$\mathcal{Q}$-anonymous social welfare relations on infinite utility streams*

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Abstract

This paper studies a class of social welfare relations (SWRs) on the set of infinite utility streams. In particular, we examine the SWRs satisfying $\mathcal{Q}$-Anonymity, an impartiality axiom stronger than Finite Anonymity, as well as Strong Pareto and a certain equity axiom. First, we characterize the extension of the generalized Lorenz SWR by combining $\mathcal{Q}$-Anonymity with Strong Pareto and Pigou-Dalton Equity. Second, we replace Pigou-Dalton Equity with Hammond Equity for characterizing the extended leximin SWR. Third, we give an alternative characterization of the extended utilitarian SWR by substituting Incremental Equity for Pigou-Dalton Equity.

Keywords: $\mathcal{Q}$-Anonymity, Intergenerational equity, Generalized Lorenz criterion, Leximin principle, Utilitarianism, Simplified criterion

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1 Introduction

In exploring a social welfare relation (SWR) on infinite utility streams, Strong Pareto and Finite Anonymity are usually employed as basic principles.[1] These axioms lead

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[1]See, for example, the seminal work by Diamond (1965).

us to the infinite-horizon variant of Suppes (1966) and Sen's (1970) grading principle (Svensson 1980; Asheim et al. 2001). The Suppes-Sen grading principle formalizes a quite intuitive but fairly weak value judgment. To establish further evaluation criteria beyond the Suppes-Sen grading principle, recent contributions in the literature propose and characterize (in terms of a subrelation) several plausible SWRs that satisfy certain desirable properties in addition to Strong Pareto and Finite Anonymity. Basu and Mitra (2007) characterize the utilitarian SWR by adding the informational invariance axiom called Partial Unit Comparability. In Asheim and Tungodden (2004), they impose the equity axiom called Hammond Equity and characterize the two versions of leximin principle with one of two alternative preference-continuity axioms as well as Hammond Equity. The weaker equity axiom, which we call Pigou-Dalton Equity, is examined by Bossert et al. (2007). They characterize the generalized Lorenz criterion with Pigou-Dalton Equity and the leximin principle with Hammond Equity.[2]

Instead of adding auxiliary axioms, Mitra and Basu (2007) strengthen a notion of anonymity beyond Finite Anonymity in a strongly Paretian SWR.[3] They propose the extended anonymity, called $\mathcal{Q}$-Anonymity, that is defined by a *group* of *cyclic* permutations which contains all finite permutations. While it is well-known that the anonymity axiom defined by all possible permutations comes in conflict with Strong Pareto (van Liedekerke 1995; Lauwers 1997a), Mitra and Basu show that any (and only) group(s) of cyclic permutations can define the anonymity axiom consistent with a strongly Paretian SWR. Using $\mathcal{Q}$-Anonymity and Strong Pareto, Banerjee (2006a) characterizes the extended Suppes-Sen grading principle. Furthermore, he also characterizes the extended utilitarian SWR, called $\mathcal{Q}$-utilitarian SWR, by strengthening Finite Anonymity to $\mathcal{Q}$-Anonymity (with a certain restriction) in the list of the axioms in Basu and Mitra's (2007) characterization of the utilitarian SWR.

The principal task of this paper is to examine whether $\mathcal{Q}$-Anonymity is consistent with a strongly Paretian and equitable SWR. The results obtained in this paper are positive. We define the extensions of the generalized Lorenz and the leximin SWRs, called $\mathcal{Q}$-generalized Lorenz criterion and $\mathcal{Q}$-leximin principle, in the same way as Mitra and Basu (2007) and Banerjee (2006a) have done for the Suppes-Sen grading principle. Then, we show that each of the $\mathcal{Q}$-generalized Lorenz criterion and the $\mathcal{Q}$-leximin principle is well-defined as a SWR and that the former is characterized in terms of Strong Pareto, $\mathcal{Q}$-Anonymity and Pigou-Dalton Equity and the latter by replacing Pigou-Dalton Equity with Hammond Equity. In this paper, we also provide a new characterization of the $\mathcal{Q}$-utilitarian SWR by using the equity axiom called Incremental

[2]The logical relationship among the two versions of the leximin principle characterized by Asheim and Tungodden (2004) and the leximin principle in Bossert et al. (2007) is the same as the one among the catching-up and the overtaking SWRs and the utilitarian SWR in Basu and Mitra (2007).

[3]See also Lauwers (1997b; 2006) and Fleurbaey and Michel (2003).

Equity. This characterization result is established without the restriction employed by Banerjee (2006a) on the permissible permutations considered in $\mathcal{Q}$-Anonymity. We show that the $\mathcal{Q}$-utilitarian SWR is well-defined even without Banerjee's restriction on the permissible permutations. The direct counterpart of Banerjee's (2006a) characterization result is also established.

The next section introduces notation and definitions. Section 3 presents axioms and establishes the characterizations of the $\mathcal{Q}$-generalized Lorenz criterion and the $\mathcal{Q}$-leximin principle. Section 4 provides two characterizations of the $\mathcal{Q}$-utilitarian SWR. Section 5 concludes. All proofs are relegated to Appendix.

