International Negotiations for Reducing Greenhouse Gases with Emission Permits Trading*

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Abstract

We build a three-stage game model of international negotiations on regulation of global emissions of greenhouse gases, and examine the Pareto optimality of an equilibrium allocation. First, we derive the condition for Pareto optimal allocations, which is an extension of the celebrated Samuelson condition. Next, we show that although production efficiency of a final allocation is always met at an equilibrium of the game, overall Pareto optimality may not be satisfied. This is because in negotiations on the level of global emissions in the first stage of the game, countries make expectations on the effect of the total supply of emission permits on the revenue from or the expenditure for emission permits in a later stage.

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1 Introduction

The Kyoto Protocol to the United Nations Framework Convention on Climate Change stipulates the limitation or reduction of greenhouse gas emissions in the developed countries and others for five years from 2008 to 2012 according to the quantified commitments. The limitation of greenhouse gas emissions after this period, however, is not determined yet, and it will soon become the most important issue of international negotiations. In this paper, we build a model of international negotiations for reduction of greenhouse gas emissions with emission permits trading, and examine whether we could attain a Pareto optimal allocation through the negotiations.

Although every country places a positive value on reduction of greenhouse gas emissions, marginal willingness to sacrifice its own consumption for improvement of the environment may vary among countries.[1] For such a profile of welfare functions of the countries over consumption and greenhouse gas emissions, we first derive the condition for the Pareto optimal levels of greenhouse gas emissions, production and consumption, which is an extension of the celebrated Samuelson condition (Samuelson, 1954).

Next, we introduce a three-stage game of international negotiations and emission permits trading. In the first stage, all the countries negotiate on the total amount of greenhouse gas emission permits, given the distribution rule of the permits. In the second stage, each country determines its domestic rule on requirements for emission permits or greenhouse gas taxation. In the third stage, the market of emission permits is open, and an equilibrium price of a permit is established. At the equilibrium, each country produces and consumes commodities, and emits the amount of greenhouse within the limit determined in the first stage. Hence, the total amount of emission permits determines the equilibrium level of consumption of each country, and thereby the final welfare level of each country.

We analyze carefully how the limitation of the total amount of greenhouse gas emissions affects the consumption of each country. In particular, we take into account the "feedback effect": an increase in the amount of emission permits for a country may raise the consumption of the country,

[1] Ahlheim and Schneider (2002) write "All the more it is amazing that in spite of the impossibility of personal perception of most greenhouse gases, in many countries people are ready or eager to make personal sacrifices for the sake of greenhouse gas reductions." Then they argue why people are willing to sacrifice private consumption for environmental improvements.

but not as much as the same amount because the increase in consumption itself accompany greenhouse gas emissions, requiring additional permits.

Based on the analysis, we draw the welfare possibility frontier of the countries by tracing the welfare vector at each level of the total emission permits. We show that a welfare vector on the possibility frontier of the international negotiations is not necessarily Pareto optimal. It is true that production efficiency (or equivalently cost minimization of a given amount of reduction of greenhouse gases) is ensured through emission permits trading. In negotiations about the total supply of emission permits, however, the countries make expectations on the effects of the total supply on the net revenue from emission permits at market equilibria, a bargaining outcome may not give rise to a Pareto optimal allocation.

There are some related works on international negotiations for abatement of global warming. Okada (2003) presents a similar two-stage game model of international negotiations on emission permits. However, he assumes that the total amount of emission permits is fixed, and considers negotiations on distributions of the fixed total amount among countries. Helm (2003) considers a three-stage game as follows. In the first stage, countries decide whether they establish a regime with permits trading or a regime without trading by a unanimous agreement. If a trading system is approved, then in the second stage, each country (as a sovereign state) chooses its own emission allowances. In the third stage, the allowances are traded in an international market, and an equilibrium allocation is established. Compared with these models, the model in this paper has the following characteristics.[2] First, we consider signatories of the Kyoto protocol as the participants in negotiations. Hence, they have already approved a regime with permits trading, and they negotiate about a target level of global emissions in the period after 2012, assuming that emission permits are tradable. Second, a distribution rule of permits among countries is assumed to be given. There have been long debates on which distribution rules are fair or just, a proportional rule to past emissions, to GDPs, to populations, or to the costs of reducing emissions? Instead of predicting a consequence of these debates on distribution rules, this paper focuses on negotiations that should follow a settlement of the debates, and examine whether the countries could succeed in attaining a

[2]Contributions which shed light on other aspects of international negotiations and agreements on climate changes are Carraro and Siniscalco (1993), Barrett (1994), Ahlheim and Schneider (2002), Asako, K. and M. Kuninori (2001), and Lange and Vogt (2003).

Pareto optimal allocation through negotiations about a target level of global emissions.

The rest of the paper is organized as follows. The next section presents the basic assumptions on technology and preferences of each country. In section 3, we show the extension of the Samuelson condition on Pareto optimal allocations. In section 4, we present and analyze the three-stage game on emission permits. Section 5 shows that a bargaining outcome may not give rise to a Pareto optimal allocation, and section 6 concludes.

