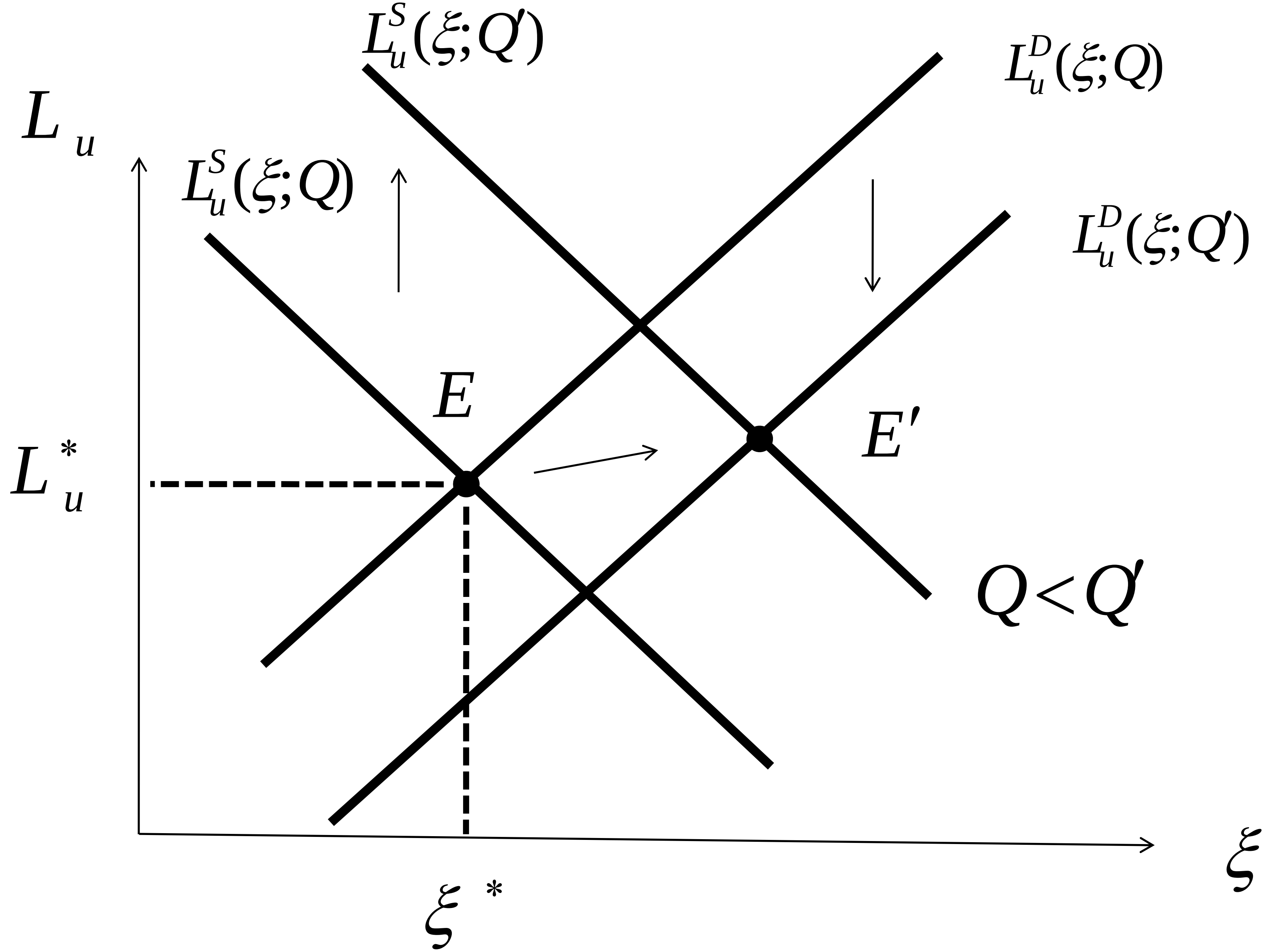



Figure 6: Unskilled Labor Market

