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Spatial Competition and Accumulation of Public Capital *

Abstract

This paper examines the effect of public capital accumulation on private sectors' productivity in a general equilibrium model where a public capital, such as a transportation infrastructure, affects households' disutility of moving. The focus is on indirect channels through which it affects the productivity. The study finds that the accumulation of public capital does not necessarily enhance the productivity of private sectors when there are plenty of initial public capital or the productivity of public sectors is low. However, it also finds that there are cases where public capital accumulation improves social welfare even if it reduces the productivity.

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1 Introduction

In the literature on macroeconomics, researchers often use theoretical models where government activities are assumed to enhance the productivity of private sectors.[1] In such models there is an underlying assumption that government expenditures are used to maintain and improve public infrastructure, or to accumulate public capital for supporting private sectors' activities.

However, empirical researches report mixed results. As for the U.S. economy, Aschauer (1989a) and others[2] show that the capital accumulation in public sectors has a significant impact on private sector productivity. On the other hand, Holtz-Eakin (1994) shows that there is no relationship between the accumulation of public capital and the productivity of private sectors when unobserved state-specific characteristics are controlled. Then, Holtz-Eakin concludes that researches without controls for these effects only find the fact that more prosperous states are likely to spend more on public capital.[3]

As for the Japanese economy, many researches estimate the productivity effect of public capital, too. [4] However, another reason prevents us from estimating the effect correctly. That is, in Japan the central government provides a subsidy to local government in poor regions to help finance public investment in the regions. Thus, since poorer regions are likely to spend more on public capital, it is difficult to detect the productivity effect of public capital without controlling such a region-specific effect. Iwamoto et al. (1996) conduct an empirical analysis with controls for the effect. However, they do not find the productivity effect of public capital during the period from 1975 to 1984. They interpret this result as a consequence that the central government attaches greater importance to the effect on income reallocation among regions than to that on

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[1] See Barro (1990) and Futagami, Morita and Shibata (1993) for models of economic growth, and Baxter and King (1993) for a business cycle model, for example. Neill (1996), Devarajan, Xie and Zou (1998) and Fisher and Turnovsky (1998) are recent researches adopting this assumption.

[2] See Munnell (1990) Ford and Poret (1991).

[3] See also a survey article by Gramlich (1994) for logical and econometric problems laid in the researches at earlier stage. Glomm and Ravikumar (1997) is another survey article that emphasizes on theoretical aspects. Sturm, Jacobs and Groote (1999) is a recent example of empirical researches.

[4] See Asako et al. (1994), Mitsui, Takezawa and Kawachi (1995) and Iwamoto et al. (1996) for example.

the productivity during this period.

Previous researches, whether theoretical or empirical, typically use an aggregate production function that has government expenditure or public capital as input.[5] That is, they *assume* that government activities have a direct effect on technologies of all private sectors. Under this assumption, we cannot analyze through what channels the government activities affect the productivity of private sectors.

Holtz-Eakin and Lovely (1996) adopt a somewhat different approach. They use a Dixit-Stiglitz-Ethier model[6] where a final goods sector only uses intermediate goods while public capital has a direct cost-saving effect on an intermediate goods sector, not on the final goods sector. Then the researchers find that the accumulation of public capital may not enhance the productivity of the final goods sector, depending on the degree of market powers which firms in the intermediate goods sector possess. Moreover, they conduct an empirical analysis using state-level panel data and confirm their theoretical findings. In addition, recently Chandra and Thompson (2000), after examining the effect of public capital on a market structure with a partial equilibrium model, a variation of Hotelling's (1929) spatial competition, offer an interesting hypothesis. They propose that a public infrastructure investment such as highway construction has a differential impact across industries and regions. Then, the researchers obtain a finding to support their hypothesis in their empirical analysis. These results indicate that public capital accumulation does not necessarily improve *aggregate* performance in private sectors, while there are *specific* sectors receiving benefits from it.

In this paper, we theoretically examine the productivity effect of public capital in depth, by focusing on indirect channels through which public capital affect the productivity of private sectors. Specifically, combining a Dixit-Stiglitz-Ethier model with a Salop-Weitzman model of

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[5] There also exists a model where government expenditures affect private investment technology. See Aschauer (1989b) and Glomm and Ravikumar (1997) for example.

