Compatibility and the Product Life Cycle in Two-Sided Markets

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Abstract

We consider a case in which two competing suppliers of hardware devices and content each chooses whether to make its content compatible with the other’s device. Our main result is that the outcome of these choices depends upon whether the firms’ major source of profit lies in the sale of hardware devices or in royalties from the sale of content. If the hardware is the main source of profit then incompatibility is a dominant strategy. If royalties are the main source of profit then compatibility is the dominant strategy. Which of these situations attains is likely to change over the product life cycle. We add to the literature by showing the equilibrium structure of compatibility in a two-sided market.

KEYWORDS: two-sided markets, platform, compatibility, product life cycle
JEL Classification Number: D43, L13, L14, L15, L22
1 Introduction

We consider a duopoly model with two competing platforms each of whom chooses whether to make its content compatible with the rival’s hardware device. Our main result is that the outcome of these choices depends on the stage of the product life cycle in the industry. Specifically, at the early stage of the product life cycle firms derive most of their profit from the sale of the hardware devices because few customers will have yet purchased them. At this early stage, making content compatible with the rival’s hardware device is unprofitable because it increases the attractiveness of the rival’s hardware device by wider availability of content. Therefore, incompatibility is the dominant strategy. But at a mature stage of the product life cycle, many customers already own hardware devices and royalties from the sale of content will have become the major profit center. At the mature stage, making content compatible with the rival’s hardware device is the dominant strategy because it supports a wider demand for the content the firm is selling. These results are derived in a stylized model in which the subgame-perfect equilibrium attains one of five possible configurations depending on parameters. Asymmetric equilibria arise when the platform is at an intermediate stage of the product life cycle and there is a large difference of market shares for installed customer base, so that the large firm chooses to make its content incompatible with the other’s hardware while the small rival does the opposite.

The sort of example we have in mind is that of an electronic book reader such as the Amazon kindle or Apple iPad tablet computer and the electronic books that are viewed on these devices. Amazon has chosen to make its electronic “kindle” books compatible with the iPad tablet computer. While Apple has so far not made its “iBooks” compatible with the Amazon kindle. Other instances of hardware devices and related content include audio or video players that can but need not be outfitted with a converter, translator, emulator, adapter, or gizmo supplied by a rival that makes the content of the rival compatible with the device.

2 Related Literature

The seminal research on compatibility with rival’s products focused on network effects.¹ For example the value of a telephone network to any user becomes

greater, the larger the number of other users. If subscribers to the telephone service of one supplier are able to call subscribers to rival telephone services, then subscriptions to either service are more valued. These network effects are called “direct network externalities”. Katz and Shapiro (1985) showed that direct network externalities incline the largest incumbent supplier against making its service compatible with the service of rivals. Crémer et al. (2000) adapted the model of Katz and Shapiro (1985) to incorporate locked-in, installed base customers who will not switch to other firms. They showed that a firm with a large installed base prefers to reduce the degree of compatibility with its smaller rivals. Malueg and Schwartz (2006) extended the model of Crémer et al. (2000) to the case in which the largest firm faces any number of small rivals and the incompatibility could produce tipping (all new customers join one network). They showed that a firm with a large installed base is likely to gain from compatibility in growing industries, but not in relatively mature industries. All of this literature studies the issue of compatibility between competitors in settings with single-sided platforms and direct network externalities.

The research on product compatibility most closely related to our model focuses on the strategic aspects of compatibility with complementary products. Matutes and Regibeau (1988, 1992) and Economides (1989) provided the mix and match framework for analysis of compatibility with rival complementary products in a single-sided market without network externalities. The main concern in those papers, and in this one, is not the size of the network that confers consumer benefits when the supplier chooses compatibility, but the wider availability of complementary products. These network effects are called “indirect network externalities”. Our model extends the concept of compatibility with rival complementary products to the case of competing platforms in two-sided markets. Rochet and Tirole (2003) and Armstrong (2006), among others, developed the basic analysis of two-sided markets, defined as any in which a supplier’s sales to one set of demanders also affects its income from a different set of demanders.\footnote{See also the seminal papers by Caillaud and Jullien (2003), Rochet and Tirole (2006), and Armstrong and Wright (2007). For surveys on two-sided markets, see Roson (2005) and Rysman (2009).} So for instance an internet platform provider supplies services to its users, and the number of such users also affects the sale of internet advertising by the same platform provider. In our framework, the competing platforms supply hardware devices (e-book readers) and the number of users of the devices also affects the income from royalties collected from original copyright holders of the content supplied for use with the devices.
There have been a number of papers that address the issue of compatibility in two-sided markets. Doganoglu and Wright (2006) examined the effect of consumers’ multi-homing on compatibility between networks. They showed that platforms have an insufficient incentive to choose compatibility in the presence of multi-homing. Miao (2009) developed the model of two-sided markets and showed that a monopoly platform has an incentive to foreclose competition in the complementary market by committing to maintain incompatibility. Casadesus-Masanell and Ruiz-Aliseda (2008) examined the case of duopoly. They show that incompatibility gives rise to the situation in which a dominant platform earns more than under compatibility. Viecens (2011) examined platform competition under the assumption that the degree of application compatibility is an exogenous parameter and showed that a small firm will always demand that application compatibility be enforced but a large firm never will.

In sum, contrary to the previous literature that focuses on competition given the structure of compatibility, the unique theoretical contribution of our work lies in showing the equilibrium structure of application compatibility in a two-sided market with indirect network externalities. To the best of our knowledge, this paper is the first one which shows the interesting point that the equilibrium structure of compatibility changes over the product life cycle, and we will provide the example of the market for electronic book readers such as those of Amazon and Apple which matches this theoretical result.