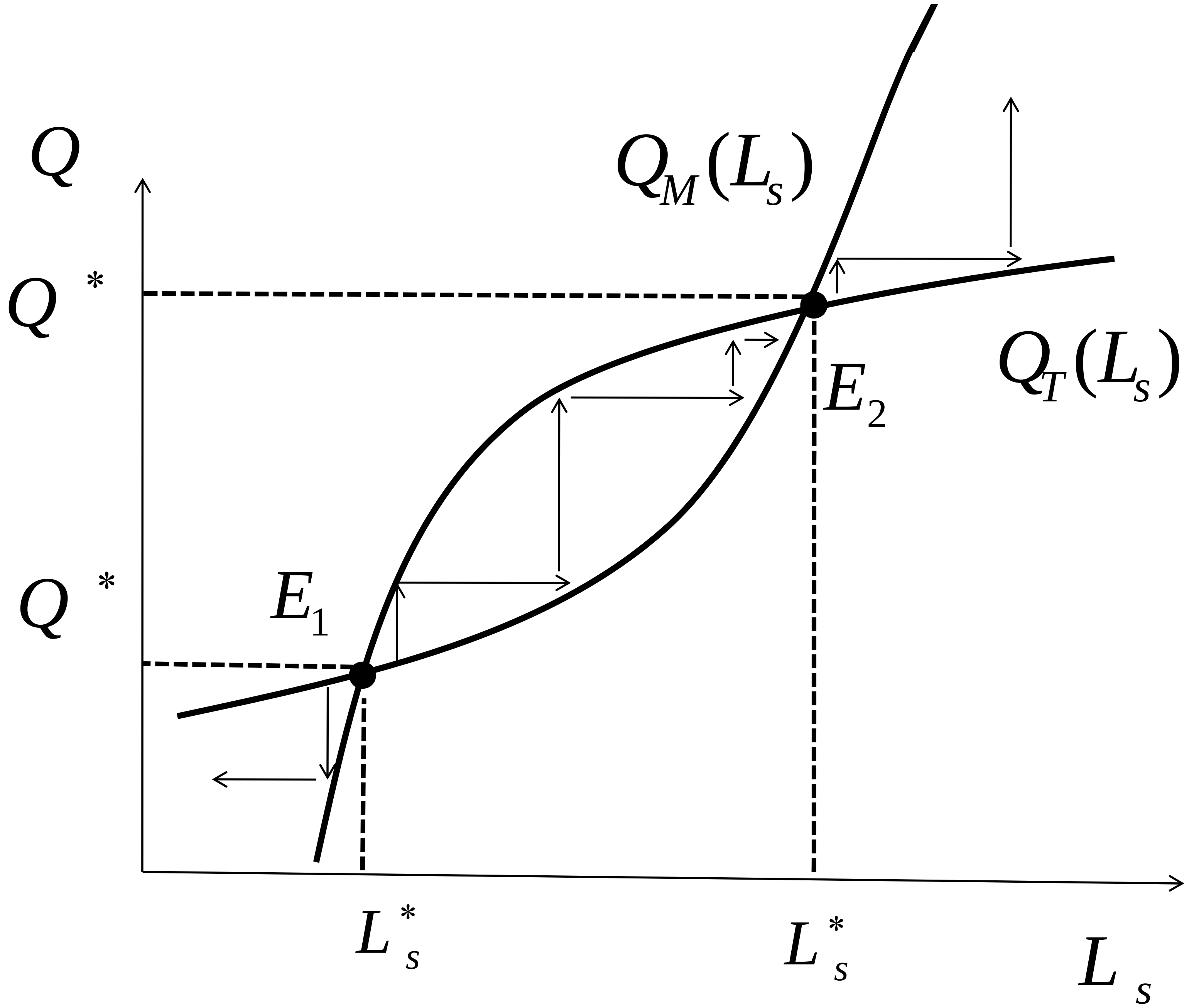



Figure 7: Equilibrium Quality and Skilled Labor