2 Notation and definitions

Let $\mathbb{R}$ (resp. $\mathbb{R}_{++}$) be the set of all (resp. all positive) real numbers and $\mathbb{N}$ the set of all positive integers. Let $X = \mathbb{R}^{\mathbb{N}}$ be the set of all utility streams $\boldsymbol{x} = (x_1, x_2, \dots)$. For all $i \in \mathbb{N}$, x_i is interpreted as the utility level of the ith generation. For all $\boldsymbol{x} \in X$ and all $n \in \mathbb{N}$, we write $\boldsymbol{x}^{-n} = (x_1, \dots, x_n)$ and $\boldsymbol{x}^{+n} = (x_{n+1}, x_{n+2}, \dots)$. For all $\boldsymbol{x} \in X$ and all $n \in \mathbb{N}$, $\left(x^{-n}_{(1)}, \dots, x^{-n}_{(n)}\right)$ denotes a rank-ordered permutation of $\boldsymbol{x}^{-n}$ such that $x^{-n}_{(1)} \leq \cdots \leq x^{-n}_{(n)}$, ties being broken arbitrarily. For all $\boldsymbol{x}, \boldsymbol{y} \in X$, we write $\boldsymbol{x} \geqslant \boldsymbol{y}$ if $x_i \geq y_i$ for all $i \in \mathbb{N}$, and $\boldsymbol{x} > \boldsymbol{y}$ if $\boldsymbol{x} \geqslant \boldsymbol{y}$ and $\boldsymbol{x} \neq \boldsymbol{y}$. Negation of a statement is indicated by the symbol $\neg$.

A SWR is a reflexive and transitive binary relation, $\succsim$, on X. Let $\succ$ (resp. $\sim$) be the asymmetric (resp. symmetric) part of $\succsim$. A SWR $\succsim_A$ is a subrelation of a SWR $\succsim_B$ if (i) $\boldsymbol{x}, \boldsymbol{y} \in X$, $\boldsymbol{x} \succsim_A \boldsymbol{y} \Rightarrow \boldsymbol{x} \succsim_B \boldsymbol{y}$ and (ii) $\boldsymbol{x}, \boldsymbol{y} \in X$, $\boldsymbol{x} \succ_A \boldsymbol{y} \Rightarrow \boldsymbol{x} \succ_B \boldsymbol{y}$.

A permutation matrix is an infinite matrix $\boldsymbol{P} = (p_{ij})_{i,j \in \mathbb{N}}$ such that (i) for all $i \in \mathbb{N}$, there exists $j(i) \in \mathbb{N}$ such that $p_{ij(i)} = 1$ and $p_{ij} = 0$ for all $j \neq j(i)$; and (ii) for all $j \in \mathbb{N}$, there exists $i(j) \in \mathbb{N}$ such that $p_{i(j)j} = 1$ and $p_{ij} = 0$ for all $i \neq i(j)$. Let $\mathcal{P}$ be the set of all permutation matrices. Note that, for all $\boldsymbol{x} \in X$ and all $\boldsymbol{P} \in \mathcal{P}$, the product $\boldsymbol{P}\boldsymbol{x} = (Px_1, Px_2, \dots)$ belongs to X, where $Px_i = \sum_{k \in \mathbb{N}} p_{ik} x_k$ for all $i \in \mathbb{N}$. For any $\boldsymbol{P} \in \mathcal{P}$, let $\boldsymbol{P}'$ be the inverse of $\boldsymbol{P}$ satisfying $\boldsymbol{P}'\boldsymbol{P} = \boldsymbol{P}\boldsymbol{P}' = \boldsymbol{I}$, where $\boldsymbol{I}$ is the infinite identity matrix.[4] For all $\boldsymbol{P} = (p_{ij})_{i,j \in \mathbb{N}} \in \mathcal{P}$ and all $n \in \mathbb{N}$, let $\boldsymbol{P}(n)$ denote the $n \times n$ matrix $(p_{ij})_{i,j \in \{1, \dots, n\}}$. A matrix $\boldsymbol{P} = (p_{ij})_{i,j \in \mathbb{N}} \in \mathcal{P}$ is a finite permutation matrix if there exists $n \in \mathbb{N}$ such that $p_{ii} = 1$ for all $i > n$. Let $\mathcal{F}$ be the set of all finite permutation matrices.

Let $\boldsymbol{e^i}$ be the stream in X with 1 in the ith place and 0 elsewhere, i.e. the ith unit vector in X. A permutation $\boldsymbol{P} \in \mathcal{P}$ is said to be *cyclic* if, for any $i \in \mathbb{N}$, there exists $k(i) \in \mathbb{N}$ such that $\boldsymbol{P}^{k(i)}\boldsymbol{e^i} = \boldsymbol{e^i}$, where $\boldsymbol{P}^{k(i)}$ denotes the $k(i)$ times iterated multiplication of $\boldsymbol{P}$. Note that if $\boldsymbol{P} = (p_{ij})_{i,j \in \mathbb{N}}$ is cyclic then, for all $i \in \mathbb{N}$, there exists

[4]For any $\boldsymbol{P}, \boldsymbol{Q} \in \mathcal{P}$, the product $\boldsymbol{P}\boldsymbol{Q}$ is defined by $(r_{ij})_{i,j \in \mathbb{N}}$ with $r_{ij} = \sum_{k \in \mathbb{N}} p_{ik} q_{kj}$.

a $k'(i)$-dimensional ($k'(i) \leq k(i)$) vector $(i_1, \ldots, i_{k'(i)})$ of *distinct* positive integers with $i_1 = i$ and $p_{i_2 i_1} = \cdots = p_{i_{k'(i)} i_{k'(i)-1}} = p_{i_1 i_{k'(i)}} = 1$. While $\mathcal{P}$ and $\mathcal{F}$ define a *group* with respect to the matrix multiplication, a special class of cyclic permutations does not (e.g. the class of all cyclic permutations).[5]

3 Strong impartiality and consequentialist equity

We examine the possibility of a strongly Paretian and equitable SWR that reflects impartiality stronger than Finite Anonymity. We begin with Strong Pareto and the extended anonymity called $\mathcal{Q}$-Anonymity. In what follows, let $\mathcal{Q}$ be some fixed group of cyclic permutations with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$.

Strong Pareto (SP): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if $\boldsymbol{x} > \boldsymbol{y}$, then $\boldsymbol{x} \succ \boldsymbol{y}$.