2 Technology and Preferences

There are n countries, $N = \{1, \ldots, n\}$. Let $y_i \in \mathbb{R}_+$ denote the gross domestic product (GDP) of country $i \in N$, $c_i \in \mathbb{R}_+$ the consumption of country i.

Both production and consumption are accompanied by emissions of greenhouse gases. Let $x_i^p \in \mathbb{R}_+$ denote the emission of greenhouse gases from production. The relation of x_i^p and y_i is represented by the function $x_i^p = f_i(y_i)$, where $f_i' > 0$, $f_i'' > 0$. Let $x_i^c \in \mathbb{R}_+$ be the emission of greenhouse gases from consumption. The relation of x_i^c and c_i is represented by the function $x_i^c = g_i(c)$, where $g_i' > 0$, $g_i'' \geq 0$. Let $x_i = x_i^p + x_i^c$ be the total emission of greenhouse gases of country i, and let $X := \sum_{i \in N} x_i$ be the global emission of greenhouse gases.

Each country i has preferences over pairs (c_i, X) of its own consumption and a total amount of emission of greenhouse gases. The preferences are represented by a continuously differentiable and strictly quasi-concave function $V_i : \mathbb{R}_+^2 \to \mathbb{R}$. We call the function V_i *the welfare function of country* i.

The partial derivative of V_i with respect to the variable a is denoted by $D_a V_i$. We assume that $D_c V_i > 0$ and $D_X V_i < 0$. We also assume that (i) for every $c_i \in \mathbb{R}_+$, $\lim_{X \to 0} \frac{D_X V_i(c_i, X)}{D_c V_i(c_i, X)} = 0$, and (ii) for every $X \in r_+$, $\lim_{c_i \to 0} \frac{D_X V_i(c_i, X)}{D_c V_i(c_i, X)} = 0$. These assumptions mean that the marginal willingness to sacrifice consumption for reduction of greenhouse gas emissions approaches zero as the level of greenhouse gases goes to zero or as the level of consumption goes to zero.

3 Pareto optimal allocations

A greenhouse gas is a public "bad" in the sense that an increase in greenhouse gas emissions makes every country worse off. There are several features of emission of greenhouse gases that are distinct from other public goods or bads. First, every country emits greenhouse gases inevitably. That is, every country produces the public bad through both production and consumption of commodities. Second, the relation of emission of greenhouse gases to production or consumption varies widely among countries, depending on technology. For instance, to attain a given level of production, Japan emits a relatively smaller amount of greenhouse gases than Russia, thanks to its advanced technology to save oil and other resources.

Samuelson (1954) established the celebrated condition of Pareto optimal allocations in an economy with public goods, saying that at an optimum, the sum of the marginal rates of substitution between the (composite) commodity and the public good over all individuals should be equal to the marginal rate of transformation between the two goods. Next we derive the condition of the Pareto optimal level of greenhouse gas emissions, taking account of the essential features of greenhouse gas emissions as described above. Our condition differs from Samuelson's condition due to the fact that consumption of commodities must accompany greenhouse gas emissions. Samuelson did not consider the case where consumption has such an external effect. Without the external effect, our condition coincides with Samuelson's.

An *allocation* is a vector $(y, c, x) := (y_1, \ldots, y_n; c_1, \ldots, c_n; x_1, \ldots, x_n) \in \mathbb{R}^{3n}_+$. An allocation (y, c, x) is *technologically feasible* if and only if
(i) $\sum_{h \in N} y_h = \sum_{h \in N} c_h$ and
(ii) for every $j \in N$, $x_j = f(y_j) + g(c_j)$.

We say that an allocation $(\hat{y}, \hat{c}, \hat{x})$ *Pareto dominates* an allocation (y, c, x) if and only if
(i) for every $j \in N$, $V_j(\hat{c}_j, \sum_{h \in N} \hat{x}_h) \geq V_j(c_j, \sum_{h \in N} x_h)$, and
(ii) for some $j \in N$, $V_j(\hat{c}_j, \sum_{h \in N} \hat{x}_h) > V_j(c_j, \sum_{h \in N} x_h)$.
An allocation (y^*, c^*, x^*) is *Pareto optimal* if and only if it is technologically feasible and there is no technologically feasible allocation that Pareto dominates it.