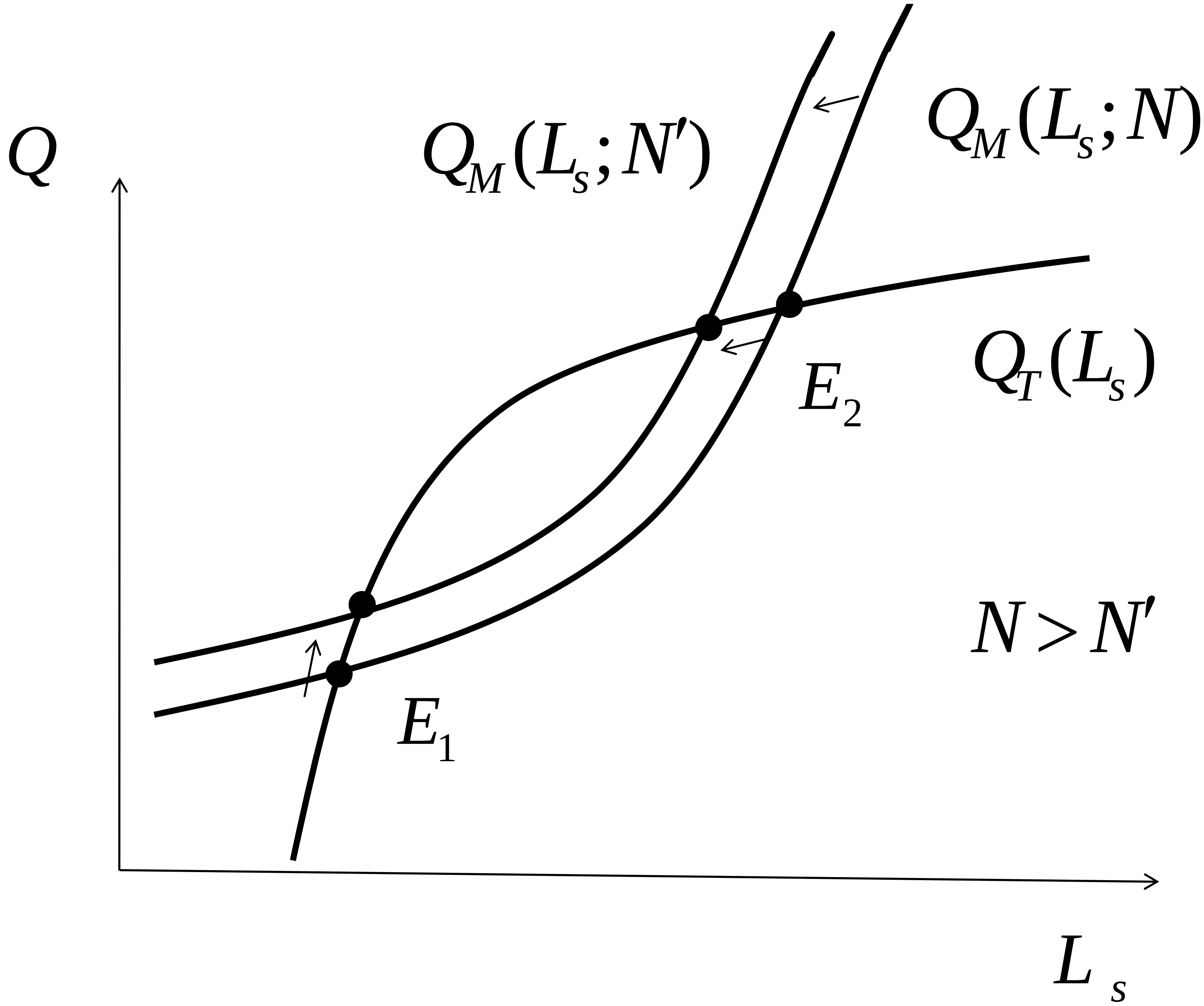



Figure 8: Declining Population