$\mathcal{Q}$-Anonymity (QA): For all $\boldsymbol{x} \in X$ and all $\boldsymbol{P} \in \mathcal{Q}$, $\boldsymbol{Px} \sim \boldsymbol{x}$.

In the case of $\mathcal{Q} = \mathcal{F}$, $\mathcal{F}$-Anonymity is equivalent to Finite Anonymity (**FA**). When $\mathcal{Q}$ is the class of *fixed step permutations* $\mathcal{Q}_{\text{fix}} = \{\boldsymbol{P} \in \mathcal{P}$: there exists $k \in \mathbb{N}$ such that, for each $n \in \mathbb{N}$, $\boldsymbol{P}(nk)$ is a finite dimensional permutation matrix$\}$, $\mathcal{Q}_{\text{fix}}$-Anonymity corresponds to Fixed Step Anonymity in Lauwers (1997b).[6]

The following equity axioms formalize the transfer principle due to Pigou (1912) and Dalton (1920) and the stronger equity principle by Hammond (1976).

Pigou-Dalton Equity (PDE): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if there exist $i, j \in \mathbb{N}$ and $\epsilon \in \mathbb{R}_{++}$ such that $x_i = y_i + \epsilon \leq y_j - \epsilon = x_j$ and $x_k = y_k$ for all $k \in \mathbb{N} \setminus \{i, j\}$, then $\boldsymbol{x} \succ \boldsymbol{y}$.

Hammond Equity (HE): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if there exist $i, j \in \mathbb{N}$ such that $y_i < x_i \leq x_j < y_j$ and $x_k = y_k$ for all $k \in \mathbb{N} \setminus \{i, j\}$, then $\boldsymbol{x} \succ \boldsymbol{y}$.

Both two axioms are widely used in the extensive literature on social choice theory, and we omit a detailed explanation for the sake of brevity.[7]

We introduce two SWRs satisfying **SP**, **FA** and **PDE** (and **HE**). For all $n \in \mathbb{N}$, let $\succsim_G^n$ be the finite-horizon generalized Lorenz SWR defined on $\mathbb{R}^n$: for all $\boldsymbol{x}^{-n}, \boldsymbol{y}^{-n} \in \mathbb{R}^n$, $\boldsymbol{x}^{-n} \succsim_G^n \boldsymbol{y}^{-n}$ iff $\sum_{i=1}^k x_{(i)}^{-n} \geq \sum_{i=1}^k y_{(i)}^{-n}$ for all $k \in \{1, \ldots, n\}$, and let $\succsim_L^n$ be the finite-horizon leximin SWR on $\mathbb{R}^n$: for all $\boldsymbol{x}^{-n}, \boldsymbol{y}^{-n} \in \mathbb{R}^n$, $\boldsymbol{x}^{-n} \succsim_L^n \boldsymbol{y}^{-n}$ iff $\boldsymbol{x}^{-n}$ is a permutation of $\boldsymbol{y}^{-n}$, or there exists $m \in \{1, \ldots, n\}$ such that $x_{(i)}^{-n} = y_{(i)}^{-n}$ for all

[5] For any $\mathcal{Q} \subseteq \mathcal{P}$, $\mathcal{Q}$ is said to define a group w.r.t. the matrix multiplication if (i) for all $\boldsymbol{P}, \boldsymbol{Q} \in \mathcal{Q}$, $\boldsymbol{PQ} \in \mathcal{Q}$, (ii) there exists $\boldsymbol{I} \in \mathcal{Q}$ such that for all $\boldsymbol{P} \in \mathcal{Q}$, $\boldsymbol{IP} = \boldsymbol{PI} = \boldsymbol{P}$, (iii) for all $\boldsymbol{P} \in \mathcal{Q}$, there exists $\boldsymbol{P}' \in \mathcal{Q}$ such that $\boldsymbol{P}'\boldsymbol{P} = \boldsymbol{P}\boldsymbol{P}' = \boldsymbol{I}$, and (iv) for all $\boldsymbol{P}, \boldsymbol{Q}, \boldsymbol{R} \in \mathcal{Q}$, $(\boldsymbol{PQ})\boldsymbol{R} = \boldsymbol{P}(\boldsymbol{QR})$.

[6] See also Fleurbaey and Michel (2003), Banerjee (2006a), Lauwers (2006), and Mitra and Basu (2007).

[7] See d'Aspremont and Gevers (2002) and Bossert and Weymark (2004) as well as Asheim and Tungodden (2004) and Bossert et al. (2007). A weaker version of **HE** is proposed by Asheim and Tungodden (2005) under the name Hammond Equity for the Future.

$i < m$ and $x_{(m)}^{-n} > y_{(m)}^{-n}$. The generalized Lorenz and the leximin SWRs, denoted $\succsim_G$ and $\succsim_L$ respectively, are defined by: for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_G \boldsymbol{y} \Leftrightarrow \text{there exists } n \in \mathbb{N} \text{ such that } \boldsymbol{x}^{-n} \succsim_G^n \boldsymbol{y}^{-n} \text{ and } \boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}; \tag{1}$$

$$\boldsymbol{x} \succsim_L \boldsymbol{y} \Leftrightarrow \text{there exists } n \in \mathbb{N} \text{ such that } \boldsymbol{x}^{-n} \succsim_L^n \boldsymbol{y}^{-n} \text{ and } \boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}. \tag{2}$$

Bossert et al. (2007) show that the class of all SWRs that include $\succsim_G$ (resp. $\succsim_L$) as a subrelation is characterized by **SP**, **FA**, and **PDE** (resp. **HE**) (see Table 1 in Sect. 4).