Let $(y^*, c^*, x^*) \gg 0$ be a Pareto optimal allocation. For each $j \in N$, define $\bar{V}_j := V_j(c^*_j, \sum_{h \in N} x^*_h) \in \mathbb{R}$. Let $i \in N$ be given. Then, (y^*, c^*, x^*) is a solution for the following constrained maximization problem:

$$\max_{(y,c,x)\in\mathbb{R}^{3n}_{+}} V_i(c_i, \sum_{i\in N} x_i)$$

subject to

$$\forall j \in N,\ x_j = f(y_j) + g(c_j) \tag{1}$$

$$\sum_{h\in N} y_h = \sum_{h\in N} c_h \tag{2}$$

$$\forall j \in N, j \neq i,\ V_j(c_j, \sum_{h\in N} x_h) = \bar{V}_j \tag{3}$$

Define the Lagrangean function as

$$\begin{aligned} & L((c_h)_{h\in N}, (x_h)_{h\in N}, (y_h)_{h\in N}, (\lambda_j)_{j\in N, j\neq i}, (\gamma_h)_{h\in N}, \delta) \\ = \ & V_i(c_i, \sum_{h\in N} x_h) - \sum_{j\neq i} \lambda_j (V_i(c_j, \sum_{h\in N} x_h) - \bar{V}_j) \\ & - \sum_{j\in N} \gamma_j (x_j - f_j(y_j) - g_j(c_j)) - \delta(\sum_{h\in N} y_h - \sum_{h\in N} c_h). \end{aligned}$$

From the first order conditions,

$$D_{c_i}L = D_{c_i}V_i(c_i^*, \sum_{h\in N} x_h^*) + \gamma_i g_i'(c_i^*) + \delta = 0 \tag{4}$$

$$D_{x_i}L = D_X V_i(c_i^*, \sum_{h\in N} x_h^*) - \sum_{j\neq i} \lambda_j D_X V_j(c_j^*, \sum_{h\in N} x_h^*) - \gamma_i = 0 \tag{5}$$

$$D_{y_i}L = \gamma_i f_i'(y_i^*) - \delta = 0 \tag{6}$$

and for each $j \neq i$,

$$D_{c_j}L = -\lambda_j D_{c_j}V_i(c_i^*, \sum_{h\in N} x_h^*) + \gamma_j g_j'(c_j^*) + \delta = 0 \tag{7}$$

$$D_{x_j}L = D_X V_i(c_i^*, \sum_{h\in N} x_h^*) - \sum_{j\neq i} \lambda_j D_X V_j(c_j^*, \sum_{h\in N} x_h^*) - \gamma_j = 0 \tag{8}$$

$$D_{y_j}L = \gamma_j f_j'(y_j^*) - \delta = 0 \tag{9}$$

From equations (5) and (7), $\gamma_i = \gamma_j$ for all $i, j \in N$. Hence, from equations (6) and (9), $f_i'(y_i^*) = f_j'(y_j^*)$ for all $i, j \in N$. Thus, we have the following result.

Proposition 1 Production Efficiency. *At a Pareto optimal allocation, the marginal emission from production should be the same for all the countries.*

Solving the system of equations (4)-(9), we have

$$\sum_{j \in N} \frac{D_X V_j(c_j^*, \sum_{h \in N} x_h^*)}{D_{c_j} V_j(c_j^*, \sum_{h \in N} x_h^*)} \cdot (f_j'(y_j^*) + g_j'(c_j^*)) = -1$$

Define

$$\eta_i(c_i, X) := \left| \frac{D_X V_i(c_i, X)}{D_c V_i(c_i, X)} \right| \tag{10}$$

The value $\eta_i(c_i, X)$ is the absolute value of country i's marginal rate of substitution of its own consumption for global emission of greenhouse gases. Then,

$$\sum_{j \in N} \eta_j(c_j^*, \sum_{h \in N} x_h^*) \cdot (f_j'(y_j^*) + g_j'(c_j^*)) = 1 \tag{11}$$

Proposition 2 Extension of the Samuelson Condition. *At a Pareto optimal allocation, the weighted sum of marginal rates of substitution of consumption for global emission of greenhouse gases over all the countries is equal to one where each weight is the sum of marginal emission from production and from consumption in each country.*

From Proposition 1, $f_i'(y_i^*) = f_j'(y_j^*)$ for all $i, j \in N$. Hence, *if there is no external effect of consumption of commodities*, that is, $g_i' = 0$ for all $i \in N$, then equation (11) becomes

$$\sum_{j \in N} \eta_j(c_j^*, \sum_{h \in N} x_h^*) = t \quad \text{where } t = \frac{1}{f_1'(y_1^*)} = \cdots = \frac{1}{f_n'(y_n^*)} \tag{12}$$

The number t is the marginal rate of transformation between consumption of commodities and emission of greenhouse gases, and the above equation is nothing but the celebrated Samuelson condition. In other words, equation (11) is an extension of the Samuelson condition to the case where consumption of the (composite) commodity has an external effect on the production of the public bad (or good).

4 Three-stage game on emission permits

In this section, we consider a three-stage game on emission permits. We assume that that a distribution rule of emission permits among countries is given. The rule may be the proportional rule to the levels of GDP at a benchmark year, the proportional rule to populations, or the proportional rule to the costs of reducing greenhouse gas gases, etc. In the first stage, all the countries negotiate on the total amount of emissions of greenhouse gases, given the distribution rule of emission permits. In the second stage, each country determines the domestic rule on the requirement for emission permits or greenhouse gas taxes. In the third stage, firms and consumers act so as to maximize their objectives, and the levels of production, consumption and emissions of greenhouse gases are determined at a market equilibrium.