The remainder of this paper is organized as follows. In Section 3, we develop the simplest possible model of compatibility decisions by platforms in the two-sided markets. Section 4 presents the equilibrium outcomes of subgames and comparative static analyses. Section 5 derives the subgame-perfect equilibrium. Section 6 extends the basic model and Section 7 summarizes the key findings and concludes the paper by providing directions for further research.

3 Model

3.1 Platforms

Suppose that there are two platforms, \( i = 1, 2 \). Each platform provides a hardware device \( i \) at a price \( p_i \) and operates its marketplace \( i \) that distributes content for its own hardware device. We consider that there are two kinds of content, \( i = 1, 2 \), and that content \( i \) is exclusively supplied to marketplace \( i \) at price \( \rho_i \) by the independent content provider and is designed to work only on
hardware device $i$.\textsuperscript{3} We suppose that each platform chooses whether to make its content compatible with the other’s hardware device. In this paper we use the term application compatibility, which means that a platform provides a converter (also known as translator, emulator, or adapter) which enables users of other hardware devices to use content (application software) that it supplies in its marketplace.\textsuperscript{4} We assume that each unit of content provides an equal benefit for any consumer, and that the price of a unit of content is the same for any content, $\rho_i = \rho$ ($i = 1, 2$). Each platform charges a royalty rate $r$ ($0 \leq r \leq 1$) for each unit of content sold at its marketplace. Platforms also earn revenue by selling their hardware devices to consumers. We assume that each unit of content provides an equal benefit for any consumer, and that the price of a unit of content is the same for any content, $\rho_i = \rho$ ($i = 1, 2$). Each platform charges a royalty rate $r$ ($0 \leq r \leq 1$) for each unit of content sold at its marketplace. We assume that each unit of content provides an equal benefit for any consumer, and that the price of a unit of content is the same for any content, $\rho_i = \rho$ ($i = 1, 2$). Each platform charges a royalty rate $r$ ($0 \leq r \leq 1$) for each unit of content sold at its marketplace.

Platforms also earn revenue by selling their hardware devices to consumers. We treat the price of content $\rho$ and royalty rate $r$ as exogenous variables in our basic model. In Section 6, we relax this assumption and endogenize the determination of content price $\rho$, and show that our basic insights continue to hold. The assumption of a common fixed royalty rate is somewhat unusual, but is true for the electronic book (ebook) industry. The owners of digital contents, in other words book publishers, adopt the agency pricing model. They set the prices of their ebooks, and distributors, like Amazon and Apple, get a fixed percentage fee from the publishers for every book sold (about 30%) (See Jiang, 2012; Knowledge@Wharton, 2012).

The profit function of platform $i$ is given by

$$\pi_i = r \rho d_i + p_i D_i, \quad i = 1, 2,$$

where $d_i$ denotes the demand for content supplied at marketplace $i$ and $D_i$ denotes the demand for the hardware device. We assume a constant marginal cost of supplying hardware devices which is normalized to zero.

### 3.2 Consumers

We consider two groups of consumers: existing customers (the installed customer base of each platform), and new customers. The mass of consumers is normalized to 1. Denote by $\alpha$ the fraction of the consumers who are existing customers and by $1 - \alpha$ the remaining fraction who are new customers.

\textsuperscript{3}It is assumed that contents available on the two platforms are mutually exclusive. In reality, however, there are common contents supplied to several platforms (e.g. Amazon and Apple have a large overlap in their selection of e-books). Even when we relax this assumption and allow $n$ kinds of common contents between two platforms in addition to exclusive contents, we can demonstrate the results qualitatively similar to those described in the propositions in Section 5. Proof is available upon request.

\textsuperscript{4}Farrell and Saloner (1992) study the economics of converters in single-sided markets with direct network externalities.
Further stipulate that the fraction $\beta$ of the existing customers have adopted hardware device 1 and the remaining fraction $1 - \beta$ customers have adopted hardware device 2 in the previous period. These customers are the installed bases of the platforms. While the existing customers already own hardware devices and hence only demand content, new customers must buy both hardware devices and content. Platforms compete to add new customers to their installed bases.

To analyze the consumers’ choice of platform, we use a Hotelling model of product differentiation. The hardware devices are differentiated along the unit interval $[0, 1]$, with hardware 1 located at 0, and hardware 2 at 1. Each new customer buys one hardware device only. Ideal points of consumers are distributed uniformly on the unit interval with a unit density, and each consumer incurs a constant proportional disutility $t$ per unit length. We assume that the benefit derived from consumption of the hardware device (that is, the consumer’s stand-alone valuation for the hardware device) $v$ is large enough for every new customer to buy one hardware device. Denote by $B$ the utility that any consumer derives from a unit of content, which is assumed to be the same for any content and for any consumer, and satisfies the condition $B > \rho$. Thus, we consider that each consumer buys any usable content.

The utility function of a new customer who is located at $x$, buys a hardware device $i$, and uses its available contents is written as

$$u_i = N_i(B - \rho) + v - p_i - t|x - x_i|,$$

where $N_i$ is the amount of available content for hardware device $i$, and $x_i$ is the location of hardware device $i$. We will use the notation, $b = B - \rho$.

### 3.3 Game structure

We consider a two-stage game. In stage one, the two platforms independently and simultaneously choose between application compatibility and incompatibility. In stage two, platforms independently and simultaneously set their hardware prices $p_1$ and $p_2$. Then existing customers purchase content, and new customers purchase both hardware and content.