We now extend the SWRs $\succsim_G$ and $\succsim_L$ to satisfy **QA**. For any SWR $\succsim$ on X, define the $\mathcal{Q}$-*closure* of $\succsim$, denoted $\succsim_Q$, as follows:[8] for all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_Q \boldsymbol{y} \Leftrightarrow \text{there exists } \boldsymbol{P} \in \mathcal{Q} \text{ such that } \boldsymbol{P}\boldsymbol{x} \succsim \boldsymbol{y}. \tag{3}$$

Let $\succsim_{GQ}$ (resp. $\succsim_{LQ}$) denote the $\mathcal{Q}$-closure of $\succsim_G$ (resp. $\succsim_L$). We call $\succsim_{GQ}$ $\mathcal{Q}$-*generalized Lorenz criterion* and $\succsim_{LQ}$ $\mathcal{Q}$-*leximin principle*. Each of $\succsim_{GQ}$ and $\succsim_{LQ}$ is well-defined as a SWR on X (see Lemma 1 in Appendix).

The following theorems identify the SWRs satisfying **SP**, **QA** and the equity axiom(s).

Theorem 1. *Let $\mathcal{Q}$ be a group of cyclic permutations with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$. Then, a SWR $\succsim$ on X satisfies **SP**, **QA**, and **PDE** if and only if $\succsim_{GQ}$ is a subrelation of $\succsim$.*

Theorem 2. *Let $\mathcal{Q}$ be a group of cyclic permutations with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$. Then, a SWR $\succsim$ on X satisfies **SP**, **QA**, and **HE** if and only if $\succsim_{LQ}$ is a subrelation of $\succsim$.*

As discussed by Basu and Mitra (2007) and Banerjee (2006a), Theorem 1 (resp. 2) tells that $\succsim_{GQ}$ (resp. $\succsim_{LQ}$) is the minimum element w.r.t. set inclusion among all the SWRs satisfying the axioms.

4 $\mathcal{Q}$-utilitarian SWR and 2-generation conflicts

In this section, we generalize Banerjee's (2006a) $\mathcal{Q}$-closure of the utilitarian SWR that is originally defined by $\mathcal{Q}$ with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{Q}_{\text{fix}}$. As shown by Lauwers (2006), $\mathcal{Q}_{\text{fix}}$ is not maximal (w.r.t. set inclusion) within the groups of cyclic permutations. We reformulate the $\mathcal{Q}$-closure of the utilitarian SWR for $\mathcal{Q}$ with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$ and provide two characterizations of it with (i) an equity axiom or (ii) an invariance property.

Let $\succsim_U^n$ denote the finite-horizon utilitarian SWR defined on $\mathbb{R}^n$: for all $\boldsymbol{x}^{-n}, \boldsymbol{y}^{-n} \in \mathbb{R}^n$, $\boldsymbol{x}^{-n} \succsim_U^n \boldsymbol{y}^{-n} \Leftrightarrow \sum_{i=1}^n x_i \geq \sum_{i=1}^n y_i$. The utilitarian SWR $\succsim_U$ is defined as: for

[8] The term "$\mathcal{Q}$-closure" is suggested by a referee of this journal. The $\mathcal{Q}$-closure of the Suppes-Sen SWR and the $\mathcal{Q}_{\text{fix}}$-closure of the utilitarian SWR are proposed by Mitra and Basu (2007) and Banerjee (2006a) respectively. See also Lauwers (1997b) and Fleurbaey and Michel (2003) for other $\mathcal{Q}_{\text{fix}}$-anonymous SWRs.

all $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim_U \boldsymbol{y} \Leftrightarrow \text{there exists } n \in \mathbb{N} \text{ such that } \boldsymbol{x}^{-n} \succsim_U^n \boldsymbol{y}^{-n} \text{ and } \boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}. \tag{4}$$

Let $\succsim_{UQ}$ denote the $\mathcal{Q}$-closure of $\succsim_U$. We call $\succsim_{UQ}$ *$\mathcal{Q}$-utilitarian relation*. The relation $\succsim_{UQ}$ is well-defined as a SWR on X (see Lemma 1 in Appendix).

The following axiom deals with 2-generation conflicts similar to **PDE** and **HE**.[9]

Incremental Equity (IE): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, if there exist $i, j \in \mathbb{N}$ such that $x_i - y_i = y_j - x_j$ and $x_k = y_k$ for all $k \in \mathbb{N} \setminus \{i, j\}$, then $\boldsymbol{x} \sim \boldsymbol{y}$.

IE asserts that, for any utility transfer between two generations, the pre-transfer utility stream and the post-transfer stream are equally good. In contrast to **PDE**, the value judgment by **IE** is made without any reference to the relative utility levels of the two generations. Note that **IE** implies **FA**.[10]

The next proposition characterizes $\succsim_U$ with **IE**.

Proposition 1. *A SWR $\succsim$ on X satisfies **SP** and **IE** if and only if $\succsim_U$ is a subrelation of $\succsim$.*

As shown below, using **IE** as the resolution to 2-generation conflicts, all SWRs satisfying **SP** and **QA** are solely those including $\succsim_{UQ}$ as a subrelation.

Theorem 3. *Let $\mathcal{Q}$ be a group of cyclic permutations with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$. Then, a SWR $\succsim$ on X satisfies **SP**, **QA**, and **IE** if and only if $\succsim_{UQ}$ is a subrelation of $\succsim$.*

Next, we introduce an invariance axiom employed by Banerjee (2006a).

Partial Translation Scale Invariance (PTSI): For all $\boldsymbol{x}, \boldsymbol{y} \in X$, all $\boldsymbol{\alpha} \in \mathbb{R}^{\mathbb{N}}$, and all $n \in \mathbb{N}$, if $\boldsymbol{x}^{+n} = \boldsymbol{y}^{+n}$ and $\boldsymbol{x} \succsim \boldsymbol{y}$, then $\boldsymbol{x} + \boldsymbol{\alpha} \succsim \boldsymbol{y} + \boldsymbol{\alpha}$.