4.1 Equilibrium in the market of emission permits

To analyze the game backward, we first focus on market equilibria of emission permits in the third stage of the game. At a given price of the permit, how is the demand for emission permits determined?

Consider following policies of the government of country $i \in N$.

(A) The government requires that each producer should obtain emission permits at the international price for the amount of greenhouse gases emitted in the process of production, and that each consumer should also obtain permits for the amount emitted in the process of consumption of the commodity.

(B) The government imposes the greenhouse gas tax on each producer for the amount emitted in the process of production, and on each consumer for the amount emitted in the process of consumption. The tax rate is equal to the international price of an emission permit.

(C) The government requires that each producer should obtain emission permits at the international price for the amount emitted in the process of production, and imposes a greenhouse gas tax on each consumer for the amount emitted in the process of consumption. The tax rate is equal to the international price of an emission permit.

A point in the three policies is that each producer should be responsible only for greenhouse gases emitted in the process of production, while each consumer should be responsible for those emitted in the process of consumption of the product. Later we will see that any of the three policies is an optimal choice of the government at the second stage.

Let $q \in \mathbb{R}_+$ be the international price of an emission permit. Then, the marginal revenue of production is one since the price of the (composite) commodity is one, while the marginal cost of production due to the requirement for emission permits or the greenhouse gas tax is $q\,f'(y)$ under either of the policies (A), (B) or (C). Hence, from profit maximization of firms, the level of production in country i, y_i^*, is determined by

$$1 = q\,f'(y_i^*). \tag{13}$$

Given a level of production, the level of consumption is determined by the equivalence between the national income and the national expenditure.[3] Let $\bar{x}_i \in \mathbb{R}_+$ be the initial assignment of emission permits to country $i \in N$. Let $q \in \mathbb{R}_+$ be a given international price of an emission permit. When the levels of production and consumption are y_i and c_i, respectively, the total amount of emissions of greenhouse gases is $f_i(y_i) + g_i(c_i)$. Then, the revenue from (or the expenditure for, if it is negative) emission permits is equal to $q\,(\bar{x}_i - f_i(y_i) - g_i(c_i))$. Thus, the national income is given by $y_i + q\,(\bar{x}_i - f_i(y_i) - g_i(c_i))$. This should be equal to the national expenditure c_i. We therefore say that a consumption-production pair (c_i, y_i) is *feasible with emission permits trading* for country i at q and $\bar{x}_i$ if and only if

$$c_i = y_i + q\,(\bar{x}_i - f_i(y_i) - g_i(c_i)). \tag{14}$$

For each $c_i \in \mathbb{R}_+$, since $1 + q\,g_i'(c_i) > 0$, by the implicit function theorem, equation (14) can be locally solved for c_i as a function of y_i. Denote the function $c_i(y_i)$. For each $y_i \in \mathbb{R}_+$, $c_i(y_i) \in \mathbb{R}_+$ is the level of consumption when the level of production is y_i.

Let us turn to the second stage of the game in which the government of each country determines the domestic rule on emission permits trading or greenhouse gas taxes. We assume that for each $i \in N$, the objective of the government of country i is to maximize its preferences V_i. In the second stage of the game, however, the government of country i regards the global emission of greenhouse gases as fixed since it is determined in international negotiations in the first stage. Hence, in order to maximize V_i, it should choose a policy or a rule that will maximize its domestic consumption in the third stage.

To find such a rule or a policy, it is necessary to examine how the level of consumption changes as the level of production changes. By differentiating

[3] Notice that in the present model, there is only one (composite) commodity.

the function $c_i(\cdot)$,

$$c_i'(y_i) = \frac{1 - q\, f_i'(y_i)}{1 + q\, g_i'(c_i(y_i))} < 1 \tag{15}$$

and

$$c_i''(x_i) = \frac{A}{[1 + q\, g_i'(c_i(y_i))]^2} \tag{16}$$

where

$$A = -q\, f_i''(y_i)[1+q\, g_i'(c_i(y_i))] - f_i'(y_i) - g_i(c_i(y_i)) - q\, g_i''(c_i(y_i))c_i'(y_i)[1-q\, f_i'(y_i)].$$

If there were no requirement to obtain emission permits, an increase in y_i raises consumption by the same amount. With the requirement for emission permits, however, a unit of increase in y_i induces more payment for emission permits, and therefore can increase consumption by less than one unit.

