It Takes a Village - Network Effect of Child-rearing

Morihiro Yomogida
Graduate School of Economics
Hitotsubashi University*

Reiko Aoki†
Institute of Economic Research
Hitotsubashi University

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Abstract

We explore an economy where number of children is endogenously determined and the cost of raising children is determined by the total number of children in the economy. We show that number of children will be too small compared to the social optimum and that the network effect may magnify the decline of "birthrate". Our analysis demonstrates that public policies to increase birthrate must take this into account when determining subsidies.

*2-1 Naka, Kunitachi, Tokyo 186-8601, Japan. E-mail: myom@econ.hit-u.ac.jp

†2-1 Naka, Kinitachi, Tokyo 186-8603, Japan. E-mail: aokirei@ier.hit-u.ac.jp

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1 Introduction

In this paper, we construct a simple model to analyze the network effect of raising children. Networks among parents seem to play an important role in determining the cost of child rearing for individual families. For instance, the existence of such network would facilitate helping each other and exchanging information. We adopt a static partial equilibrium model, and show that the network effect may magnify the decline of "birthrate", i.e., the number of children in each household.

We also extend the partial equilibrium model to a general equilibrium setting. By using this extended model, we consider a small open economy that is allowed to trade consumption goods and capital with the rest of the world. We examine how the reduction in the size of population affects the supply side of this economy and the choice of fertility in each household.

There are many theoretical studies on fertility and population. Among them, this paper is related to Becker and Barro (1988). They develop a model of fertility choices, in which the opportunity costs of child-rearing plays a crucial role in determining the optimal choice of fertility. This paper builds on their work, but we incorporate the network effect of raising children into the model. As a result, we can analyze the fertility choice of each households in the presence of the network-effect of child-rearing.

The rest of this paper is organized as follows. In Section 2, we develop a partial equilibrium model with network effect of child-rearing. We examine the effect of wage and population on the equilibrium number of children. We also show that the equilibrium number of children is insufficient as compared to the social optimum. In Section 3, we extend the model to a general equilibrium setting and examine the free trade equilibrium in a small open economy. In Section 4, we close this paper with a brief summary.

2 A Simple Model with Network Effect

Let us consider a simple model of fertility choice. There are N identical households in an economy. They enjoy raising children as well as consuming a good. Let n denote the number of children and c denote the consumption of a good in each household. Preferences are represented by a Cobb-Douglas utility function,

$$u(n, c) = n^{\theta} c^{1-\theta},$$

where $\theta \in (0, 1)$. Each household has one unit of time and allocates its time to earning wages and rasing children. We assume that each child costs β in time, and thus $n\beta$ is the total time cost of raising children. In addition to its time, each household is endowed with k units of capital. Let w denote the wage rate and r denote the (gross) rental rate of capital. The budget constraint for each household is

$$c + w\beta n = w + rk. \tag{1}$$

Note that the price of the consumption good equals one since it is nemeraire. Each household maximizes the utility function subject to the budget constraint. Let u_n and u_c denote the partial derivatives of u with respect to n and c respectively. The first order condition for this problem is

$$\frac{u_n}{u_c} = \frac{\theta c}{(1-\theta)n} = w\beta. \tag{2}$$

The higher wage rate leads to the larger opportunity cost of raising children. Thus, as the wage rate increases, the number of children per consumption good, n/c, declines in each household. The choice of fertility also depends on the time cost of rasing children β. An increase in β implies that it is more time-consuming to raise children. Thus, a larger β leads to smaller number of children per consumption good, n/c, in each household.

2.0.1 The Network Effect

We take the view that there are networks among parents having children, and the networks can play an important role in determining the cost of rasing children. For instance, in a city with a large population of families, parents may help each other when their children are sick but they cannot be absent from work. Parents can also exchange information about raising children such as the quality of a day-care center or a pediatrician. This kind of network would facilitate raising children for each household by lowering the cost of raising children. Also, the network effect would be stronger as the population size of families becomes larger.

There also may be other reasons why cost of raising children decreases with number of children. When there are many children, there will be more doctors, day-care, and other service providers for children. Transportation cost would be lower as distribution of service providers become more dense. One can also argue search costs would decline but this effect may be indistinguishable from the aforementioned network effect.

We now introduce the network effect of raising children into the model. For this purpose, let $\beta = \beta(Nn)$ and $\beta'(Nn) < 0$ *i.e.* the cost of raising children negatively depends on the total number of children in the economy. We also assume that each household does not recognize the network effect, that is, it is a kind of a positive externality.