PTSI is interpreted as saying that utility differences of finitely many generations are comparable but utility levels are not.[11]

The following result generalizes Banerjee's (2006a) characterization of $\succsim_{UQ}$ to the case of $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$.

Theorem 4. *Let $\mathcal{Q}$ be a group of cyclic permutations with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$. Then, a SWR $\succsim$ on X satisfies **SP**, **QA**, and **PTSI** if and only if $\succsim_{UQ}$ is a subrelation of $\succsim$.*

[9] **IE** was first proposed by Blackorby et al. (2002) in a finite population framework. See also d'Aspremont and Gevers (2002) and Lemma 2 in Asheim and Tungodden (2004).

[10] A permutation exchanging only two generations entails utility transfer between them, and any finite permutation is represented by a finite composition of permutations exchanging only two generations.

[11] For details, see d'Aspremont and Gevers (2002) and Bossert and Weymark (2004). In Basu and Mitra (2007), **PTSI** is called Partial Unit Comparability and is used to characterize $\succsim_U$ (see Table 1).

Table 1: Characterizations of $\mathcal{F}$-anonymous SWRs and $\mathcal{Q}$-closures

SWR	*efficiency*	*impartiality*		*equity*			*invariance*	characterization
(minimum)	**SP**	**FA**	**QA**	**PDE**	**HE**	**IE**	**PTSI**	
$\mathcal{Q}$-G-Lorenz	$\oplus$	+	$\oplus$	$\oplus$		–	–	Theorem 1
G-Lorenz	$\oplus$	$\oplus$		$\oplus$		–	–	Bossert et al. (2007)
$\mathcal{Q}$-leximin	$\oplus$	+	$\oplus$	+	$\oplus$	–	–	Theorem 2
Leximin	$\oplus$	$\oplus$		+	$\oplus$	–	–	Bossert et al. (2007)
$\mathcal{Q}$-utilitarian	$\oplus$	+	$\oplus$	–	–	$\oplus$	+	Theorem 3
	$\oplus$	+	$\oplus$	–	–	+	$\oplus$	Theorem 4/Banerjee (2006a)
Utilitarian	$\oplus$	+		–	–	$\oplus$	+	Proposition 1
	$\oplus$	$\oplus$		–	–	+	$\oplus$	Basu and Mitra (2007)

Table 1 summarizes our five characterizations and compares them with the related results by Banerjee (2006a), Basu and Mitra (2007), and Bossert et al. (2007). For each row in Table 1, the class of SWRs that includes the SWR in the first column as a subrelation is characterized by the axioms indicated by $\oplus$, and furthermore, each SWR in the class satisfies (resp. violates) the axioms indicated by + (resp. –).

5 Conclusion

We characterized the three classes of strongly Paretian and $\mathcal{Q}$-anonymous SWRs in terms of **PDE**, **HE** and **IE**. We also generalized Banerjee's (2006a) characterization of $\succsim_{UQ}$ to any group $\mathcal{Q}$ of cyclic permutations that includes finite permutations $\mathcal{F}$. For each theorem, it follows from Arrow's (1963) variant of Szpilrajn's (1930) lemma that there exists an ordering on X satisfying the axioms.[12] Thus, in Theorems 1 and 2, the escape route by Bossert et al. (2007) from the impossibilities of an equitable (and continuous or representable) ordering (Sakai 2003, 2006; Banerjee 2006b; Hara et al. 2008) is refined with stronger impartiality.

Theorems 1 to 4 are established for an arbitrary group $\mathcal{Q}$ of cyclic permutations with $\mathcal{F} \subseteq \mathcal{Q} \subseteq \mathcal{P}$, i.e. applicable even to maximal forms of **QA**. Lauwers (2006) shows that a maximal group of cyclic permutations involves an ultrafilter on the lattice of partitions of $\mathbb{N}$ and is nonconstructive. Consequently, the $\mathcal{Q}_{\text{fix}}$-closures, which have explicit descriptions, will be plausible extensions for a practical purpose, though $\mathcal{Q}_{\text{fix}}$-Anonymity is not the strongest anonymity consistent with a strongly Paretian SWR. An issue to be addressed in future work is to test the usefulness of our newly defined $\mathcal{Q}_{\text{fix}}$-closures on an intergenerational resource allocation model.

[12]However, these orderings cannot have an explicit description (Lauwers 2006; Zame 2007).

Appendix: Proofs

A finite-horizon SWR is a reflexive and transitive binary relation on a finite dimensional Euclidean space. We provide two lemmata which are stated for the SWRs defined in terms of the Pareto criterion and a sequence of finite-horizon SWRs satisfying the following properties common to $(\succsim_G^n)_{n\in\mathbb{N}}$, $(\succsim_L^n)_{n\in\mathbb{N}}$, and $(\succsim_U^n)_{n\in\mathbb{N}}$:

P1: For all $n \in \mathbb{N}$ and all $\boldsymbol{x}^{-n}, \boldsymbol{y}^{-n} \in \mathbb{R}^n$, if $x_i^{-n} \geq y_i^{-n}$ for all $i \in \{1, \ldots, n\}$ and $\boldsymbol{x}^{-n} \neq \boldsymbol{y}^{-n}$, then $\boldsymbol{x}^{-n} \succ^n \boldsymbol{y}^{-n}$;

P2: For all $n \in \mathbb{N}$, all $\boldsymbol{x}^{-n}, \boldsymbol{y}^{-n} \in \mathbb{R}^n$, and all $r \in \mathbb{R}$, if $\boldsymbol{x}^{-n} \succsim^n \boldsymbol{y}^{-n}$ then $(\boldsymbol{x}^{-n}, r) \succsim^{n+1} (\boldsymbol{y}^{-n}, r)$;

P3: For all $n \in \mathbb{N}$ and all $\boldsymbol{x}^{-n}, \boldsymbol{y}^{-n} \in \mathbb{R}^n$, if $\boldsymbol{x}^{-n}$ is a permutation of $\boldsymbol{y}^{-n}$, then $\boldsymbol{x}^{-n} \sim^n \boldsymbol{y}^{-n}$.