Let y_i^* be the level of production of country i that maximizes its own consumption. By equation (16), $c_i''(y_i) < 0$ whenever $c_i'(y_i) = 0$. Hence, the necessary and sufficient condition for a local optimum, y_i^*, is $c_i'(y_i^*) = 0$. By equation (15), we have

$$q = \frac{1}{f_i'(y_i^*)}. \tag{17}$$

In general, one unit of increase in production generates $f_i'(y_i)$ units of emission of greenhouse gases, which increases the payment for (or decreases the revenue from) emission permits by $q\, f_i'(y_i)$. If consumption also increased, then more payment for emission permits would be necessary. At the optimum, however, consumption should never increase by the change in production. Hence, the increase in production should be just offset by the increase in the payment for (or the decrease in the revenue from) emission permits from production. That is why $q\, f_i'(y_i) = 1$ and equation (17) hold at the optimal production y_i^*.

However, as one notice by comparing equation (17) with equation (13), the optimal amount y_i^* is exactly the amount of production attained through profit maximization of firms under any of the policies (A), (B) and (C). Therefore, to choose any of the three policies is indeed an optimal choice of the government in the second stage of the game.

Proposition 3 *Consider country $i \in N$. In the second stage of the game, any of the three policies (A), (B) and (C) is an optimal choice of the government of country i whose objective is to maximize the preferences V_i. Under* *the policy, and at any given price of emission permit, the amount of production y_i^* of country i is determined by*

$$q = \frac{1}{f_i'(y_i^*)}.$$

Then the amount of consumption $c_i(y_i^*)$ is the maximal amount under the price q and the initial assignment $\bar{x}_i$ of emission permits.

From Proposition 3, we have $f_i'(y_i^*) = f_j'(y_j^*) = \frac{1}{q}$ for all $i, j \in N$. Hence, we obtain the following corollary.

Corollary 1 *Any allocation attained in the third stage of the game satisfies the condition of production efficiency.*

4.2 The demand function for emission permits

Having examined how the production and the consumption of country i are determined in the third stage of the game, this subsection studies the property of the demand function for emission permits of country i. That is, we investigate how the demand for emission permits changes as the price of a permit changes.

For every $q \in \mathbb{R}_+$, let $y_i(q) \in \mathbb{R}_+$ be the amount of production of country i at q in the third stage of the game. Since $f''(y_i) > 0$ for all $y_i \in \mathbb{R}_+$, the function $\frac{1}{f'(y_i)}$ is decreasing in y_i. From Proposition 3, $y_i(q)$ is decreasing in q.

Define $x_i^p(q) := f_i(y_i(q))$. $x_i^p(q)$ is the emission from production of country i at q. Define $x_i^c(q) := g_i(c_i(y_i(q)))$. $x_i^c(q)$ is the emission from consumption of country i at q. Let $x_i(q) := x_i^p(q) + x_i^c(q)$. Then, $x_i(q)$ is the gross demand for emission permits of country i at q.

We would like to examine whether the gross demand for emission permits is decreasing in q or not. As we have already seen, if q rises, then $y_i(q)$ decreases, and hence $x_i^p(q)$ also decreases. We need to check whether $x_i^c(q)$ is decreasing in q or not.

By definition, for all $q \in \mathbb{R}_{++}$,

$$c_i(y_i(q_i)) = y_i(q) + q\,[\bar{x}_i - f_i(y_i(q)) - g_i(c_i(y_i(q)))].$$

Differentiating both sides,

$$\begin{aligned} c_i'(y_i(q))y_i'(q) &= [1 - q\, f_i'(y_i(q))]y_i'(q) + \bar{x}_i - f_i(y_i(q)) - g_i(c_i(y_i(q))) \\ &\quad - q\, g_i'(c_i(y_i(q)))c_i'(y_i(q))y_i'(q). \end{aligned}$$

Since $1 - q\, f_i'(y_i(q)) = 0$ from Proposition 3, we have

$$\begin{aligned} [1 + q\, g_i'(c_i(y_i(q)))]c_i'(y_i(q))y_i'(q) &= \bar{x}_i - f_i(y_i(q)) - g_i(c_i(y_i(q))) \\ &= \bar{x}_i - x_i^p(q) - x_i^c(q). \end{aligned}$$

Hence,

$$\frac{dc}{dq} = c_i'(y_i(q))y_i'(q) = \frac{\bar{x}_i - x_i^p(q) - x_i^c(q)}{1 + q\, g_i'(c_i(y_i(q)))}. \tag{18}$$

Because $g_i'(c_i) > 0$ for all $c_i \in \mathbb{R}_+$, it follows that

$$\frac{dc}{dq} \geq 0 \Leftrightarrow \bar{x}_i - x_i^p(q) - x_i^c(q) \geq 0$$

and

$$\frac{dc}{dq} \leq 0 \Leftrightarrow \bar{x}_i - x_i^p(q) - x_i^c(q) \leq 0.$$

This means that if country i is a *net supplier* of emission permits, then an increase in the price q induces higher consumption through the income effect, which then raises the emission from consumption, x_i^c. That is, $x_i^c(q)$ is increasing in q. Therefore, it is ambiguous whether the total demand for emission permits of country i, $x_i(q) = x_i^p(q) + x_i^c(q)$, is decreasing or not. If $|x_i^{p\prime}(q)| < |x_i^{c\prime}(q)|$, then the total demand is increasing in q.