The model is a one-period and static game, but we focus on the equilibrium of compatibility choices of platforms in different stages of the product life cycle. We use $\alpha$ (the fraction of existing customers) as a measure of the product life cycle in the industry. If $\alpha$ is large, almost all customers have purchased hardware devices, and this industry is in the mature stage of the product life cycle. Otherwise, if $\alpha$ is small, the industry is in the introductory stage of the product life cycle.
4 Equilibrium and comparative statics

Given the compatibility decisions in stage one, it follows that there are four possible market structures. The first is one of incompatible platforms in which both platforms choose incompatibility, (IC, IC). The second is one of compatible platforms in which both platforms choose compatibility, (C, C). And the third and fourth are asymmetric market structures in which one platform chooses incompatibility and the other chooses compatibility, (IC, C) and (C, IC). For example, Figure 1 illustrates the asymmetric market structure in which firm 1 chooses incompatibility and firm 2 compatibility. In this section, we will derive the equilibrium prices, demands, profits, consumer surplus, and social surplus under the various market structures.

![Diagram of market structures](image)

Figure 1: Market structure in which firm 2 has chosen compatibility, but firm 1 has chosen incompatibility.

4.1 Incompatible platforms

In this case, for each hardware device, only a single content is available. The utility function of a new customer who is located at $x$ can be written as

$$u_i = b + v - p_i - t|x - x_i|, \quad (i = 1, 2).$$

Let $\hat{x}$ be the location of a new customer who is indifferent between the two hardware devices, and thus it equals the proportion of consumers who buy hardware 1:

$$u_1 = u_2 \Rightarrow \hat{x} = \frac{t - p_1 + p_2}{2t}.$$
Hence, the demand for hardware device $i$ is

$$D_i = \frac{(t - p_i + p_j)(1 - \alpha)}{2t} \quad (i = 1, 2, \ j \neq i).$$

Platform 1 maximizes its profit

$$\pi_1 = r\rho(D_1 + \alpha\beta) + p_1D_1 = \frac{(p_1 + r\rho)(t - p_1 + p_2)(1 - \alpha)}{2t} + \alpha\beta r\rho$$

with respect to its hardware price $p_1$. Similarly, platform 2 maximizes its profit

$$\pi_2 = r\rho(D_2 + \alpha(1 - \beta)) + p_2D_2 = \frac{(p_2 + r\rho)(t - p_2 + p_1)(1 - \alpha)}{2t} + \alpha(1 - \beta)r\rho$$

with respect to its hardware price $p_2$. Taking the first-order conditions and solving for prices, we have the equilibrium prices as follows:

$$p_1(\text{IC, IC}) = p_2(\text{IC, IC}) = t - r\rho.$$

We can derive the equilibrium demands, profits, consumer surplus, and social surplus as shown in Table 1.

### 4.2 Compatible platforms

When both platforms choose compatibility, all content is usable on either hardware device. In this case, the utility function of a new customer is

$$u_i = 2b + v - p_i - t|x - x_i|, \quad (i = 1, 2).$$

From this we can derive the demand for hardware as follows:

$$D_i = \frac{(t - p_i + p_j)(1 - \alpha)}{2t} \quad (i = 1, 2, \ j \neq i).$$

The profit function of platform $i$ is

$$\pi_i = r\rho + p_iD_i = \frac{p_i(t - p_i + p_j)(1 - \alpha)}{2t} + r\rho \quad (i = 1, 2, \ j \neq i).$$

From the first-order conditions for profit maximization, we have the equilibrium hardware prices, demands, profits, consumer surplus, and social surplus as shown in Table 1.
4.3 Incompatible-compatible platforms

When platform 1 chooses incompatibility and platform 2 chooses compatibility, all content is usable with hardware device 1, but only content supplied by firm 2 is usable with hardware device 2. In this case, the utility functions of new customers are

\[
\begin{align*}
  u_1 &= 2b + v - p_1 - tx \\
  u_2 &= b + v - p_2 - t(1 - x).
\end{align*}
\]

From these we can derive the demands for hardware devices as follows:

\[
D_1 = \frac{(b + t - p_1 + p_2)(1 - \alpha)}{2t}, \quad D_2 = \frac{(-b + t - p_1 + p_2)(1 - \alpha)}{2t}.
\]

The profit functions of the platforms are

\[
\begin{align*}
  \pi_1 &= r\rho(D_1 + \alpha\beta) + p_1D_1 = \frac{(p_1 + r\rho)(b + t - p_1 + p_2)(1 - \alpha)}{2t} + \alpha\beta r\rho, \\
  \pi_2 &= r\rho + p_2D_2 = \frac{p_2(-b + t - p_1 + p_2)(1 - \alpha)}{2t} + r\rho.
\end{align*}
\]

From the first-order conditions for profit maximization, we have the equilibrium prices, demands, profits, consumer surplus, and social surplus as shown in Table 1. The equilibrium in the other asymmetric case is similar.

In this paper, we exclude cases in which all new customers choose the same hardware device ("tipping"). This requires us to assume that the premise of the following Lemma is true, that the hardware devices of the two firms are inherently differentiated.

**Lemma 1.** If the hardware of the two firms is sufficiently differentiated that \( t > \frac{(b + r\rho)}{3} \), then there exists an equilibrium in which both platforms have positive market shares for new customers.

**Proof.** Tipping is the situation in which the market structure is asymmetric and the incompatible platform captures all the new customers. Therefore, in order to exclude tipping it must hold that the market share of the incompatible platform is less than one:

\[
\frac{1}{2} + \frac{b + r\rho}{6t} < 1 \iff \frac{b + r\rho}{6t} < \frac{1}{2} \iff t > \frac{b + r\rho}{3}
\]
Table 1: Equilibrium price, demands, profits, consumer surplus, and social surplus.