Let $(\succsim^n)_{n\in\mathbb{N}}$ be a sequence of finite-horizon SWRs (one for each time horizon $n \in \mathbb{N}$) satisfying **P1** to **P3**. Define the binary relation $\succsim$ on X by, for any $\boldsymbol{x}, \boldsymbol{y} \in X$,

$$\boldsymbol{x} \succsim \boldsymbol{y} \Leftrightarrow \text{there exists } n \in \mathbb{N} \text{ such that } \boldsymbol{x}^{-n} \succsim^n \boldsymbol{y}^{-n} \text{ and } \boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}.\text{[13]} \tag{5}$$

As shown in Claim 1 below, the relation $\succsim$ is a SWR on X. Recall that $\succsim_Q$ denotes the $\mathcal{Q}$-closure of $\succsim$.

We owe a lot to Banerjee's (2006a) work in establishing the following lemmata.

Lemma 1. *$\succsim_Q$ is reflexive and transitive.*

Proof of Lemma 1. The proof proceeds through two claims.

Claim 1. *The binary relation defined in (5) is reflexive and transitive.*

Reflexivity is obvious. To prove $\succsim$ is transitive, consider any $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$ with $\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{y} \succsim \boldsymbol{z}$. By (5), there exist $n, n' \in \mathbb{N}$ such that (i) $\boldsymbol{x}^{-n} \succsim^n \boldsymbol{y}^{-n}$ and $\boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}$ and (ii) $\boldsymbol{y}^{-n'} \succsim^{n'} \boldsymbol{z}^{-n'}$ and $\boldsymbol{y}^{+n'} \geqslant \boldsymbol{z}^{+n'}$. Let $\bar{n} = \max\{n, n'\}$. We only provide the proof for the case of $\bar{n} = n'$. By (i) and **P2**, $(\boldsymbol{x}^{-n}, y_{n+1}, \ldots, y_{\bar{n}}) \succsim^{\bar{n}} \boldsymbol{y}^{-\bar{n}}$. By **P1** and reflexivity, $\boldsymbol{x}^{-\bar{n}} \succsim^{\bar{n}} (\boldsymbol{x}^{-n}, y_{n+1}, \ldots, y_{\bar{n}})$, thus $\boldsymbol{x}^{-\bar{n}} \succsim^{\bar{n}} \boldsymbol{y}^{-\bar{n}}$ by transitivity. By transitivity, $\boldsymbol{x}^{-\bar{n}} \succsim^{\bar{n}} \boldsymbol{z}^{-\bar{n}}$. Since $\boldsymbol{x}^{+\bar{n}} \geqslant \boldsymbol{z}^{+\bar{n}}$, $\boldsymbol{x} \succsim \boldsymbol{z}$ by (5).

Claim 2. *The binary relation defined in (5) satisfies the following properties: for any $\boldsymbol{P} \in \mathcal{Q}$ and any $\boldsymbol{x}, \boldsymbol{y} \in X$, $\boldsymbol{x} \succsim \boldsymbol{y}$ if and only if $\boldsymbol{P}\boldsymbol{x} \succsim \boldsymbol{P}\boldsymbol{y}$.*

To prove the only-if-part, take any $\boldsymbol{P} = (p_{ij})_{i,j\in\mathbb{N}} \in \mathcal{Q}$ and let $\boldsymbol{x} \succsim \boldsymbol{y}$. By (5), there exists $n \in \mathbb{N}$ such that $\boldsymbol{x}^{-n} \succsim^n \boldsymbol{y}^{-n}$ and $\boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}$. Let $\bar{n} = \max\{i \in \mathbb{N} : p_{ij} = 1,\ j \in \{1, \ldots, n\}\}$. Define M by $M = \{i \in \{1, \ldots, \bar{n}\} : p_{ij} = 1 \text{ for } j \in \{n+1, n+2, \ldots\}\}$, and let $\tilde{i}$ denote the ith smallest number in M. Note

[13] d'Aspremont (2007) refers to this type of binary relation as *simplified criterion*.

that $\{x_1, \dots, x_n\} \subseteq \{Px_1, \dots, Px_{\bar{n}}\}$ for all $x \in X$. Define $\boldsymbol{w}^{-\bar{n}}, \boldsymbol{z}^{-\bar{n}} \in \mathbb{R}^{\bar{n}}$ by (i) $\boldsymbol{w}^{-n} = \boldsymbol{x}^{-n}$ and $\boldsymbol{z}^{-n} = \boldsymbol{y}^{-n}$ and (ii) $w_{n+i} = Px_{\bar{i}}$ and $z_{n+i} = Py_{\bar{i}}$ for all $i \in \{1, \dots, \bar{n} - n\}$. Since $\boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}$, $\boldsymbol{w}^{-\bar{n}} \succsim^{\bar{n}} \boldsymbol{z}^{-\bar{n}}$ by **P1** and **P2**. By **P3**, $\boldsymbol{w}^{-\bar{n}} \sim^{\bar{n}} \boldsymbol{P}\boldsymbol{x}^{-\bar{n}}$ and $\boldsymbol{z}^{-\bar{n}} \sim^{\bar{n}} \boldsymbol{P}\boldsymbol{y}^{-\bar{n}}$. By transitivity, $\boldsymbol{P}\boldsymbol{x}^{-\bar{n}} \succsim^{\bar{n}} \boldsymbol{P}\boldsymbol{y}^{-\bar{n}}$. Since $\boldsymbol{P}\boldsymbol{x}^{+\bar{n}} \geqslant \boldsymbol{P}\boldsymbol{y}^{+\bar{n}}$ holds, $\boldsymbol{P}\boldsymbol{x} \succsim \boldsymbol{P}\boldsymbol{y}$ by (5). The if-part is proved by using the inverse $\boldsymbol{P}'$ in the only-if-part.