On the other hand, if country i is a *net demander* of emission permits, then both $x_i^{p\prime}(q)$ and $x_i^{c\prime}(q)$ are negative. Thus, the total demand for emission permits of country i is decreasing in q.

What about the *aggregate demand* for emission permits? Let $X^D(q)$ denote the aggregate demand for emission permits at q. Since $x_i(q)$ may be increasing or decreasing in q for each $i \in N$, it is ambiguous in general whether the aggregate demand $X^D(q)$ is decreasing in q or not. However, under an additional assumption, we can determine the sign of $X^{D\prime}(q)$ *at an equilibrium price.*

Assumtion L (Linearity in Emission from Consumption):
For some constant $\alpha \in \mathbb{R}_{++}$, $g_i(c_i) = \alpha c_i$ for all $i \in N$.

Assumption L means that (i) emissions from consumption are proportional to the levels of consumption, and (ii) per unit emissions from consumption are the same among countries. While the relations of emissions of greenhouse gases with production vary widely among countries, depending on the production technologies to save energies, emissions from consumption may be nearly proportional to the amount of consumption, and moreover, the differences among countries in per unit emissions from consumption seem small. For example, emissions of greenhouse gases from cooking, heating, driving, exercising, etc., would be proportional to the amount of consumption, and there is little difference in per unit emissions among countries.

Under Assumption L, we have

$$\begin{aligned} X^{D\prime}(q) &= \sum_{i \in N} {x_i}'(q) \\ &= \sum_{i \in N} x_i^{p\prime}(q) + \sum_{i \in N} x_i^{c\prime}(q) \\ &= \sum_{i \in N} x_i^{p\prime}(q) + \sum_{i \in N} g_i'(c_i(y_i(q)))\, c_i'(y_i(q))\, y_i'(q) \\ &= \sum_{i \in N} x_i^{p\prime}(q) + \alpha \sum_{i \in N} c_i'(y_i(q))\, y_i'(q). \end{aligned}$$

By equation (18),

$$X^{D\prime}(q) = \sum_{i \in N} x_i^{p\prime}(q) + \frac{\alpha}{1 + \alpha\, q} \sum_{i \in N} [\bar{x}_i - x_i^p(q) - x_i^c(q)]. \tag{19}$$

Let $q^* \in \mathbb{R}_+$ be an equilibrium price of the emission permit when the total supply of emission permits is a given $\bar{X} \in \mathbb{R}_+$. Then,

$$\begin{aligned} \sum_{i \in N} [\bar{x}_i - x_i^p(q^*) - x_i^c(q^*)] &= \sum_{i \in N} \bar{x}_i - \sum_{i \in N} x_i(q^*) \\ &= \bar{X} - X^D(q^*) \\ &= 0 \end{aligned} \tag{20}$$

Substituting (20) into (19),

$$X'(q^*) = \sum_{i \in N} x_i^{p\prime}(q^*) < 0.$$

Thus, we have established the following proposition.

Proposition 4 *Let $q^* \in \mathbb{R}_+$ be an equilibrium price of the emission permit. Under Assumption L, the aggregate demand function for emission permits is decreasing in a small neighborhood of q^*.*

4.3 Maximization of preferences of each country in international negotiations

The previous subsection showed the property of the aggregate demand function for emission permits. Based on the property, this subsection examines how the equilibrium consumption of each country changes as the total supply of emission permits changes. This reveals a trade-off between reduction of greenhouse gases and consumption of each country. Then, for each country, the condition for an optimal amount of total supply of emission permits is determined, taking the trade-off into account.

Let $q(X)$ be the equilibrium price when the total supply of emission permits is X. It follows form Proposition 4 that $q'(X) < 0$ under Assumption L.

In the first stage of the game, each country claims how much the total supply of emission permits should be. Each country tries to maximize its preferences $V_i(c_i, X)$, taking account of the fact that, once the total supply X is determined in the first stage, the production and the consumption of country i are determined at the equilibrium price $q(X)$ in the final stage.

We assume that the assignment of emission permits to each country is proportional to the total supply of emission permits. Let $(\theta_1, \ldots, \theta_n) \in]0, 1[^n$ with $\sum_{i \in N} \theta_i = 1$ be the proportional factors.

If a total supply of emission permits X is given, then the equilibrium price, the production level and the consumption level of each country i are determined, which then determines the welfare level of country i. Hence, the welfare of each country is a function of X.