<table>
<thead>
<tr>
<th>(IC, IC)</th>
<th>(C, C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p_1 )</td>
<td>( t - \frac{b + \rho}{2\rho} )</td>
</tr>
<tr>
<td>( p_2 )</td>
<td>( t - \frac{b + \rho}{2\rho} )</td>
</tr>
<tr>
<td>( D_1 )</td>
<td>( \frac{1}{2}(1 - \alpha) )</td>
</tr>
<tr>
<td>( D_2 )</td>
<td>( \frac{1}{2}(1 - \alpha) )</td>
</tr>
<tr>
<td>( \pi_1 )</td>
<td>( \frac{1}{4}(1 - \alpha) + \alpha \beta \rho )</td>
</tr>
<tr>
<td>( \pi_2 )</td>
<td>( \frac{1}{4}(1 - \alpha) + \alpha \beta \rho )</td>
</tr>
<tr>
<td>( CS )</td>
<td>( (v - \frac{1}{4}t)(1 - \alpha) + \frac{1}{2}(b + \rho) )</td>
</tr>
<tr>
<td>( SS )</td>
<td>( (v - \frac{1}{4}t)(1 - \alpha) + \frac{1}{2}(b + \rho) )</td>
</tr>
</tbody>
</table>
4.4 Comparative statics

We next compare the equilibrium prices, demands, profit, consumer surplus, and social surplus under the various market structures. First, we assert the following proposition regarding prices and demands.

**Proposition 1.** When the benefit of content is large enough to satisfy the condition \( b > 2 \rho \), then the equilibrium prices and demands are ordered as follows:

\[
\begin{align*}
    p_2(I_C, C) &= p_1(C, I_C) < p_i(I_C, I_C) < p_i(C, C) = p_2(C, I_C), \\
    D_2(I_C, C) &= D_1(C, I_C) < D_i(I_C, I_C) = D_i(C, C) < D_1(I_C, C) = D_2(C, I_C).
\end{align*}
\]

**Proof.** These results can be easily shown from the results in Table 1.

This proposition shows the natural result that when consumers get much benefit from content, the relative price and demand for a hardware device increases as it has more usable content compared to competing devices: \( p_1(C, I_C) < p_2(I_C, C) \) and \( D_1(C, I_C) < D_2(C, I_C) \). It is also the case that the prices tend to be higher when both devices are compatible with the others’ content compared to the case in which both are incompatible: \( p_i(I_C, I_C) < p_i(C, C) \). That is, compatibility softens price competition. A similar result has been shown in the indirect network externalities literature (Schiff, 2003; Doganoglu and Wright, 2006; Miao, 2010). And Matutes and Regibeau (1988) and Economides (1989) showed that even in the absence of network externalities, when firms sell compatible components, cutting the hardware price will increase the demand for hybrid systems that use not only the one firm’s component but also the rival firm’s component (i.e., rival’s content). In this way, some benefit from increased demand accrues to the rival firm, which weakens the one firm’s incentive to cut its price. The logic behind our result is subtly different and may be given as follows. Consider the situation of incompatible platforms (I_C, I_C) and suppose that one platform cuts its hardware price. Then the demand for its hardware increases, which also increases the demand for its content. Because of the increase in revenue from both sides of the two-sided market (the revenue from selling its own hardware and the royalties from selling its content), a firm selling incompatible hardware has a strong incentive to cut its price. However, when the platforms are both compatible (C, C), and one of the firms cuts its hardware price, then the demand for its hardware increases but, in our model, the demand for its content does not increase. This is because of our assumption that any consumer purchases all available content for his hardware device. By this assumption, under compatible platforms, the total demand for either firm’s content is constant and equal to 1. Thus, the
firms have less incentive to price their hardware aggressively under compatible platforms (C, C) than they would do under incompatible platforms (IC, IC).

We find the same rank ordering for prices and demands under asymmetric market structures. This allows us to derive the relations \( p_1(C, IC)D_1(C, IC) < p_1(IC, IC)D_1(IC, IC) \) and \( p_1(C, C)D_1(C, C) < p_1(IC, C)D_1(IC, C) \). Thus we obtain the following corollary.

**Corollary 1.** Consider a single-sided market in which platforms do not charge royalties for content and earn revenue only by selling their hardware devices to consumers. Then, incompatibility is the dominant strategy for both platforms.

That is, if a platform cannot get profit from selling content in its market-place, then the only effect of compatibility is to make the rival platform more attractive to consumers. Thus in this single-sided market, it is the dominant strategy equilibrium for both platforms to choose incompatible platforms.

However, by comparing the profits of platforms summarized in Table 1, we have the following proposition.

**Proposition 2.** It follows that \( \pi_i(IC, IC) < \pi_i(C, C) \).

The similar result has been shown in Schiff (2003) and Doganoglu and Wright (2006). Then from Corollary 1 and Proposition 2 we can find that the equilibrium in the single-sided market, that is, (IC, IC), forms a prisoner’s dilemma situation and leads to lower profits for both platforms.

Next, when we compare the equilibrium consumer surplus under the various market structures, we have the following result.

**Proposition 3.** It follows that

If \( b > 7rp/5 \) and \( t > (b + rp)/3 \), then the consumer surplus is ordered as

(i) \( CS^N(IC, IC) < CS^N(IC, C) = CS^N(C, IC) < CS^N(C, C) \) for new customers, and

(ii) \( CS^E(IC, IC) < CS^E(IC, C) = CS^E(C, IC) < CS^E(C, C) \) for existing customers,

where \( CS^N \) denotes consumer surplus for new customers, and \( CS^E \) for existing customers.

**Proof.** See Appendix 1 for the proof.

While we find in Proposition 1 that compatibility leads to higher hardware device prices compared to incompatibility, it also increases the variety of available content. Thus some consumers are better off with compatibility,
while other consumers are worse off. As Proposition 3 part (ii) shows, existing customers are always better off when compatibility prevails, because they benefit from wider availability of content without paying higher hardware prices. What about the new customers? Proposition 3 part (i) shows that when the benefit of content is large enough, then compatible platforms provide more benefit for new consumers than do incompatible platforms.

Finally, we will compare the social surplus among the different market structures. We assert the following proposition.