We now prove Lemma 1. Since $\boldsymbol{I} \in \mathcal{Q}$ and $\succsim$ is reflexive, $\succsim_Q$ is reflexive. To prove $\succsim_Q$ is transitive, consider $\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{z} \in X$ with $\boldsymbol{x} \succsim_Q \boldsymbol{y}$ and $\boldsymbol{y} \succsim_Q \boldsymbol{z}$. By (3), there exist $\boldsymbol{P}, \boldsymbol{Q} \in \mathcal{Q}$ such that $\boldsymbol{P}\boldsymbol{x} \succsim \boldsymbol{y}$ and $\boldsymbol{Q}\boldsymbol{y} \succsim \boldsymbol{z}$. By Claim 2, $\boldsymbol{Q}(\boldsymbol{P}\boldsymbol{x}) \succsim \boldsymbol{Q}\boldsymbol{y}$. Since $\succsim$ is transitive (by Claim 1), $\boldsymbol{Q}(\boldsymbol{P}\boldsymbol{x}) \succsim \boldsymbol{z}$. Since $\boldsymbol{Q}\boldsymbol{P} \in \mathcal{Q}$, $\boldsymbol{x} \succsim_Q \boldsymbol{z}$ by (3). $\square$

Lemma 2. *For any $\boldsymbol{x}, \boldsymbol{y} \in X$,*

$$\begin{cases} \boldsymbol{x} \succ_Q \boldsymbol{y} \text{ if and only if there exists } \boldsymbol{P} \in \mathcal{Q} \text{ such that } \boldsymbol{P}\boldsymbol{x} \succ \boldsymbol{y}; & \text{(6a)} \\ \boldsymbol{x} \sim_Q \boldsymbol{y} \text{ if and only if there exists } \boldsymbol{P} \in \mathcal{Q} \text{ such that } \boldsymbol{P}\boldsymbol{x} \sim \boldsymbol{y}. & \text{(6b)} \end{cases}$$

Proof. First, we prove the only-if-part of (6a) by contradiction. Assume $\boldsymbol{x} \succ_Q \boldsymbol{y}$. By (3), there exists $\boldsymbol{P} \in \mathcal{Q}$ such that $\boldsymbol{P}\boldsymbol{x} \succsim \boldsymbol{y}$ and $\neg(\boldsymbol{Q}\boldsymbol{y} \succsim \boldsymbol{x})$ for all $\boldsymbol{Q} \in \mathcal{Q}$. Suppose that there is no $\boldsymbol{P} \in \mathcal{Q}$ such that $\boldsymbol{P}\boldsymbol{x} \succ \boldsymbol{y}$. Then, $\boldsymbol{P}\boldsymbol{x} \sim \boldsymbol{y}$. By Claim 2, $\boldsymbol{x} = \boldsymbol{P}'(\boldsymbol{P}\boldsymbol{x}) \sim \boldsymbol{P}'\boldsymbol{y}$, which contradicts that $\neg(\boldsymbol{Q}\boldsymbol{y} \succsim \boldsymbol{x})$ for all $\boldsymbol{Q} \in \mathcal{Q}$.

Next, to prove the if-part of (6a), assume that there exists $\boldsymbol{P} \in \mathcal{Q}$ such that $\boldsymbol{P}\boldsymbol{x} \succ \boldsymbol{y}$. By (3), $\boldsymbol{x} \succsim_Q \boldsymbol{y}$. We show $\neg(\boldsymbol{y} \succsim_Q \boldsymbol{x})$ by contradiction. Suppose $\boldsymbol{y} \succsim_Q \boldsymbol{x}$. By (3), there exists $\boldsymbol{Q} \in \mathcal{Q}$ such that $\boldsymbol{Q}\boldsymbol{y} \succsim \boldsymbol{x}$. By Claim 2, $\boldsymbol{P}(\boldsymbol{Q}\boldsymbol{y}) \succsim \boldsymbol{P}\boldsymbol{x}$. Let $\boldsymbol{R} = (r_{ij})_{i,j\in\mathbb{N}}$ denote the composition $\boldsymbol{P}\boldsymbol{Q}$. Note that $\boldsymbol{R} \in \mathcal{Q}$. By transitivity, $\boldsymbol{R}\boldsymbol{y} \succ \boldsymbol{y}$. By (5), **P1** and **P2**, two cases are now possible: (i) $\boldsymbol{R}\boldsymbol{y}^{-n} \succ^n \boldsymbol{y}^{-n}$ and $\boldsymbol{R}\boldsymbol{y}^{+n} = \boldsymbol{y}^{+n}$ for some $n \in \mathbb{N}$ or (ii) for some $n \in \mathbb{N}$, $\boldsymbol{R}\boldsymbol{y}^{-n} \succsim^n \boldsymbol{y}^{-n}$ and $\boldsymbol{R}\boldsymbol{y}^{+n} > \boldsymbol{y}^{+n}$ holds with an infinite number of strict inequality. First, consider the case (i). Take any $i \in \{1, \dots, n\}$. Since $\boldsymbol{R} \in \mathcal{Q}$, there exists a finite dimensional vector $(i_1, \dots, i_k)$ of distinct positive integers with $i_1 = i$ and $r_{i_2 i_1} = \dots = r_{i_k i_{k-1}} = r_{i_1 i_k} = 1$. Define h by $h = \min\{t \in \{2, \dots, k\} : i_t \in \{1, \dots, n\}\}$ if $\{i_2, \dots, i_k\} \cap \{1, \dots, n\} \neq \varnothing$; and $h = 1$ otherwise. Since $\boldsymbol{R}\boldsymbol{y}^{+n} = \boldsymbol{y}^{+n}$, $y_i = Ry_{i_2} = y_{i_2} = Ry_{i_3} = \dots = Ry_{i_h}$. Since the number i_h must differ for different $i \in \{1, \dots, n\}$ (otherwise, $\boldsymbol{R}$ fails to be a permutation matrix), $\{Ry_1, \dots, Ry_n\} = \{y_1, \dots, y_n\}$. By **P3**, $\boldsymbol{R}\boldsymbol{y}^{-n} \sim^n \boldsymbol{y}^{-n}$, a contradiction. Next, consider the case (ii). Note that the cardinality of $M = \{i \in \mathbb{N} : Ry_i > y_i\}$ is infinite. Take any $i \in M$. By the same argument as in the case (i), there exists a vector $(i_1, \dots, i_k)$ of distinct numbers with $i_1 = i$ and $r_{i_2 i_1} = \dots = r_{i_1 i_k} = 1$. Since $Ry_i > y_i$, $Ry_j < y_j$ holds for some $j \in \{i_2, \dots, i_k\}$. Applying the same argument to $i' \in M \backslash \{i_1, \dots, i_k\}$, we have $Ry_{j'} < y_{j'}$ for some j' with $j' \neq j$. Consequently, the cardinality of $\{j \in \mathbb{N} : Ry_j < y_j\}$ must be infinite, which contradicts that the cardinality of $\{j \in \mathbb{N} : Ry_j < y_j\}$ is at most $n-1$.