With slight abuse of notation, let $c_i(X)$ be the consumption level of country $i \in N$ when the total supply of emission permits is X. For each $i \in N$, define

$$W_i(X) := V_i(c_i(X), X).$$

Then,

$$W_i'(X) = D_c V_i(c_i, X) c_i'(X) + D_X V_i(c_i, X). \tag{21}$$

A change in the total supply of emission permits, X, affects the welfare of country i through two routes: one is the direct effect represented by

$D_X V_i(c_i, X)$, and the other is the indirect effect through the change in consumption, $D_c V_i(c_i, X) c_i'(X)$. To examine how the consumption changes with a change in X, notice first that by feasibility,

$$c_i(X) = y_i(q(X)) + q(X)[\theta_i X - f_i(y_i(q(X))) - g_i(c_i(y_i(q(X)))).$$

Differentiating,

$$\begin{aligned} c_i'(X) &= y_i'(q)q'(X) + q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)] \\ &\quad + q(X)[\theta_i - f_i'(y_i)y_i'(q)q'(X) - g_i'(c_i)c_i'(X)]. \end{aligned} \tag{22}$$

The terms of equation (22) represent the following effects of an increase in X on c_i.
(i) $y_i'(q)q'(X) > 0$: the increase in production due to the fall in q.
(ii) $q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)]$: the change in revenue from (or the payment for) emission permits due to the fall in q. If country i is a net supplier of emission permits, then $q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)] < 0$, and if it is a net demander, $q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)] > 0$.
(iii) $q(X)\,\theta_i > 0$: the increase in the revenue from (or the decrease in payment for) emission permits due to the increase in the initial assignment of emission permits to country i.
(iv) $-q(X)[f_i'(y_i)y_i'(q)q'(X)] < 0$: the decrease in the revenue from (or the increase in payment for) emission permits due to the increase in emission through production.
(v) $-q(X)[g_i'(c_i)c_i'(X)]$: the change in the revenue from (or the payment for) emission permits due to the change in emission through consumption.

By Proposition 3, $q(X)f'(y_i(q(X))) = 1$. Hence, the effect (i) is just offset by the effect (iv). This is because each country maximizes its consumption in the second stage of the game. Therefore,

$$c_i'(X) = \frac{\theta_i q(X) + q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)]}{1 + q(X)g_i'(c_i)} \tag{23}$$

The value $c_i'(X)$ may be called the *marginal opportunity cost of reduction of greenhouse gases for country i in terms of its own consumption under the system of emission permits trade.*

Substituting (23) and (10) into (21), we have

$$\frac{W_i'(X)}{D_c V_i(c_i, X)} = \frac{\theta_i q(X) + q'(X)[\theta_i X - f_i(y_i) - g_i(c_i)]}{1 + q(X)g_i'(c_i)} - \eta_i(c_i, X). \tag{24}$$

Let X_i^* be the amount of total supply of emission permits that is optimal for country i, and let $y_i^* := y_i(q(X_i^*))$ and $c_i^* := c_i(X_i^*)$. If $X = 0$, then $c_i = 0$, which is clearly not optimal for country i. Hence, $X_i^* > 0$ and $W_i'(X_i^*) = 0$. From (24),

$$\eta_i(c_i^*, X_i^*) = \frac{\theta_i q(X_i^*) + q'(X_i^*)[\theta_i X_i^* - f_i(y_i^*) - g_i(c_i^*)]}{1 + q(X_i^*) g_i'(c_i^*)} \tag{25}$$

Let us summarize the analysis as follows.

Proposition 5 *Suppose that the initial assignments of emission permits to the countries are proportional to the total supply X. Let X_i^* be the amount of the total supply of emission permits that maximizes the welfare of country i, and let c_i^* be the level of consumption at $q(X_i^*)$. At (c_i^*, X_i^*), country i's marginal rate of substitution of its own consumption for global emission of greenhouse gases is equal to country i's marginal opportunity cost of reduction of greenhouse gases in terms of its own consumption under the system of emission permits trade. The latter is equal to*

$$\frac{\theta_i q(X_i^*) + q'(X_i^*)[\theta_i X_i^* - f_i(y_i^*) - g_i(c_i^*)]}{1 + q(X_i^*) g_i'(c_i^*)}$$

where $\theta_i \in]0, 1[$ is the proportion of the initial assignment of emission permits to country i.

5 Non-optimality of bargaining outcomes

As the analyses in the previous sections have shown, once the amount of the total supply of emission permits is determined in international negotiations in the first stage, an equilibrium price of the emission permit is established in the market of emission permits, which then determines the level of consumption of each country. Hence, the welfare level of each country depends solely on the total supply of emission permits. In other words, by tracing the vector of welfare levels attained by all the countries at each amount of the total supply of emission permits, we can draw the welfare possibility frontier in the international negotiations. This gives the set of feasible welfare vectors in the theory of bargaining à la Nash (1950), and by applying one of various bargaining solutions proposed in the literature, we may predict the outcome of the international negotiations.

However, as shown next, vectors on the welfare possibility frontier in these international negotiations are not necessarily Pareto optimal. That is, there exists a technologically feasible allocation for which the welfare of every country is at least as good as at the vector on the frontier, and the welfare of some country is strictly higher. We show this fact by focusing on the the level of the total supply of emission permits that maximizes the welfare of one country. Such a level of the total supply clearly supports a welfare vector on the frontier, but the associated allocation is not Pareto optimal except for a rare case. We first consider the rare case, and then the general case.