[6] See Dixit and Stiglitz (1977) and Ethier (1982).

retail competition,[7] a variant of Hotelling's model, we construct a general equilibrium model where public capital, such as a transportation infrastructure affecting households' disutility of moving to retail stores, indirectly influences the productivity of manufacturers through equilibrium interactions. It should be emphasized that our model differs from Holtz-Eakin and Lovely (1996) and Chandra and Thompson (2000) in important ways: our model does not assume any direct cost-reducing effect of public capital on the supply side, while Holtz-Eakin and Lovely (1996) do; it considers a general equilibrium effect of infrastructure investment, while Chandra and Thompson's (2000) can not in their partial equilibrium framework.

The rest of this paper is organized as follows. Section 2 presents our analytical framework. Section 3 analyzes equilibrium allocation and examines the effect of public capital on the productivity. Section 4 investigates a welfare effect of public capital. Section 5 contains concluding remarks.

2 The Model

Our economy consists of households, manufacturers, retailers, and the government. Households consume a single final good made from a variety of differentiated intermediate goods. Manufacturers are classified into two types, one that produces final goods and the other that does intermediate goods. We call the former "final goods manufacturers" and the latter "intermediate goods manufacturers." Retailers buy final goods from the final goods manufacturers and sell them to the households. The government accumulates a public capital. In what follows, we characterize these economic agents respectively.

Household

Households are uniformly distributed around a circle of unit circumference. The density of households is N around this circle and each household supplies one unit of labor inelastically.

[7] See Salop (1979) and Weitzman (1982). Strictly speaking, they deal with models of horizontal product differentiation. The former deals with a partial equilibrium model while the latter does a general equilibrium model. We apply the model of Weitzman (1982) to an analysis of spatial competition in a retail market.

Thus, the total labor supply equals N. If a household buys c units of the final goods from a retailer located at a distance i away and consumes them, it achieves a utility[8] ,

$$u(c,i) = c - \phi(\gamma)\, i, \tag{1}$$

where $\phi(\gamma)$ is the unit disutility of moving to a store which is assumed to be related to a public capital, γ. We explain it in detail later.

Final goods manufacturer

A production function for a representative final goods manufacturer takes a form of symmetric CES;

$$Y = \left\{ \int_0^n [x(z)]^{1-\frac{1}{\sigma}} dz \right\}^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1, \tag{2}$$

where $x(z)$ is the amount of intermediate goods z employed in production, $[0,n]$ represents the range of the intermediate goods available in the marketplace, and σ is the direct partial elasticity between each pair of intermediates. The cost function and an input demand associated with (2) are given as

$$C^Y = PY \tag{3}$$

$$x(z) = \left(\frac{p(z)}{P} \right)^{-\sigma} Y, \tag{4}$$

respectively, where $p(z)$ denotes price of intermediate goods z, while P is a unit cost defined as follows

$$P \equiv \left\{ \int_0^n [p(z)]^{1-\sigma} dz \right\}^{\frac{1}{1-\sigma}}. \tag{5}$$

We assume that a wholesale market of final goods where final goods manufacturers sell them to retailers is perfectly competitive.

[8] The specification (1) is the same as in Weitzman (1982). However, Solow (1986) asserts that (1) is inappropriate for a product differentiation model because the marginal utility of consumption is independent of the character of goods consumed. Instead, he proposes a specification, $u(c,i) = c\exp(-\mu i)$, for example. Taking this into account, Weitzman (1994) adopts the latter. However, the former is appropriate for a retail market model because it is plausible to consider that the (marginal) utility of consumption is independent of the disutility of moving.

Intermediate goods manufacturer

A differentiated intermediate good is produced by an intermediate goods manufacturer. When intermediate goods manufacturer z supplies $x(z)$ units, it requires

$$L_z^I = a^I x(z) + F^I, \quad z \in [0, n] \tag{6}$$

units of labor where a^I is a marginal labor requirement and F^I is a fixed labor input. We assume that a market of intermediate goods where intermediate goods manufacturers sell them to final goods manufacturers is monopolistically competitive since the intermediate goods are differentiated from one another.

Retailer

There are m retailers whose stores are located around the circle. Retailer j must hire

$$L_j^R = F^R, \quad j = 1, 2, ..., m$$

units of labor to set up its own store. If retailer j distributes y_j units of final goods bought from manufacturers to households, its cost function is represented as

$$C_j^R = p^W y_j + w L_j^R, \tag{7}$$

where p^W is the wholesale price of the final goods and w is the nominal wage. We assume that a retail market of the final goods is monopolistically competitive since the retailers have a spatial market power.