Using the budget constraint (1) and first order condition (2), we can derive the demand for children,

$$n(\beta) = \frac{\theta(w + rk)}{w\beta}. \tag{3}$$

If β is constant, the demand for children is determined for the given values of w and r. However, β is not constant in the presence of the network effect since it depends on the total number of children. For the simplification of the analysis, let us consider a specific function of β,

$$\beta(Nn) = \frac{\overline{\beta}}{(Nn)^{\alpha}}, \tag{4}$$

where $\alpha \in (0, 1)$ and $\overline{\beta} > 0$. Clearly, β is decreasing in the total number of children. It can be shown that β is convex in n since $\frac{d^2\beta}{dn^2} > 0$. For the given wage and rental rate, the number of children and the cost of raising children are determined by the two equations (3) and (4). Let n_e and β_e denote the equilibrium values of n and β. Then, we can derive

$$n_e = \left[\frac{\theta\,(w + rk)}{w\overline{\beta}}\right]^{\frac{1}{1-\alpha}} N^{\frac{\alpha}{1-\alpha}},$$
$$\beta_e = \overline{\beta}^{\frac{1}{1-\alpha}} \left[\frac{\theta\,(w + rk)}{w}\right]^{-\frac{\alpha}{1-\alpha}} N^{-\frac{\alpha}{1-\alpha}}.$$

Figure 1 shows the determination of equilibrium values of n and β. At the equilibrium point (n_e, β_e), the curve of the demand for children is steeper than that of the cost of raising children. This case is guaranteed by the assumption that α is smaller than 1 in the cost function (4). Under this assumption, it can be show that the equilibrium is stable. To confirm this point, let us consider the following two scenarios.

2.1 The Adjustment Process of Birthrate

Suppose that the cost of child rearing is given by $a > \beta_e$ in Figure 2. For this value of the cost, the number of children chosen by each household is $n(a)$. However, when the number of children per household is $n(a)$, the actual cost of child rearing is $\beta(n(a))$ due to the network externality. Since $\beta(n(a))$ is smaller than a, each household would increase the the number of children. Again, β declines due to the increase in n, and so on. This process continues until the equilibrium is reached. If the cost of raising children is higher than the equilibrium value, the number of children per household would increase in the adjustment process.

In contrast, suppose that the cost of child rearing is given by $b < \beta_e$ in Figure 2. For each household, it is optimal to choose $n\,(b)$. For this number of children, the network effect is too weak to keep the cost of child rearing as low as b. As a result, the cost rises up to $\beta(n(b))$, under which each household chooses the smaller number of children. Then, the decline in n raises β further more, and this process continues until the equilibrium is reached. If the cost of raising children is smaller than the equilibrium value, each household would reduce the number of children during the adjustment process.

2.2 The Effect of the Wage Rate

Let us examine the impact of a wage increase on the number of children in each household. If the wage rate increases, then the opportunity cost of child rearing would rise. Thus, each household would choose to have fewer children. This is confirmed by $\frac{dn_e}{dw} < 0$. The point is that the network externality can magnify the impact of the wage increase. In Figure 3, the curve of the demand for children shifts down due to an increase in the wage rate. If there were no network effect, the cost of child rearing would be constant, and thus the decline in the number of children would be smaller than that in the presence of the network effect. This implies that the network effect will magnify a decline in "birthrate".

2.3 The Effect of the Number of Households

Let us turn to a change in the number of households. In Figure 4, a fall in N shifts the curve of the cost of child rearing upward, and thus the number of children per household would decline. The reason is straightforward. The decline in the number of households weakens the network effect of child rearing, and increasing the cost of raising children. It is worth noting that this effect does not appear in the absence of the network effect. This result also implies that a decline in the number of households reduces more than proportionally the total number of children in the economy.

2.4 Socially Optimal Number of Children

We determine the relationship between the equilibrium and socially optimal number of children. Since all households are identical, social welfare is,

$$W(n, c) = Nu(n, c).$$

The resource constraint is simply N times 1) with β replaced by the function $\beta(Nn)$. The social planner takes the externality into account. The first order condition is,

$$\frac{u_n}{u_c} = \frac{N\theta c}{N(1-\theta)n} = w\beta(Nn) + w\beta'(Nn)N. \tag{5}$$

Since $\beta'(Nn) < 0$, comparison with (2) shows that the positive externality actually makes the cost of an extra child smaller. Using the explicit formulation (4), we get the relationship between β and n,

$$n = \frac{\theta(w+rk)}{w\beta} + \alpha N(1-\theta). \tag{6}$$

The second term utilizes

$$\beta'(Nn) = -\alpha\bar{\beta}N^{-\alpha}n^{-\alpha-1} = -\alpha\frac{\beta(Nn)}{n}.$$

Unlike (3), this is an implicit demand function of n. It does show the relationship between n and the value of β. This is depicted in dotted lines in Figure 1. We can see that the socially optimal number of children n^* is more than n_e and the corresponding cost will be lower, $\beta^* < \beta_e$.