**Proposition 4.** When the degree of differentiation is large enough to satisfy the condition \( t > \max\left(\frac{5(b + r\rho)}{18}, \frac{(b + r\rho)}{3}\right) \), then the equilibrium social surplus are ordered as follows:

\[
SS(\text{IC, IC}) < SS(\text{IC, C}) \leq SS(C, C), \quad SS(\text{IC, IC}) < SS(C, \text{IC}) \leq SS(C, C).
\]

**Proof.** See Appendix 2 for the proof. \(\square\)

From Propositions 3 and 4 we can find that the compatible platforms \((C, C)\) is optimal in the light of not only consumer surplus but also social surplus.

## 5 Subgame-perfect equilibrium

In the previous section, we examined the subgame equilibrium in stage two. We use the results of the analyses to derive the subgame-perfect equilibrium of the three-stage game using backward induction. To determine the equilibrium, we first compare the equilibrium profits under the various market structures in stage two. Using the equilibrium profits shown in Table 1, we have the following Lemma 2.

**Lemma 2.** It follows that

\[
\beta > \beta_1(\alpha) \iff \pi_1(\text{IC, IC}) > \pi_1(C, IC),
\]

\[
\beta > \beta_2(\alpha) \iff \pi_2(\text{IC, C}) > \pi_2(\text{IC, IC}),
\]

\[
\beta > \beta_3(\alpha) \iff \pi_1(\text{IC, C}) > \pi_1(\text{C, C}),
\]

\[
\beta > \beta_4(\alpha) \iff \pi_2(\text{C, C}) > \pi_2(\text{C, IC}),
\]

where

\[
\beta_1(\alpha) = \frac{6t(b + r\rho) - (b + r\rho)^2}{18t\rho} - \frac{6t(b - 2r\rho) - (b + r\rho)^2}{18t\rho \cdot \alpha},
\]

\[
\beta_2(\alpha) = -\frac{6t(b - 2r\rho) - (b + r\rho)^2}{18t\rho} + \frac{6t(b - 2r\rho) - (b + r\rho)^2}{18t\rho \cdot \alpha},
\]

13
β₃(α) = \frac{6t(b + rρ) + (b + rρ)^2}{18trρ} - \frac{6t(b - 2rρ) + (b + rρ)^2}{18trρ · α},

β₄(α) = -\frac{6t(b - 2rρ) + (b + rρ)^2}{18trρ} + \frac{6t(b - 2rρ) + (b + rρ)^2}{18trρ · α},

β₁ + β₂ = 1, \text{ and } β₃ + β₄ = 1.

Platforms choose between compatibility and incompatibility in stage one of the game. From Lemma 2, depending on parameter values, we can identify four possible subgame-perfect equilibria as follows.

**Lemma 3.** It follows that

(i) (IC, IC) is the equilibrium when β₁(α) ≤ β ≤ β₂(α),

(ii) (IC, C) is the equilibrium when β₂(α) ≤ β and β₃(α) ≤ β,

(iii) (C, IC) is the equilibrium when β ≤ β₁(α) and β ≤ β₄(α),

(iv) (C, C) is the equilibrium when β₄(α) ≤ β ≤ β₃(α).

These results can be immediately shown from Lemma 2.

In Lemma 2, we can see that the graph of βᵢ (i = 1, 2, 3, 4) is dependent on the sign of the second term in each equation, that is, the signs of 6t(b - 2rρ) + (b + rρ)^2 and 6t(b - 2rρ) - (b + rρ)^2. Therefore, there are three cases;

(Case 1) \quad 6t(b - 2rρ) + (b + rρ)^2 > 0 and 6t(b - 2rρ) - (b + rρ)^2 > 0
\quad \iff b > 2rρ \text{ and } t > \max\left(\frac{(b + rρ)^2}{6(b - 2rρ)}, \frac{b + rρ}{3}\right),

(Case 2) \quad 6t(b - 2rρ) + (b + rρ)^2 > 0 and 6t(b - 2rρ) - (b + rρ)^2 < 0
\quad \iff rρ < b < 5rρ \text{ and } \frac{b + rρ}{3} < t < \frac{(b + rρ)^2}{6(b - 2rρ)},

(Case 3) \quad 6t(b - 2rρ) + (b + rρ)^2 < 0 and 6t(b - 2rρ) - (b + rρ)^2 < 0
\quad \iff b < 2rρ \text{ and } t > \max\left(\frac{(b + rρ)^2}{6(b - 2rρ)}, \frac{b + rρ}{3}\right).

When we use Lemma 3 in each case, we can describe the partition of the parameter space (α, β). The parameter space (α, β) is divided into several regions which have different subgame perfect equilibria. The following proposition gives the partition of the parameter space in Case 1.

**Proposition 5.** Consider Case 1, that is, both the benefit of content and the degree of hardware differentiation are large enough to satisfy the conditions b > 2rρ and t > \max\left((b + rρ)^2/(6|b - 2rρ|), (b + rρ)/3\right). Then depending on parameter values, the equilibrium market structure is either: (A) incompatible platforms; (B) compatible platforms; (C) incompatible-compatible platforms;
(D) compatible-incompatible platforms; or (E) multiple equilibria (IC,C) and (C,IC). Figure 2 illustrates the possibilities in $(\alpha, \beta)$ space.

Figure 2: Equilibrium Market Structure of Case 1.

The result can be immediately shown from Lemma 3. Suppose that $\alpha$ is relatively small (as in the introductory stage of the product life cycle) and there are many new customers who need to purchase hardware devices. Then it is of great importance to platforms to sell hardware devices to new customers and add them to their installed bases. In such a situation, compatibility is unprofitable because it increases the attractiveness of the rival’s hardware device by providing wider availability of content. Therefore, the unique equilibrium is one in which platforms are incompatible (IC, IC).