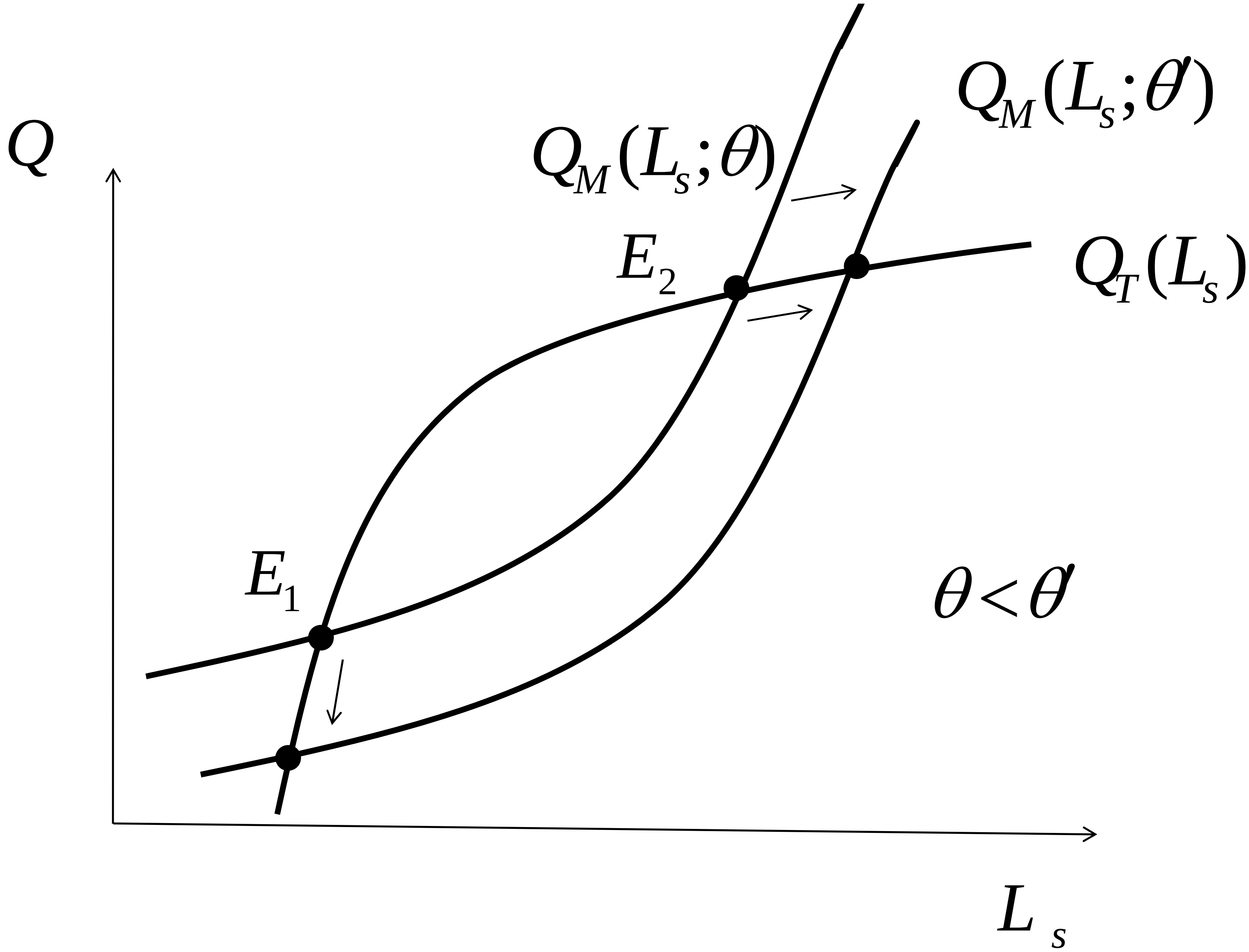
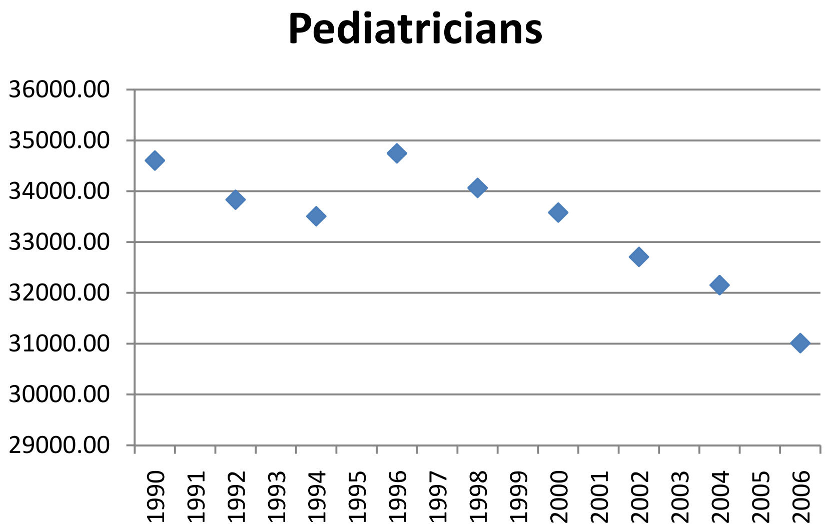