Government

The government is assumed to be able to decrease household's disutility of moving by accumulating a public capital. The government must hire

$$L^G = a^G (\gamma - \gamma_0), \tag{8}$$

units of labor to increase the public capital from the initial level, γ_0, to γ. a^G is a marginal labor requirement to increase the public capital. We assume that γ is related with $\phi(\gamma)$ such that

$d\phi/d\gamma < 0$ and $d^2\phi/d\gamma^2 > 0$. That is, the household's unit disutility, $\phi(\gamma)$, decreases when the government accumulates the public capital, γ. However, the marginal effect diminishes.[9] Finally, to pay a wage to workers employed in the public sector, the government collects lump sum tax from each household. Let T denote per capita tax. To balance the budget of the government, it must hold that $T = wL^G/N$.

3 Equilibrium

In this section, we characterize an equilibrium of the model set up in the previous section. We assume that these events occur in the following sequence: (1) the government accumulates a public capital; (2) intermediate goods manufacturers enter the market; (3) retailers enter the market; and (4) the retailers set their price to maximize their profit while each retailer's market area is determined by the households' utility maximization. On the other hand, the manufacturers maximize their profit, too.

3.1 Fourth stage: utility and profit maximization

We assume that the retailers are spaced equally around the circle. Thus the distance between any neighboring retailers is $1/m$. We limit our attention to a symmetric equilibrium.

Determination of retailer j's market area

Suppose that retailers except retailer j set their price at p^R and that retailer j sets its price at p_j^R and can attract households in a range of $-h\left(p_j^R\right)/2$ to $+h\left(p_j^R\right)/2$ centered on its location. Thus a marginal buyer, who is indifferent between purchasing from retailer j and its nearest neighbors, must be located on the site at distance $h\left(p_j^R\right)/2$ from the former and at $(1/m) - h\left(p_j^R\right)/2$ from

[9] In the product differentiation literature, von Ungern-Sternberg (1988) and Weitzman (1994) construct similar models where the degree of product differentiation is a private firm's choice variable and endogenously determined in equilibrium. Our model of spatial competition differs from theirs in that the disutility of moving is the government's choice variable.

the latter. Consequently, from (1), it must be satisfied that

$$\frac{I}{p_j^R} - \phi(\gamma)\left[\frac{h\left(p_j^R\right)}{2}\right] = \frac{I}{p^R} - \phi(\gamma)\left[\frac{1}{m} - \frac{h\left(p_j^R\right)}{2}\right], \tag{9}$$

where I is each household's income, consisting of a wage and profits except tax, given as $I = w + (1/N)\left(n\pi^I + m\pi^R\right) - T$. π^I is the profit of each intermediate goods manufacturer and π^R is that of each retailer. Rearranging (9), we obtain retailer j's market area,

$$h\left(p_j^R\right) = \frac{1}{m} + \frac{I}{\phi(\gamma)}\left(\frac{1}{p_j^R} - \frac{1}{p^R}\right). \tag{10}$$

Considering (10) and the fact that N households located at each point in the circle as well as each household in the retailer j's market area buys I/p_j^R units, we obtain a demand for the retailer j,

$$D\left(p_j^R\right) = \frac{I}{p_j^R}\left[Nh\left(p_j^R\right)\right] = \alpha\left(\frac{1}{p_j^R}\right) + \beta\left(\frac{1}{p_j^R}\right)\left(\frac{1}{p_j^R} - \frac{1}{p^R}\right), \tag{11}$$

where $\alpha = IN/m$, $\beta = I^2N/\phi(\gamma)$.

Profit maximization of retailer

Retailer j maximizes its profit with respect to its own price p_j^R given other retailers' price p^R, using its local market power. From (7) and (11), the optimal condition is given as

$$p_j^R\left(1 - \frac{1}{E}\right) = p^W. \tag{12}$$

E in (12) is the price elasticity of the demand (11), $-\left(p_j^R/D\right)D'$ where $D^{'}$ is the first derivative of D. This elasticity is given as

$$E = 1 + \frac{\beta}{\alpha p^R}, \tag{13}$$

when evaluated in a symmetric equilibrium ($p_j^R = p^R$). Hence, substituting p^R into p_j^R in (12) and rearranging it, using (13), we obtain a retail price and a demand in a symmetric equilibrium[10] ,[11]

[10] It is straightforward to show that the undercut strategy that the retailer j cuts its price big enough to capture its neighbor's entire market is unprofitable when the neighbor sets price at (14). This result is the same as Salop (1979).