In contrast, consider the situation in which $\alpha$ is large (as in the mature stage of the product life cycle) so that almost all customers are in the installed base of some platform. They already have hardware devices and only want to purchase content. Then a platform obtains most of its profit by selling content in its marketplace and obtaining royalties. In such a situation, compatibility is more profitable than incompatibility. Thus, the unique equilibrium is one in which platforms are compatible (C, C).

Suppose that $\alpha$ is at an intermediate level (i.e., in the growth stage of the product life cycle). First, consider the situation in which there is a large difference between the market shares of two firms. Then for the small platform (with a small installed customer base) the profitable choice is to make its content compatible with the larger rival’s hardware device and garner royalties.
from the sale of content to the installed base of the rival firm. However, for the large platform (with a large installed customer base), the profitable choice is to make its content incompatible with the hardware device of the smaller rival, and maintain its share of the hardware market by preserving the wider availability of content on its hardware devices, as shown in regions C and D of figure 2.

Next, consider the situation in which there is little difference between the market shares of the two firms. Then there are multiple equilibria as shown in region E in Figure 2. The market has the property of a game of “chicken”. That is, either firm will make more profit by making its content incompatible with the rival’s hardware device, given that the rival does the opposite.

Figure 2 illustrates the possibilities of equilibrium in \((\alpha, \beta)\) space for given values of the other parameters \(b\) and \(t\). Performing comparative statics with respect to the parameters \(b\) and \(t\) gives the following two corollaries.

**Corollary 2.** As the parameter \(b\) becomes larger, every function \(\beta_i\) shifts to the right. Therefore, the larger benefit from content increases the region A (equilibria with only incompatible platforms) and decreases the region B (equilibria with only compatible platforms).

*Proof.* See Appendix 3 for the proof.

The intuition for this result is the following. When the benefit of content becomes large, new customers are likely to choose the hardware device which has more usable content. Then choosing compatibility is not profitable for any firm because it provides more usable content for the rival’s hardware device.

**Corollary 3.** As the parameter \(t\) increases, the functions \(\beta_1\) and \(\beta_3\) get closer and the functions \(\beta_2\) and \(\beta_4\) also get closer. Therefore, the larger degree of hardware differentiation increases the region A (equilibria with only incompatible platforms) and the region B (equilibria with only compatible platforms).

*Proof.* See Appendix 4 for the proof.

The intuition behind the increase of region B is given as follows. The larger is \(t\), the smaller is the profit loss from hardware device derived from the compatibility decision. So given the profit gain from larger demand of content from the compatibility decision, it is more likely to use the strategy of compatibility. Thus, the region B increases.

Next, we will consider Case 2. The following proposition gives the partition of the parameter space in Case 2.
Proposition 6. Consider Case 2, that is, both the benefit of content and the degree of hardware differentiation are at an intermediate level that satisfies the conditions $r\rho < b < 5r\rho$ and $(b + r\rho)/3 < t < (b + r\rho)^2/(6|b - 2r\rho|)$. Then, depending on parameter values, the equilibrium market structure is either: (A) there are multiple equilibria (IC, C) and (C, IC); (B) compatible platforms (C, C); (C) incompatible-compatible platforms (IC, C); or (D) compatible-incompatible platforms (C, IC). Figure 3 illustrates these possibilities in $(\alpha, \beta)$ space.

Figure 3: Equilibrium Market Structure of Case 2.

They can be immediately shown from Lemma 3. The difference from Proposition 5 is that (IC, IC) is not a possible equilibrium when the degree of hardware differentiation is small. The intuition for this result is the following. When the degree of hardware differentiation is relatively small, there is keen competition in hardware pricing, which reduces the profit from selling hardware. Then platforms are more likely to make their content compatible with the rival’s hardware device to garner royalties by selling content to the rival’s installed base. Hence, even in the introductory stage of the product life cycle, incompatible platforms (IC, IC) is not an equilibrium market structure.

Finally, consider Case 3. The following proposition gives the partition of the parameter space in Case 3.

Proposition 7. Consider Case 3, that is, the benefit of content is small enough to satisfy the condition $b < 2r\rho$ and the degree of hardware differentiation is large enough to satisfy the condition $t > \text{Max}((b + r\rho)^2/(6|b - 2r\rho|), (b + r\rho)/3)$. 

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Then the unique equilibrium market structure is one with compatible platforms (C, C). Figure 4 illustrates the possibilities in (α, β) space.

![Equilibrium Market Structure of Case 3](image)

Figure 4: Equilibrium Market Structure of Case 3.

This can be immediately shown from Lemma 3. When the royalty from selling a unit of content \( r \rho \) is larger than one half of the marginal benefit from content \( b \), and the degree of hardware differentiation \( t \) is relatively large, making content compatible with the rival’s hardware device has little adverse effect on the firm’s own sale of hardware devices. In such situation, platforms are likely to make their content compatible with the rival’s hardware device, and profit by garnering royalties from the sale of content to the rival’s installed base. Thus the unique equilibrium is one with compatible platforms (C, C).

6 Model extension

In this section, we relax our assumption that the content price \( \rho \) is exogenously fixed to show the robustness of the main propositions obtained in our basic model. We allow the determination of content price by content providers.

We consider a three-stage game. That is, in stage one, the two platforms independently and simultaneously choose between application compatibility and incompatibility. In stage two, platforms independently and simultaneously set their hardware prices. Then the new consumers purchase hardware device. In stage three, content providers independently and simultaneously set their content prices. Then the customers purchase content. Following Church and
Gandal (2000), we suppose that the benefit consumers receive from consuming \( N \) varieties of content is represented by \( w(N) \). Here we assume that \( w(1) < w(2) \) and \( w(2) - w(1) < w(1) \). Solving the subgame in stage three it can be easily seen that when a single content is available, content price is set at \( \rho = w(1) \). When two content are available, content price is set at \( \rho = w(2) - w(1) \).