Figure 9: Larger Proportion of Skilled Labor



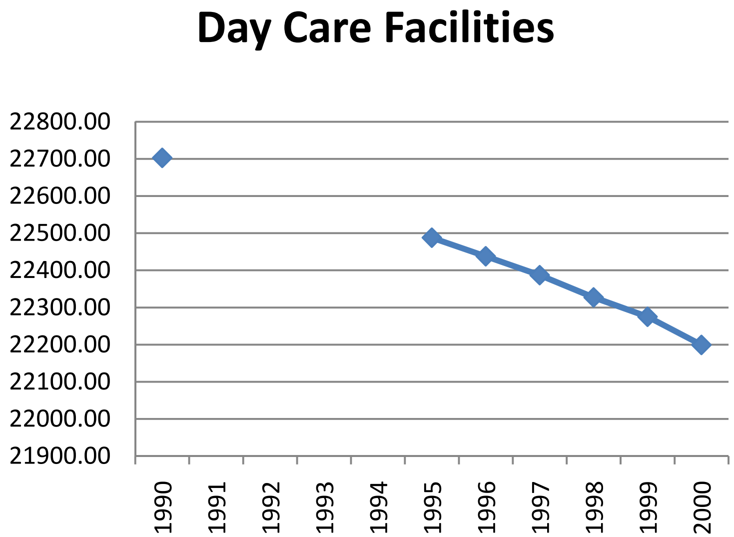




Figure 10 Number of Service Providers