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$$p^R = \frac{p^W}{1 - p^W \left(\frac{\phi(\gamma)}{mI}\right)}, \tag{14}$$

$$D = \frac{\alpha}{p^R} = \frac{IN}{mp^W}\left[1 - p^W \frac{\phi(\gamma)}{mI}\right]. \tag{15}$$

Profit maximization of manufacturer

Since the wholesale market of final goods is perfectly competitive and the market of intermediate goods is monopolistically competitive, from (3), (4) and (6) the profit maximization conditions for the final goods manufacturers and the intermediate goods manufacturers are given as

$$p^W = P, \tag{16}$$

$$p(z)\left(1 - \frac{1}{\sigma}\right) = a^I w, \tag{17}$$

respectively. Here, we use normalization such that $a^I = 1 - 1/\sigma$.[12] Thus, (17) is reduced to

$$p = w, \tag{18}$$

independent of the type of intermediate goods, z. Hereafter, we take labor as numeraire and set the nominal wage equal to one ($w = 1$). When we substitute (18) into (16) taking (5) and $w = 1$ into account, we obtain

$$p^W = n^{\frac{1}{1-\sigma}}. \tag{19}$$

[11] For the equilibrium to be well-defined, that is, for the price and the demand in the symmetric equilibrium to be positive, it must hold that $1 - p^W [\phi(\gamma) / (mI)] > 0$, which we can show under the condition that N is sufficiently large or γ is in the vicinity of γ_0.

[12] This normalization, which is also employed in Matsuyama (1995), is used to simplify the notation without affecting any result obtained later.

3.2 Third stage: entry of retailers

When we substitute (14) and (15) into retailer's profit, $\pi^R = \left(p^R - p^W\right) D - F^R$, taking (19) into account, we obtain $\pi^R = \left[N\phi\left(\gamma\right) n^{\frac{1}{1-\sigma}}\right]/m^2 - F^R$. Given n, which is already determined in the second stage, it holds that $d\pi^R/dm < 0$. That is, the entry-exit process of retailers is stable. Ignoring an integer constraint, we obtain the number of retailers determined by zero profit condition, $\pi^R = 0$,

$$m^*\left(\gamma, n\right) = \sqrt{\frac{N\phi\left(\gamma\right) n^{\frac{1}{1-\sigma}}}{F^R}}. \tag{20}$$

From (20), we find that $\partial m^*/\partial\gamma < 0$ and $\partial m^*/\partial n < 0$. These are explained as follows. If more public capital accumulates, the competition in the retail market becomes fierce as the result of small disutility of moving. Thus, the retailers cut their price and earn small gross profit (i.e. profit excluding the fixed cost, F^R). Consequently, the number of retailers, m^*, becomes small. On the other hand, if the number of intermediate goods manufacturers, n, is large, from (19) we find that the marginal cost of each retailer becomes small. Thus the retailers cut their price and sell large amounts. However, the revenue is constant in the equilibrium and the variable cost is large because the retailers sell such large amounts. Therefore, the gross profit becomes small and, as a result, m^* becomes small too when n is large.

3.3 Second stage: entry of intermediate goods manufacturers

When we substitute (18) into intermediate goods manufacturer's profit, $\pi^I = px - \left(ax + F^I\right)$ taking $w = 1$ and $a^I = 1 - 1/\sigma$ into account, we obtain $\pi^I = x/\sigma - F^I$.