We now prove the equivalence assertion in (6b). First, assume $\boldsymbol{x} \sim_Q \boldsymbol{y}$. By (3), there exists $\boldsymbol{P} \in \mathcal{Q}$ such that $\boldsymbol{Px} \succsim \boldsymbol{y}$. If $\boldsymbol{Px} \succ \boldsymbol{y}$, then $\boldsymbol{x} \succ_Q \boldsymbol{y}$ by (6a), which contradicts $\boldsymbol{x} \sim_Q \boldsymbol{y}$. Thus, $\boldsymbol{Px} \sim \boldsymbol{y}$. Next, assume that there exists $\boldsymbol{P} \in \mathcal{Q}$ such that $\boldsymbol{Px} \sim \boldsymbol{y}$. By Claim 2, $\boldsymbol{x} = \boldsymbol{P}'(\boldsymbol{Px}) \sim \boldsymbol{P}'\boldsymbol{y}$. Then, by (3), $\boldsymbol{x} \succsim_Q \boldsymbol{y}$ and $\boldsymbol{y} \succsim_Q \boldsymbol{x}$, i.e., $\boldsymbol{x} \sim_Q \boldsymbol{y}$. □

We now provide the proofs of Theorem 1 and Proposition 1. Theorems 2 to 4 are proved by the same argument as in the proof of Theorem 1 by using the existing characterization results of $\succsim_L$ (by Bossert et al. (2007)) and $\succsim_U$ (by Basu and Mitra (2007)) and Proposition 1.

Proof of Theorem 1. The proof of the if-part is easy and omitted. Assume $\boldsymbol{x} \succ_{GQ} \boldsymbol{y}$. By (6a), there exists $\boldsymbol{P} \in \mathcal{Q}$ such that $\boldsymbol{Px} \succ_G \boldsymbol{y}$. Note that, from the characterization of $\succsim_G$ by Bossert et al. (2007), $\succsim_G$ is now a subrelation of $\succsim$. Thus, $\boldsymbol{Px} \succ \boldsymbol{y}$. By **QA**, $\boldsymbol{x} \sim \boldsymbol{Px}$, and $\boldsymbol{x} \succ \boldsymbol{y}$ by transitivity. Using (6b), the same argument proves that $\boldsymbol{x} \sim \boldsymbol{y}$ whenever $\boldsymbol{x} \sim_{GQ} \boldsymbol{y}$. □

Proof of Proposition 1. The if-part is straightforward and omitted. If $\boldsymbol{x} \sim_U \boldsymbol{y}$, then $\boldsymbol{x} \sim \boldsymbol{y}$ follows from (4) and the well-known implication of **IE** that $\boldsymbol{x} \sim \boldsymbol{y}$ holds whenever $\sum_{i=1}^n x_i = \sum_{i=1}^n y_i$ and $\boldsymbol{x}^{+n} = \boldsymbol{y}^{+n}$ for some $n \in \mathbb{N}$ (See Lemma 3 in Asheim and Tungodden (2004) and also Theorem 10 in Blackorby et al. (2002)). Next, assume $\boldsymbol{x} \succ_U \boldsymbol{y}$. By (4), we can find $n \in \mathbb{N}$ such that $\sum_{i=1}^n x_i > \sum_{i=1}^n y_i$ and $\boldsymbol{x}^{+n} \geqslant \boldsymbol{y}^{+n}$. Take $\boldsymbol{z} \in X$ such that $z_1 = y_1 + \sum_{i=1}^n (x_i - y_i)$, $z_i = y_i$ for all $i \in \{2, \dots, n\}$, and $z_j = x_j$ for all $j \in \{n+1, \dots\}$. By **SP** and the implication of **IE** stated above, $\boldsymbol{z} \succ \boldsymbol{y}$ and $\boldsymbol{x} \sim \boldsymbol{z}$, thus $\boldsymbol{x} \succ \boldsymbol{y}$. □

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