2.5 Optimal Subsidy

In this section, we analyze the government's optimal subsidy. Suppose that the government provides a subsidy s per child and levies a lump-sum tax t on income. Then, the budget constraint for each household is

$$c + (w\beta - s)n = w + rk - t.$$

Under this budget constraint, each household maximizes the utility. It can be shown that the first order condition is

$$\frac{\theta c}{(1-\theta)n} = w\beta - s.$$

The subsidy stimulates the demand for children by reducing the cost of child-rearing. By using the first order condition with the budget constraint, we can obtain the following condition:

$$n\beta = \frac{\theta(w + rk - t)}{w} + \frac{sn}{w}. \tag{7}$$

Under the subsidy, the demand for children must satisfy this condition. The balanced budget condition for the government implies that

$$Nsn = Nt.$$

With this condition, we can rearrange (7) as

$$n\beta = \frac{\theta(w + rk - t)}{w} + \frac{(1-\theta)sn}{w}. \tag{8}$$

In the previous section, we derived the optimal condition (6). We can rewrite (6) as follows:

$$n^*\beta^* = \frac{\theta(w + rk)}{w} + \alpha N(1-\theta)\beta^*. \tag{9}$$

If the government chooses the subsidy optimally, then (n^*, β^*) must satisfy (8) under the optimal subsidy s^*. Thus, by using (8) and (9), we have

$$\frac{\theta(w + rk)}{w} + \frac{(1-\theta)s^*n^*}{w} = n^*\beta^* = \frac{\theta(w + rk)}{w} + \alpha N(1-\theta)\beta^*.$$

By solving for s^*, we can derive the optimal subsidy,

$$s^* = \frac{w\alpha N\beta^*}{n^*} = \frac{\alpha w\overline{\beta}N^{1-\alpha}}{n^{*1+\alpha}},$$

where the second equality is obtained by (5). We can show that the optimal subsidy is positively related to the wage rate. In Figure 1, an increase in the wage rate shifts the dotted line upward, and thus the optimal number of children falls. Then, we can easily see that an increase in wage rate raises the optimal subsidy. The intuition is straightforward. A rise in the wage rate increases the opportunity costs for child-rearing. Thus, the government must provide the larger subsidy to stimulate birthrate.

3 Globalization and Population

In this section, we turn to the link between "globalization" and population. For this purpose, we extend the previous model to a general equilibrium setting. "Globalization" means the integration of the domestic good and capital markets to the world markets. First, we show that, in a globalized economy, declining population may "hollow out" the economy due to a reduction in the labor supply, but such a "shortage" of the labor supply may not be recovered because of a further reduction in birthrate. Second, we show that a surge in foreign investment may reduce the output of domestic production, but such "hollowing out" may have a positive impact on population.

3.1 A General Equilibrium Model

The model can be extended to a general equilibrium setting. Let Y denote the output of the consumption good. Technology is represented by a production function,

$$Y = F(K, L),$$

where K and L denote the inputs of capital and labor respectively. We assume that the production function is constant returns to scale. The capital input K equals the total endowment of capital Nk. Let F_L denote the marginal product of labor. The labor input is determined by the following first order condition,

$$F_L(Nk, L) = w.$$

Solving this condition for L, we have the labor demand function $L_d = L_d(w)$. Since the marginal product of labor is declining in the labor input, the labor demand is decreasing in the wage rate.

The labor supply of each household is given by $(1 - \beta n)$. Using (3), we can derive the total supply of labor as

$$N(1 - \beta n) = N\left[1 - \frac{\theta(w + rk)}{w}\right].$$

It can be easily shown that the total supply of labor is increasing in the wage rate. The equilibrium condition in the labor market determines the wage rate,

$$L_d(w) = N\left[1 - \frac{\theta(w + rk)}{w}\right]. \tag{10}$$

Finally, the rental rate for capital is determined by the competitive condition for the good market. Let $c(w, r)$ denote the unit cost function of the consumption good. Since the market for the consumption good is competitive, the unit cost equals the price, which is one, at equilibrium.

$$1 = c(w, r). \tag{11}$$

Let w_e and r_e denote the equilibrium wage and rental rate. Solving the conditions (10) and (11) simultaneously, we obtain w_e and r_e.