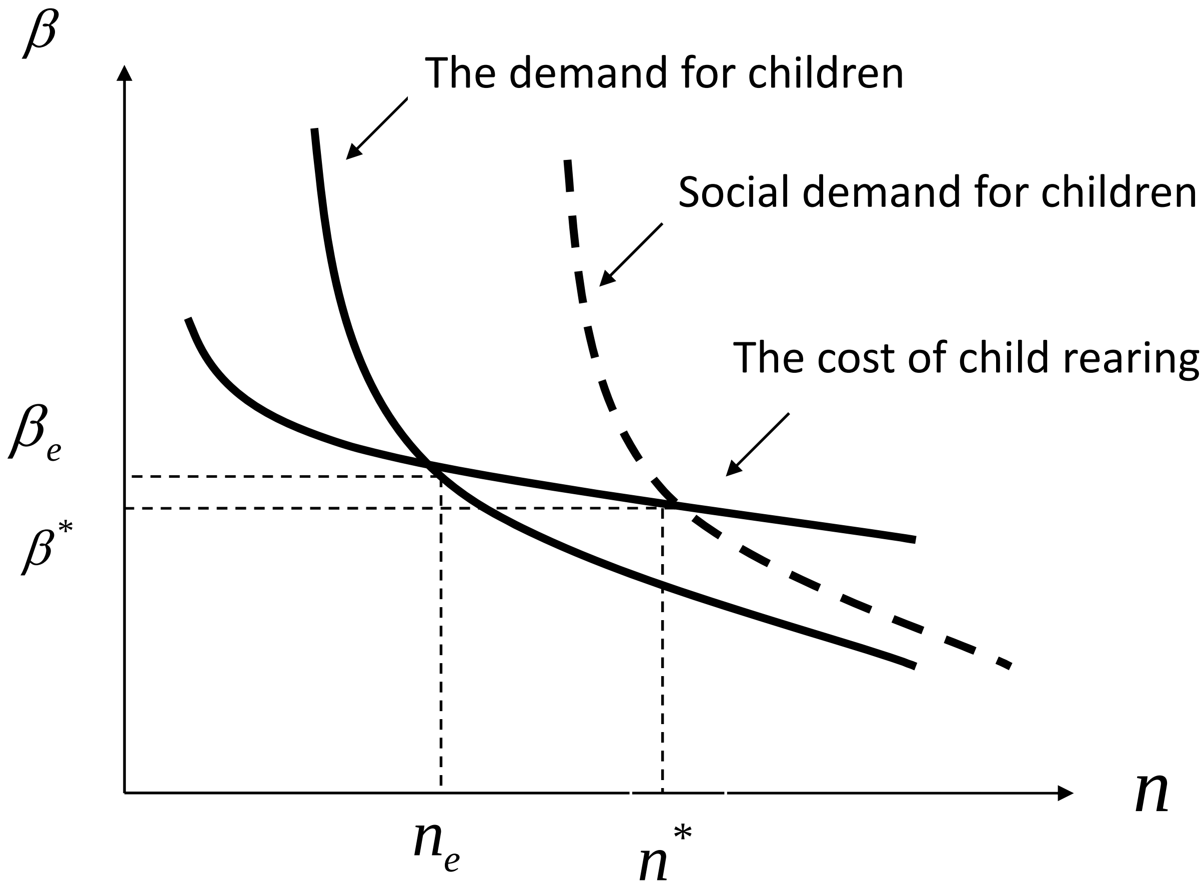



Figure 11: Determination of equilibrium number of children and cost

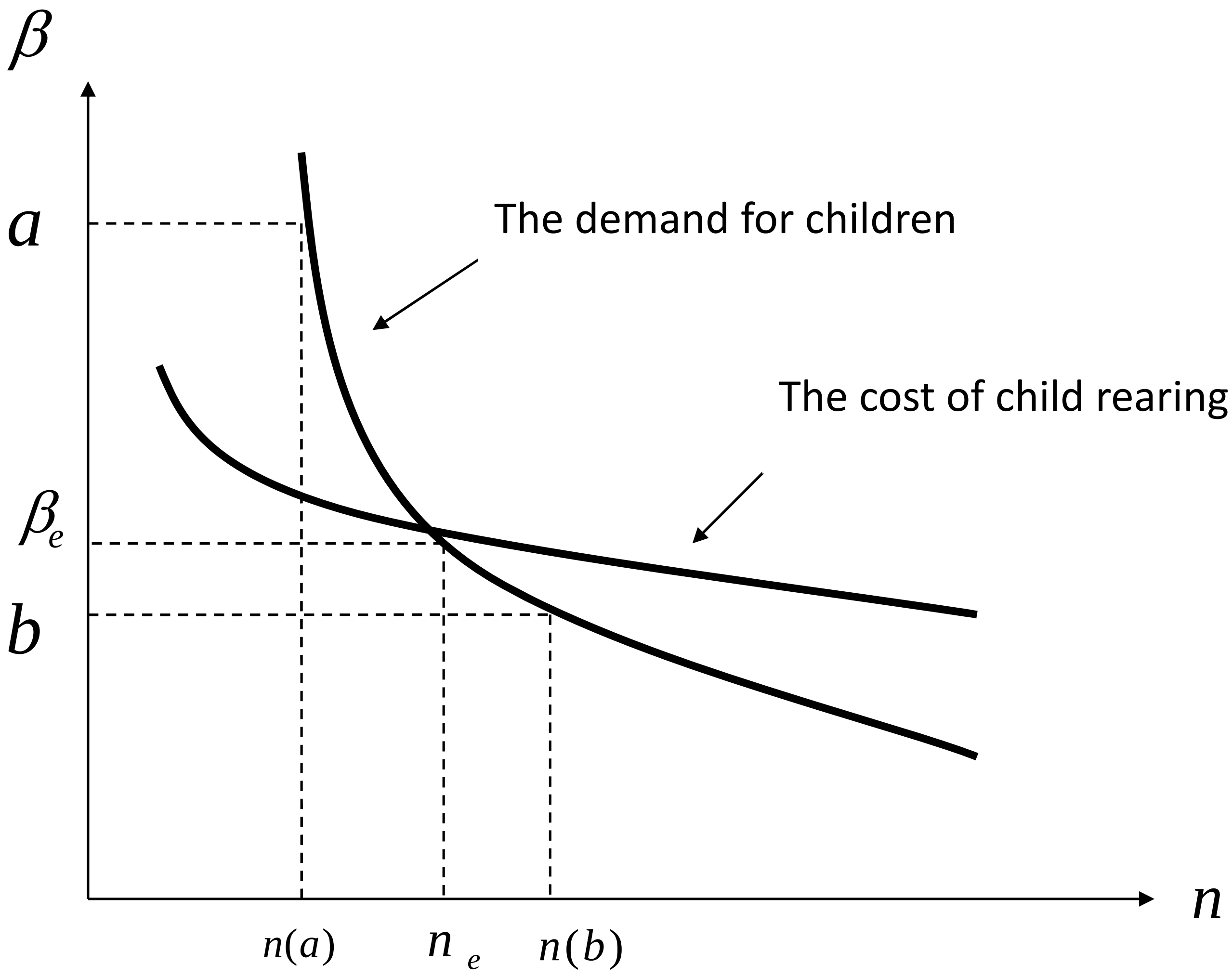