Proposition 6 *For each $i \in N$, let X_i^* be the amount of total supply of emission permits that maximizes the welfare of country i. Under Assumption L, if $X_i^* = X_j^* := X^*$ for all $i, j \in N$, then the allocation attained at X^* is Pareto optimal.*

Proof. By Proposition 5 and Assumption L, for all $i \in N$,

$$\eta_i(c_i^*, X^*) = \frac{\theta_i q(X^*) + q'(X^*)[\theta_i X^* - f_i(y_i^*) - g_i(c_i^*)]}{1 + \alpha q(X^*)}$$

where c_i^* and y_i^* are consumption and production of country i achieved at X^*. Summing up the n equations,

$$\begin{aligned}
\sum_{i \in N} \eta_i(c_i^*, X^*) &= \frac{1}{1 + \alpha q(X^*)} \left(q(X^*) \sum_{i \in N} \theta_i + q'(X^*)[X^* \sum_{i \in N} \theta_i - \sum_{i \in N}(f_i(y_i^*) + g_i(c_i^*))] \right) \\
&= \frac{1}{1 + \alpha q(X^*)} \left(q(X^*) + q'(X^*)[X^* - \sum_{i \in N}(f_i(y_i^*) + g_i(c_i^*))] \right) \\
&= \frac{q(X^*)}{1 + \alpha q(X^*)} \\
&= \frac{1}{\frac{1}{q(X^*)} + \alpha}
\end{aligned}$$

By Proposition 3, $\frac{1}{q(X^*)} = f_i'(y_i^*)$ for all $i \in N$. Notice also that $\alpha = g_i'(c_i^*)$ for all $i \in N$. Hence,

$$\sum_{i \in N} \eta_i(c_i^*, X^*)(f_i'(y_i^*) + g_i'(c_i^*)) = 1$$

■

Proposition 7 *Let $N = \{1, 2\}$. For each $i \in N$, let X_i^* be the amount of total supply of emission permits that maximizes the welfare of country i. Under Assumption L, if $X_1^* \neq X_2^*$, then for each $i \in N$, the allocation attained at X_i^* is not Pareto optimal.*

Proof. By Proposition 5 and Assumption L,

$$\eta_1(c_1(X_1^*), X_1^*) = \frac{\theta_1 q(X_1^*) + q'(X_1^*)[\theta_1 X_1^* - f_1(y_1^*(q(X_1^*)) - g_1(c_1^*(X_1^*)]}{1 + \alpha q(X_1^*)}$$

Since $X_1^* \neq X_2^*$,

$$\eta_2(c_2(X_1^*), X_1^*) \neq \frac{\theta_2 q(X_1^*) + q'(X_1^*)[\theta_2 X_1^* - f_2(y_2^*(q(X_1^*)) - g_1(c_2(X_1^*))]}{1 + \alpha q(X_1^*)}$$

Adding both sides,

$$\sum_{i \in N} \eta_i(c_i(X_1^*), X_1^*) \neq \frac{1}{\frac{1}{q(X_1^*)} + \alpha}$$

Hence,

$$(\frac{1}{q(X_1^*)} + \alpha) \sum_{i \in N} \eta_i(c_i(X_1^*), X_1^*) \neq 1$$

By Proposition 3,

$$\sum_{i \in N} \eta_i(c_i(X_1^*), X_1^*)(f_i'(y_i(q(X_1^*))) + g_i'(c_i(X_1^*)) \neq 1$$

■

6 Conclusion

In this paper we have examined how effective an international cooperation would be for abatement of global warming. First, we have derived the condition for Pareto optimal allocations, which is an extension of the celebrated Samuelson condition to the case of greenhouse gas emissions. Second, we have built a three-stage game of international negotiations on regulation of global emissions with emission permits trading, and examined whether a final allocation is Pareto optimal or not. We have shown that production efficiency of a final allocation is always met at an equilibrium of the game. This is because once a level of global emission is determined by international negotiations in the first stage of the game, the government of each country chooses its domestic rule on emissions so as to maximize the amount of its own consumption in the second stage of the game, and firms act as profit-maximizers in the third stage. However, we have also shown that overall Pareto optimality of a final allocation may not be satisfied. The reason is because in international negotiations in the first stage of the game, countries take account of the effect of the total supply of emission permits on the revenue from or the expenditure for emission permits.

There are some limits to the analysis in this paper. First, we assume that an initial distribution rule of emission permits among countries is given. Though the *total* emissions of greenhouse gases determine the speed of global warming, it is also an important issue how the total emission permits should be distributed initially. Second, we fix the participants (signatories of the Kyoto protocol) in negotiations, and ignore the behavior of non-participants (non-signatories). In fact, before starting negotiations there should be another stage in which each country decides whether it participates in the mechanism or not. To develop a more comprehensive model which takes account of these factors is left for future researches.

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