3.2 Equilibrium in an Open Economy

Let us consider a situation in which good trade and capital mobility are allowed in the economy. Suppose that the rental rate at autarky is smaller than the rental rate at the world capital market, *i.e.* $r_e < r^*$. Then, the economy has a comparative advantage in capital and it would export capital for the import of the consumption good. We assume that the economy is a small country so that the domestic rental rate is determined by the world rental rate r^*. Since the economy exports capital, the capital inputs used in domestic production would be smaller than the endowment of capital, *i.e.* $K < Nk$. The domestic capital inputs are determined by the following first order condition,

$$F_K(K, L) = r^*, \tag{12}$$

where F_K is the marginal product of capital. Substituting the world rental rate r^* into the competitive condition (11), we can derive the wage rate at trade equilibrium. Let $w(r^*)$ denote the equilibrium wage rate. The total supply of labor is determined by the equilibrium wage rate and the world rental rate. The labor market is clear when the total supply equals the input demand,

$$L = N\left[1 - \frac{\theta(w(r^*) + r^*k)}{w(r^*)}\right]. \tag{13}$$

Substituting (13) into (12), we can obtain the input demand for capital at trade equilibrium. Finally, we can show the balance of trade by using the budget constraint,

$$\begin{aligned} cN &= w(r^*)L + r^*kN \\ &= w(r^*)L + r^*K + r^*(kN - K) \\ &= Y + r^*(kN - K). \end{aligned}$$

The last equality implies that trade is balanced since $cN - Y = r^*(kN - K)$. The consumption of each household is determined so that the value of import of the consumption good can equal the value of export of capital.

3.3 The Effect of Declining Population

Suppose that the number of households declines. Then, for the given wage rate, the total supply of labor decreases, and this puts upward pressure on the wage rate. For domestic producers to stay competitive, the domestic rental rate would fall to absorb the increase in the wage rate. This leads to the expansion of capital exports since foreign investment is more profitable. As a result, the output of domestic production declines, *i.e.* “hollowing out” occurs in the economy.

Also, as we have already shown, a decline in the number of households reduces the demand for children since the weaker network effect increases the cost of child rearing.

In sum, a decline in the population of households leads to a “shortage” of the total labor supply, and as a result, it induces an increase in capital exports and a reduction in domestic production. The “shortage” would not necessarily be recovered since the demand for children decreases due to the weaker network effect.

3.4 The Effect of a Foreign Investment Boom

Suppose that the world rental rate r^* rises for some reason. This results in a surge in foreign investment since it is more profitable than domestic investment. The capital outflows raise the domestic rental rate. Then, the competitive condition (11) implies that the wage rate falls to make the domestic production competitive in the world market. The decline in the wage rate reduces the opportunity cost of child rearing. Also, the nominal income of each household rises due to the higher returns of capital. Thus, both substitution and income effects have positive impacts on the demand for children in each household.

In sum, if the domestic capital market is integrated into the world market, an increase in the world rental rate results in the expansion of capital exports. This reallocation of capital reduces the output of domestic production, and thus the wage rate falls in the labor market. The fall in the wage rate leads to a decline in the opportunity cost of child rearing, and increasing the demand for children. The increase in the rental rate also induces each household to have more children by making them wealthier. Thus, a surge in foreign investment may lead to "hollowing out", but at the same time, it may results in an increase in birthrate.

4 Concluding Remarks

In this paper, we develop a simple model to examine the network-effect of child rearing. If the net-work effect exists, the cost of raising children decreases with the total number of children. Then, the equilibrium number of children is too small as compared to the socially optimal value, and the network effect can magnify the decline of birthrate. The government can provide the optimal subsidy to stimulate birthrate. In a high-wage country, the opportunity cost of child-rearing is large for each household. Thus, the size of the optimal subsidy would increase with income of the economy.

In a general equilibrium setting, the wage rate is determined endogenously. Then, a decline in the size of population leads to the higher wage rate by reducing the labor supply. The rise in the wage rate leads to the higher opportunity costs for raising children, and thus the demand for children declines as well. In the presence of the network effect, a decrease in population can magnify the reduction in birthrate since the cost of child-rearing increases with a decline in the size of population.

In this paper, we develop the static framework. It may be possible to extend the model to a dynamic setting. Immigration is another aspect of globalization. It is interesting to examine the impact of immigration on birthrate. These are tasks for our future research.

References

[5] Reiko Aoki, 2004. "Microeconomics of Declining Birthrate – Review of Existing Literature", PIE Discussion Paper No.228, Institute of Economic Research, Hitotsubashi University.

[5] Robert J. Barro and Gary S. Becker, 1989 "Fertility Choice and in a Model of Economic Growth," *Econometrica* 57: 481-501

[5] Gary S. Becker and Robert J. Barro, 1988. "A Reformulation of the Economic Theory of Fertility," *Quarterly Journal of Economics*, 103:1-25.

[5] Gary S. Becker, Kevin M. Murphy and Robert Tamura, 1990. "Human Capital, Fertility and Economic Growth," *Journal of Political Economy*, 98(5):812-837.

[5] Charles I. Jones, 2003, Population and Ideas: A Theory of Endogenous Growth Aghion, Frydman, Stiglitz, and Woodford (eds.) *Knowledge, Information, and Expectations in Modern Macroeconomics: In Honor of Edmund S. Phelps* (Princeton University Press)