Using these results and solving the game backward, we have the equilibrium profits of platforms under the four different possible market structures in stage two. Comparing those equilibrium profits, we can show that Lemma 2 in Section 5 is changed as follows.

**Lemma 2a**. It follows that

\[
\beta > \beta_1(\alpha) \iff \pi_1(\text{IC, IC}) > \pi_1(\text{C, IC}),
\]

\[
\beta > \beta_2(\alpha) \iff \pi_2(\text{IC, C}) > \pi_2(\text{IC, IC}),
\]

\[
\beta > \beta_3(\alpha) \iff \pi_1(\text{IC, C}) > \pi_1(\text{C, C}),
\]

\[
\beta > \beta_4(\alpha) \iff \pi_2(\text{C, C}) > \pi_2(\text{C, IC}),
\]

where

\[
\beta_1(\alpha) = \frac{6t \{(1 - r)W + r\Delta w\} - \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w},
\]

\[
\beta_2(\alpha) = -\frac{6t \{W - rw(2)\} - \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w},
\]

\[
\beta_3(\alpha) = \frac{6t \{(1 - r)W + r\Delta w\} + \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w} - \frac{6t \{W - rw(2)\} + \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w},
\]

\[
\beta_4(\alpha) = -\frac{6t \{W - rw(2)\} + \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w} + \frac{6t \{W - rw(2)\} + \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w}.
\]

\( \beta_1 + \beta_2 = 1, \beta_3 + \beta_4 = 1, W \equiv 2w(1) - w(2), \text{ and } \Delta w \equiv w(2) - w(1). \)

Using these four functions, we can separate three cases similarly in section 5, and in each case we have the following propositions which give the partition of the parameter space.
Proposition 5a. Suppose that both the benefit of content and the degree of hardware differentiation are large enough to satisfy the conditions $0 < r < \frac{2w(1) - w(2)}{w(2)}$ and $t > \frac{(1 - r)W + r\Delta w}{6\{W - rw(2)\}}$. Then, depending on parameter values, the equilibrium market structure is either: (A) incompatible platforms; (B) compatible platforms; (C) incompatible-compatible platforms; (D) compatible-incompatible platforms; or (E) multiple equilibria (IC, C) and (C, IC). Figure 2 illustrates the possibilities in $(\alpha, \beta)$ space.

Proof. Proof is available upon request.

Proposition 6a. Suppose that both the benefit of content and the degree of hardware differentiation are at an intermediate level that satisfies the conditions,

(i) $0 < r < \frac{2w(1) - w(2)}{w(2)}$ and $6 < t < \frac{(1 - r)W + r\Delta w}{6\{W - rw(2)\}}$, or

(ii) $\frac{2w(1) - w(2)}{w(2)} < r < \frac{2w(1) - w(2)}{3w(1) - w(2)}$ and $\frac{(1 - r)W + r\Delta w}{6} < t < \frac{(1 - r)W + r\Delta w}{6\{rw(2) - W\}}$.

Then, depending on parameter values, the equilibrium market structure is either: (A) there are multiple equilibria (IC, C) and (C, IC); (B) compatible platforms (C, C); (C) incompatible-compatible platforms (IC, C); or (D) compatible-incompatible platforms (C, IC). Figure 3 illustrates these possibilities in $(\alpha, \beta)$ space.

Proof. Proof is available upon request.

Proposition 7a. Suppose that the benefit of content is small enough to satisfy the condition $\frac{2w(1) - w(2)}{w(2)} < r < 1$ and the degree of hardware differentiation is large enough to satisfy the conditions $t > \max\{\frac{(1 - r)W + r\Delta w}{6\{rw(2) - W\}}, \frac{(1 - r)W + r\Delta w}{6}\}$. Then the unique equilibrium market structure is one with compatible platforms (C, C). Figure 4 illustrates the possibilities in $(\alpha, \beta)$ space.

Proof. Proof is available upon request.

By comparing Propositions 5 to 7 and Propositions 5a to 7a, we see that even when content price is determined endogenously, the equilibrium outcomes are qualitatively similar.
7 Discussion and conclusion

The purpose of this paper was to understand how the product life cycle affects the compatibility strategy of platforms in two-sided markets. To this end, we proposed a duopoly model of platforms in which the market comprises two segments of consumers: the installed base of customers and the new customers. We have developed a stylized model in which two competing suppliers of platform hardware devices and content each chooses whether to make its content compatible with the other’s hardware device. The sort of platform business we have in mind is the market for electronic books. In this market, Amazon has adopted the principle of “application compatibility”, meaning that e-books purchased in the Amazon store can be read not only on the “Kindle” hardware devices that it sells but also on the electronic readers sold by its rival Apple, the “iPad”, “iPod”, and “iPhone”. But Apple has chosen to make the electronic books that it sells incompatible with the Amazon Kindle reader. That is, electronic books purchased at the Apple iTunes store (so-called “iBooks”) can only be read on tablet-like devices also supplied by Apple. They cannot be read on the Amazon Kindle. The main question that our stylized model is intended to address is why these rivals might have chosen opposite strategies with regard to compatibility with one another’s hardware devices of the content each supplies.

This situation can be explained by using our results. Since the ebook market is now in the growth stage of the product life cycle, \( \alpha \) is at an intermediate level, and the benefit of content (ebook) \( b \) is relatively large. In this case, when there is a large difference between the market shares of the two firms, asymmetric equilibria arise, corresponding to regions C and D of Figure 2 and Figure 3, based on Proposition 5 and Proposition 6. In fact, Apple has a large installed base of iPad, iPod, and iPhone users, and there is a great difference between the market shares of hardware devices of the two firms, Apple and Amazon. For the small platform (Amazon) the profitable choice is to make its content compatible with the larger rival’s hardware device and gain royalties from expanding the sale of content to the installed base of the rival firm. However, for the large platform (Apple), the profitable choice is to make its content incompatible with the hardware device of the smaller rival, and maintain its share of the hardware market by preserving the wider availability of content on its hardware devices. These roughly correspond to the relative positions of Amazon and Apple in the market for electronic books and the hardware devices for reading them.