In the second stage, the profit of intermediate goods manufacturer must equal zero ($\pi^I = 0$) and a labor market, which is assumed to be perfectly competitive, must clears;

$$x = \sigma F^I, \tag{21}$$

$$N = \overline{L}^R + \overline{L}^I + L^G, \tag{22}$$

where $\overline{L}^R$ and $\overline{L}^I$ are the total labor demands for the retail sector and the intermediate goods sector given as

$$\overline{L}^R = m^*(\gamma, n) F^R, \tag{23}$$

$$\overline{L}^I = n\left[\left(1 - \frac{1}{\sigma}\right)x + F^I\right], \tag{24}$$

respectively. Substituting (21) into (24), we obtain

$$\overline{L}^I = n\left(\sigma F^I\right). \tag{25}$$

Thus, from (8), (22), (23) and (25), we find that n^* is determined by

$$f(n^*) = \sigma F^I, \tag{26}$$

where $f(n)$ is defined as

$$f(n) = \frac{1}{n}\left[N - a^G(\gamma - \gamma_0) - m^*(\gamma, n) F^R\right]. \tag{27}$$

That is, $f(n)$ is labor available for each of the intermediate goods manufacturer. As shown in Appendix A, $f(n)$ is single peaked under the condition that N is sufficiently large or γ is in the vicinity of γ_0.

In addition, for the entry-exit process of intermediate goods manufacturers to be stable, the number of intermediate goods determined by (26), n^*, must satisfy

$$f'(n^*) < 0, \tag{28}$$

provided that f' is the first derivative of f. Thus, if F^I is small enough, $f(n)$ and σF^I intersect with each other at two points and the right point satisfies (28). (See figure 1.)

3.4 First stage: accumulation of public capital

We consider an effect of a marginal increase in public capital from the initial level, γ_0. Using (27), we can express (26) in another form,

$$N - a^G(\gamma - \gamma_0) - m^*(\gamma, n^*) F^R = n^*\left(\sigma F^I\right). \tag{29}$$

The left hand side of (29) represents the total labor supply except the employment of the public sector and that of the retail sector. On the other hand, the right hand side of (29) represents the employment of the intermediate goods sector.

Before analyzing the effect, as a preliminary step, we first investigate the relationship between the initial level of the public capital and the number of intermediate goods available in the market when the government does not accumulate the public capital at all, which means $\gamma = \gamma_0$. Since the government does not employ any labor in this case, substituting γ_0 into γ in (29), we can reduce (29) to

$$N - m^* (\gamma_0, n^*) F^R = n^* \left(\sigma F^I\right). \tag{30}$$

The left hand side of (30) is the total labor supply minus the employment for the retail sector. Recall that, when γ is large, the equilibrium number of retailers m^* becomes small because of the fierce competition in the retail market. Thus, when the initial public capital γ_0 is large, the left hand side of (30) becomes large, too. It means that the labor available for the intermediate goods sector increases. As the result of the reallocation of resources, the number of the intermediate goods n^* increases for (30) to hold.[13]

Now, we are ready to examine the effect of the marginal increase of public capital on the economy. The marginal increase of the public capital has two effects on the left hand side of (29). First, to accumulate the public capital, the government must hire labor. Thus, the left hand side of (29) decreases by a^G, which means that the labor available for the intermediate goods sector decreases. Second, accumulating the public capital, however, makes the retail market competition more fierce and the employment for the retail sector decreases. Therefore, the left hand side of (29) increases by $(-\partial m^*/\partial\gamma) F^R$ where $\partial m^*/\partial\gamma$ is evaluated at $\gamma = \gamma_0$. This means that the intermediate goods sector can employ more labor.

[13] To be precise, we must consider a derived effect of the change of n^* on m^*. To take this effect into account, we must consider (30) as an implicit function which has n^* as its dependent variable and γ_0 as its independent variable and apply the implicit function theorem to it. After some calculation, we can find that $dn^*/d\gamma_0 > 0$, provided that the stability condition (28) holds.

Which of the above two effects dominates determines whether or not the marginal accumulation of the public capital increases the equilibrium number of the intermediate goods. If

$$\left(-\frac{\partial m^*}{\partial \gamma}\right) F^R - a^G > 0 \tag{31}$$

holds, more labor can be used for the intermediate goods sector by accumulating the public capital and n^* increases. On the other hand, if (31) does not hold, less labor can be used for it and n^* does not increase.