Figure 12: Stability of equilibrium

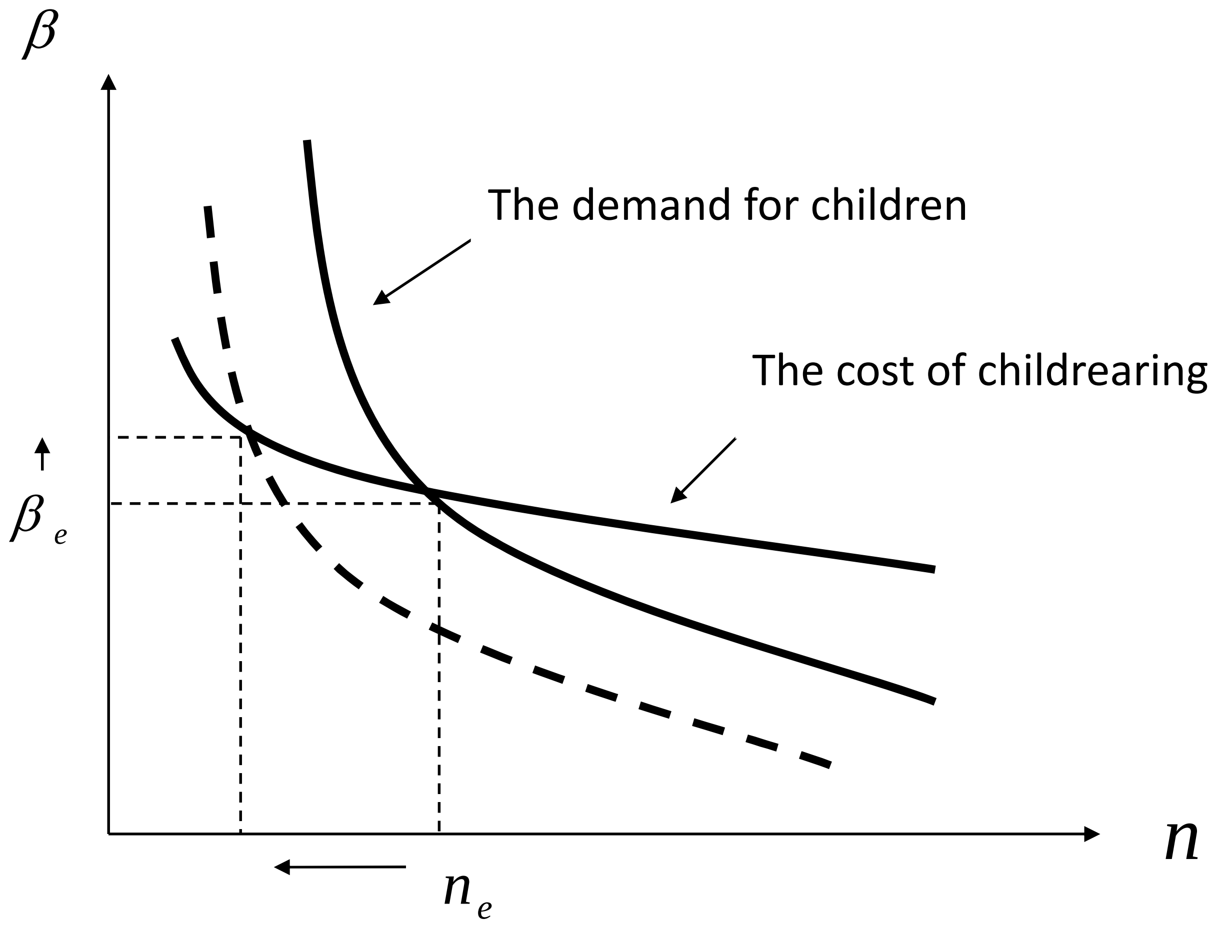



Figure 13: Effect of wage