In developing our model, we made a few simplifying assumptions. We assumed that the royalty rate was predetermined and thus unaffected by firms’
compatibility choices. We also assumed that there was no possibility of one platform fully capturing the market (tipping). A more complete analysis of compatibility decisions which includes these issues remains for future work.

8 Appendix

Appendix 1: Proof of Proposition 3
From the results in Table 1, we have

\[ CS(C,C) - CS(IC,C) = \left( v + 2b - \frac{5}{4}t \right) - \left( \frac{(b + rp)^2}{36t} + v + \frac{3}{2}b - \frac{5}{4}t + \frac{rp}{2} \right) \]

\[ = \frac{b}{2} - \frac{rp}{2} - \frac{(b + rp)^2}{36t} \]

\[ = \frac{18t(b - rp) - (b + rp)^2}{36t} \]

\[ > \frac{18 \cdot \frac{b + rp}{3} \cdot (b - rp) - (b + rp)^2}{36t} \]

\[ = \frac{1}{36t} (b + rp)(5b - 7rp) > 0 \quad (\iff b > \frac{7}{5}r) \]

\[ CS(IC,C) - CS(IC,IC) = \left( \frac{(b + rp)^2}{36t} + v + \frac{3}{2}b - \frac{5}{4}t + \frac{rp}{2} \right) - \left( v + b - \frac{5}{4}t + rp \right) \]

\[ = \frac{b}{2} - \frac{rp}{2} + \frac{(b + rp)^2}{36t} > 0 \quad (\iff b > rp) \]

\[ \square \]

Appendix 2: Proof of Proposition 4
From the results in Table 1, we have

\[ SS(C,C) - SS(IC,C) \]

\[ = \left[ \left( v - \frac{1}{4}t \right)(1 - \alpha) + 2(b + \rho) \right] \]

\[ - \left[ \left( \frac{5(b + \rho)^2}{36t} + v - \frac{1}{4}t \right)(1 - \alpha) + (b + \rho) \left( \frac{3}{2} + \alpha \left( \beta - \frac{1}{2} \right) \right) \right] \]

\[ = - \frac{5(b + \rho)^2}{36t} (1 - \alpha) + (b + \rho) \left( \frac{1}{2} - \alpha \left( \beta - \frac{1}{2} \right) \right) \]

(This is decreasing function in \( \beta \).)
\[ > - \frac{5(b + \rho)^2}{36t}(1 - \alpha) + (b + \rho) \left( \frac{1}{2} - \frac{1}{2} \alpha \right) (\beta < 1) \]
\[ = \left( \frac{1}{2} - \frac{5(b + \rho)}{36t} \right)(b + \rho)(1 - \alpha) > 0 \quad (\iff \ t > \frac{5}{18}(b + \rho)) \]

\[ SS(\text{IC}, \text{C}) - SS(\text{IC}, \text{IC}) \]
\[ = \left[ \left( \frac{5(b + \rho)^2}{36t} + v - \frac{1}{4}t \right)(1 - \alpha) + (b + \rho) \left( \frac{3}{2} + \alpha \left( \beta - \frac{1}{2} \right) \right) \right] \]
\[ - \left[ \left( v - \frac{1}{4}t \right)(1 - \alpha) + b + \rho \right] \]
\[ = \frac{5(b + \rho)^2}{36t}(1 - \alpha) + (b + \rho) \left( \frac{1}{2} + \alpha \left( \beta - \frac{1}{2} \right) \right) \]
\[ = \frac{5(b + \rho)^2}{36t}(1 - \alpha) + (b + \rho) \left( \frac{1}{2}(1 - \alpha) + \alpha \beta \right) > 0 \]

\[ \square \]

**Appendix 3: Proof of Corollary 2**

Here we define \( \tilde{\alpha} \) and \( \hat{\alpha} \) as

\[ \beta_1 = \beta_2 \iff \alpha = \frac{(b + r\rho)^2 - 3t(2b - 4r\rho)}{(b + r\rho)^2 - 3t(2b - r\rho)} \equiv \tilde{\alpha}, \]
\[ \beta_3 = \beta_4 \iff \alpha = \frac{(b + r\rho)^2 + 3t(2b - 4r\rho)}{(b + r\rho)^2 + 3t(2b - r\rho)} \equiv \hat{\alpha}. \]

The derivatives of \( \tilde{\alpha} \) and \( \hat{\alpha} \) with respect to \( b \) are given by

\[ \frac{\partial \tilde{\alpha}}{\partial b} = \frac{18tr\rho(3t - b - r\rho)}{\{b^2 - 2b(3t - r\rho) + r\rho(3t + r\rho)\}^2} > 0 \quad \text{from Lemma 1, and} \]
\[ \frac{\partial \hat{\alpha}}{\partial b} = \frac{18tr\rho(3t + b + r\rho)}{\{b^2 + 2b(3t + r\rho) - r\rho(3t - r\rho)\}^2} > 0. \]

\[ \square \]

**Appendix 4: Proof of Corollary 3**

The derivatives of \( \tilde{\alpha} \) and \( \hat{\alpha} \) with respect to \( t \) are:

\[ \frac{\partial \tilde{\alpha}}{\partial t} = \frac{9r\rho(b + \rho)^2}{\{(b + \rho)^2 - 3t(2b - r\rho)\}^2} > 0, \quad \text{and} \]
\[ \frac{\partial \hat{\alpha}}{\partial t} = -\frac{9r\rho(b + \rho)^2}{\{(b + \rho)^2 + 3t(2b - r\rho)\}^2} < 0. \]

\[ \square \]
9 References