Here, we specify the effect of the public capital on disutility of moving, $\phi(\gamma)$, such as

$$\phi(\gamma) = \frac{\theta}{\gamma}, \tag{32}$$

where θ is a parameter. Substitute (32) into (20) and differentiate it with respect to γ, we obtain

$$\left(-\frac{\partial m^*}{\partial \gamma}\right) F^R = \frac{1}{2}\left(\theta N F^R\right)^{\frac{1}{2}} \left[(n^*)^{\frac{1}{1-\sigma}}\right]^{\frac{1}{2}} (\gamma_0)^{-\frac{3}{2}}, \tag{33}$$

when $\partial m^*/\partial \gamma$ is evaluated at $\gamma = \gamma_0$. The right hand side of (33) is a decreasing function of γ_0 and converges to zero when γ_0 approaches to infinity.[14] Therefore, we find that $(-\partial m^*/\partial \gamma) F^R$ and a^G intersect only at a point.

Finally, substituting the output level of each intermediate good (21) into (2), we obtain the output level of final goods,

$$Y(n^*) = (n^*)^{\frac{\sigma}{\sigma-1}} \left(\sigma F^I\right), \tag{34}$$

which is an increasing function of n^*. Moreover, from (34), we find that $Y(n^*)/\left[n^*\left(\sigma F^I\right)\right] = (n^*)^{\frac{1}{\sigma-1}}$, that is, the (average) productivity of the final goods sector increases (or decreases) when n^* increases (or decreases).

In the end, we obtain the following proposition.[15]

[14] Note that γ_0 indirectly affects $(-\partial m^*/\partial \gamma) F^R$ through n^* since n^* is an increasing function of γ_0. A large γ_0 implies a large n^* which makes $(-\partial m^*/\partial \gamma) F^R$ decrease too, as γ_0 directly does. Hence, we find that $(-\partial m^*/\partial \gamma) F^R$ is a decreasing function of γ_0.

[15] Again, to be precise, we must consider the derived effect of the change of n^* on m^*. See Appendix B for a proof of Proposition 1 using the implicit function theorem.

Proposition 1 *If (31) is satisfied, a marginal accumulation of public capital enhances the productivity of final goods manufacturers. On the other hand, if (31) is not satisfied, it reduces the productivity of the final goods manufacturers.*

This proposition shows that the accumulation of the public capital induces the reallocation of resources among sectors but does not necessarily enhance the productivity of private sectors. Key parameters are γ_0 and a^G. If the former, the initial level of public capital, is small, as well as the latter, the marginal requirement of labor in the public sector, is also small, which implies the sector's high productivity, then (31) holds and accumulating public capital induces the private sector's productivity to rise. On the other hand, if both the former and the latter are large, (31) does not hold and accumulating public capital may reduce the private sector's productivity.[16]

The fact that the productivity effect of public capital depends on the initial level of the capital may confirm a remark in Gramlich (1994, p 1187) that "Simply saying that some capital has been productive in the past .... does not mean that future investments will also be productive." Additionally, this fact may be a reason why the productivity effect of public capital can not be found in the Japanese economy.

More generally, the results obtained in this section indicates that empirical analyses using an aggregate production function with public capital as input does not capture general equilibrium effect of the public capital accumulation through the reallocation of input. It also suggests that a relevant model must be constructed before empirically examining the productivity effect of public capital. Holtz-Eakin and Lovely (1996) conduct such an analysis and their findings are useful for considering further elaboration.

[16] Note that the parameter θ is also important because its level determines the marginal effect of the public capital on the disutility of moving. When θ is large (or small), (33) shifts upward (or downward) then the range within which the accumulation of public capital enhances the productivity of the final goods sector becomes large (or small).

4 Social Welfare

In this section we discuss welfare effect of the marginal accumulation of the public capital. First of all, we define a social welfare function as follows

$$V=\int_0^1 u_i N di, \tag{35}$$

where u_i is a household's utility located on the position $i\in[0,1]$ on the circumference of the circle. Since the distance between any neighboring retailers is $1/m^*(\gamma,n^*)$ in equilibrium, we can rewrite (35) using (1) and (32) as

$$V=m^*(\gamma,n^*)\left\{2\int_0^{\frac{1}{2m^*(\gamma,n^*)}}\left[c-\left(\frac{\theta}{\gamma}\right)i\right]Ndi\right\}, \tag{36}$$

where c is each household's consumption of the final goods. Moreover, since total demands of the final goods, Nc, must be equal to (34) in equilibrium, we can reduce (36) to

$$V(\gamma,n^*)=Y(n^*)-\frac{N(\theta/\gamma)}{4m^*(\gamma,n^*)}. \tag{37}$$

From (37), we find that the social welfare is a function of n^* and γ. Taking (20) and (37) into account, we obtain

$$\left.\frac{\partial V}{\partial\gamma}\right|_{\gamma=\gamma_0}=\frac{1}{8}\left(\theta NF^R\right)^{\frac{1}{2}}(n^*)^{\frac{1}{2(\sigma-1)}}(\gamma_0)^{-\frac{3}{2}}>0,$$

which means that the direct effect of accumulating the public capital on households' disutility of moving improves the social welfare.