increase on birth rate

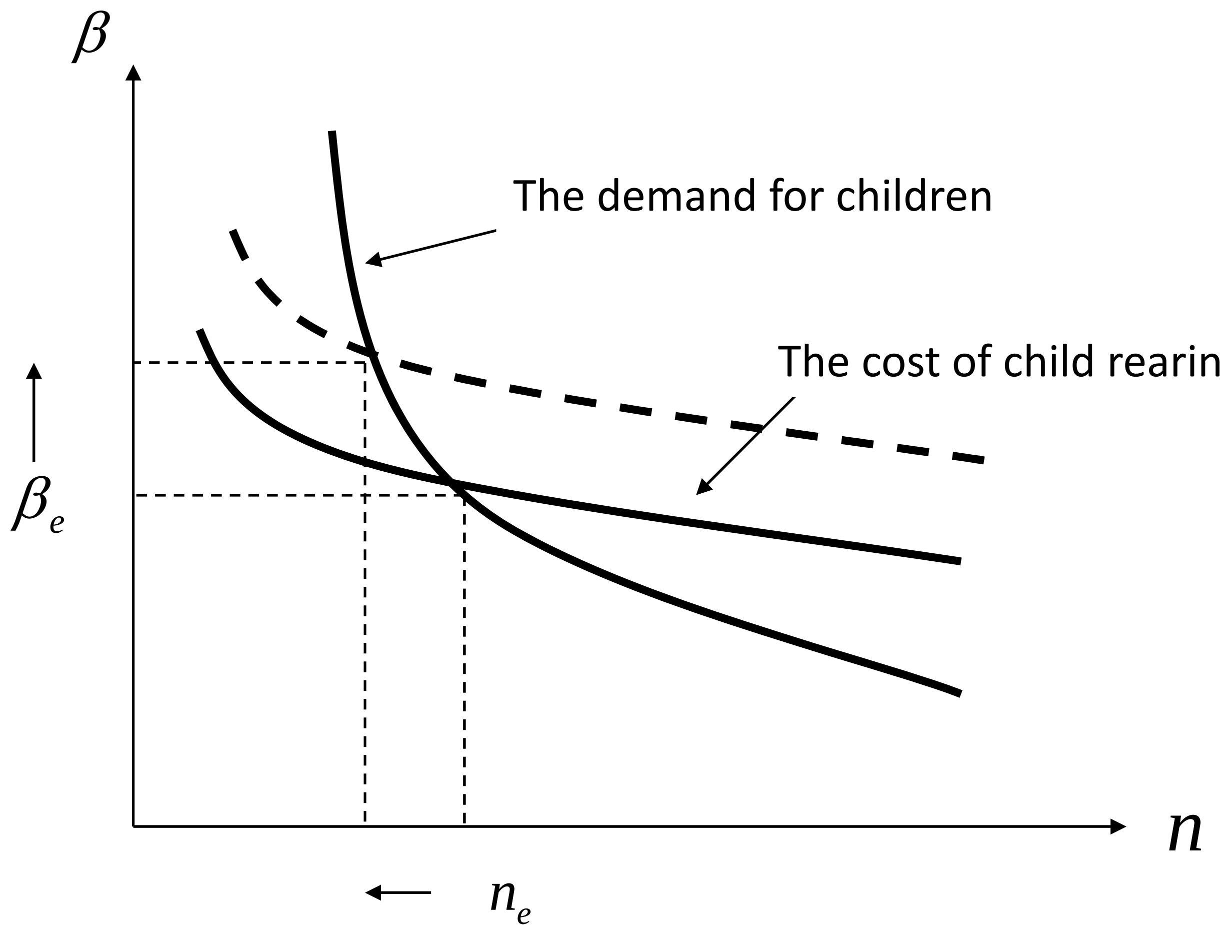



Figure 14: Effect of decrease in number of households on birth rate