However, note that n^* is a function of γ_0. We notice this fact to examine the total welfare effect of it. Differentiating (37) with respect to γ and evaluating it at $\gamma=\gamma_0$, we obtain[17]

$$\left.\frac{dV}{d\gamma}\right|_{\gamma=\gamma_0}=\frac{\partial V}{\partial n^*}\frac{1}{n^*\left[-f'(n^*)\right]}\left[(1+\eta)\left(-\left.\frac{\partial m^*}{\partial\gamma}\right|_{\gamma=\gamma_0}\right)-a^G\right], \tag{38}$$

where

$$\eta=\frac{1}{4}\frac{(n^*)^{\frac{\sigma}{\sigma-1}}\left[-f'(n^*)\right]}{\partial V/\partial n^*}.$$

[17] See Appendix C for derivation of (38).

From (38), we find that the sign of $dV/d\gamma$ is determined by the sign of $(1+\eta)(-\partial m^*/\partial\gamma)\,F^R - a^G$, since $\partial V/\partial n^* > 0$ are positive. Moreover, note that $0 < \eta < 1/4$ holds[18] and that $\lim_{\gamma_0\to\infty}(-\partial m^*/\partial\gamma) = 0$ holds from (33). Thus, we find that $(1+\eta)(-\partial m^*/\partial\gamma)\,F^R$ and a^G have at least a point of intersection.[19] Then, when the initial public capital is small,

$$(1+\eta)\left(-\frac{\partial m^*}{\partial\gamma}\right) - a^G > 0 \tag{39}$$

holds. On the other hand, when the initial public capital is large, (39) does not hold. Therefore, we finally obtain the following proposition.

Proposition 2 *If (39) is satisfied, the marginal accumulation of public capital improves social welfare. On the other hand, if (39) is not satisfied, it reduces the welfare.*

From Proposition 1 and 2, we find that if the marginal public capital accumulation enhances the manufacturer's productivity, it also does the social welfare. More interestingly, we find that there are cases where accumulating public capital improves social welfare even if it reduces the productivity of final goods manufacturers when (31) is satisfied while (39) is not. This is because the direct effect on households' disutility of moving overwhelms the indirect effect on the manufacturer's productivity.

5 Concluding Remarks

In this paper, we theoretically examine the effect of a marginal increase in public capital on the private sectors' productivity, by focusing on indirect channels through which public capital affects the productivity. Specifically, combining a Dixit-Stiglitz-Ethier model of monopolistic competition in an intermediate goods market with a Salop-Weitzman model of spatial competition in a retail market, we construct a general equilibrium model where public capital, such as a transportation infrastructure, which affects households' disutility of moving, indirectly influences the productivity of manufacturers through equilibrium interactions. The study finds that, when initial public

[18] See Appendix C to confirm that $\partial V/\partial n^* > 0$ and $0 < \eta < 1/4$.

[19] We cannot rule out the possibility that $(1+\eta)(-\partial m^*/\partial\gamma)$ and a^G have multiple points of intersection, since η is a function of γ_0.

capital is scarce or the productivity of public sectors is high, additional accumulation of public capital enhances the productivity, while the opposite is true when there are plenty of public capital at the beginning or the low productivity of public sector prevails. An effect of public capital on welfare is also examined. If the productivity rises by the public capital accumulation, welfare always improves. More interestingly, there are cases where accumulating public capital improves social welfare even if it reduces the productivity. This is because the direct effect on households' disutility of moving overwhelms the indirect effect on the manufacturer's productivity.

These results obtained in this paper suggest that approaches using an aggregate production function with public capital as input have limitations in understanding the effect of public capital accumulation. It is thus important to construct models using our approach in various fields that are related to public capital (e.g. economic growth and development). Besides, our modeling strategy may be applied to new economic geography models (e.g. Krugman (1991)) where the cost of transporting goods is a parameter and the long-run effect of decreasing this parameter is analyzed. If decreasing transportation cost requires resources as in our model, the short-run effect can be analyzed.

Appendix A

the Shape of $f(n)$

Substitute (20) into (27), we acquire

$$f(n) = \frac{1}{n}\left[A - B(n)^{\frac{1}{2(1-\sigma)}}\right], \tag{A1}$$

where $A = N - a^G(\gamma - \gamma_0)$ and $B = \left(NF^R\right)^{\frac{1}{2}}[\phi(\gamma)]^{\frac{1}{2}}$. Note that A is positive when N is large enough or γ is in the vicinity of γ_0. Rearranging (A1), we obtain

$$f(n) = A\left(\frac{1}{n}\right) - B\left(\frac{1}{n}\right)^{\frac{2\sigma-1}{2(\sigma-1)}}, \tag{A2}$$

$$= \left(\frac{1}{n}\right)^{\frac{2\sigma-1}{2(\sigma-1)}}\left\{A(n)^{\frac{1}{2(\sigma-1)}} - B\right\}. \tag{A3}$$

Recalling that $\sigma > 1$, from (A2), we find that $\lim_{n\to\infty} f(n) = 0$. Similarly, from (A3), we find that $\lim_{n\to 0} f(n) = -\infty$. Next, differentiating (A2) with respect to n, we obtain the first derivative of $f(n)$,

$$f'(n) = -An^{-2} + \frac{2\sigma - 1}{2(\sigma-1)}Bn^{-\frac{2\sigma-1}{2(\sigma-1)}-1}. \tag{A4}$$

Note that $(2\sigma-1)/[2(\sigma-1)] > 1$. Thus, from (A4), we find that, when

$$\widehat{n} = \left[\frac{2\sigma-1}{2(\sigma-1)}\frac{B}{A}\right]^{2(\sigma-1)}, \tag{A5}$$

$f'(\widehat{n}) = 0$ and $f''(\widehat{n}) < 0$ hold, where f'' is the second derivative of f . Finally, substitute (A5) into (A3), we obtain

$$f(\widehat{n}) = \left[\frac{2(\sigma-1)}{2\sigma-1}\frac{A}{B}\right]^{2\sigma-1} B\left[\frac{2\sigma-1}{2(\sigma-1)} - 1\right] > 0.$$

Therefore, from the above discussion, we can conclude that $f(n)$ is single-peaked under the condition that N is large enough or γ is in the vicinity of γ_0.

Appendix B

Proof of Proposition 1

Consider (29) as an implicit function which has n^* as its dependent variable, γ as its independent variable and γ_0 as a parameter. Then, differentiate both sides of (29) with respect to γ and evaluate it at $\gamma = \gamma_0$, we obtain

$$\left.\frac{dn^*}{d\gamma}\right|_{\gamma=\gamma_0} = \frac{\left(-\frac{\partial m^*}{\partial \gamma}\right) F^R - a^G}{\left(\sigma F^I\right) + \frac{\partial m^*(\gamma_0, n^*)}{\partial n} F^R}. \tag{A6}$$

The denominator in the right hand side of (A6) is positive because

$$f'\left(n^*\right) = -\frac{1}{n^*}\left[\left(\sigma F^I\right) + \frac{\partial m^*\left(\gamma_0, n^*\right)}{\partial n} F^R\right] < 0 \tag{A7}$$

holds from (26), (27) and (28). Therefore, finally we find that, if (31) is satisfied, n^* increases. On the other hand, if (31) is not satisfied, n^* decreases.

Appendix C

Derivation of (38)

Differentiate (37) with respect to γ and evaluate it at $\gamma = \gamma_0$, then we obtain

$$\frac{dV}{d\gamma} = \frac{\partial V}{\partial \gamma} + \frac{\partial V}{\partial n^*}\frac{dn^*}{d\gamma}, \tag{A8}$$

where

$$\frac{\partial V}{\partial \gamma} = \frac{1}{8}\left(\theta N F^R\right)^{\frac{1}{2}}\left(n^*\right)^{\frac{1}{2(\sigma-1)}}\left(\gamma_0\right)^{-\frac{3}{2}}, \tag{A9}$$

$$\frac{\partial V}{\partial n^*} = \left(n^*\right)^{\frac{1}{\sigma-1}}\left[\frac{\sigma}{\sigma-1}\left(\sigma F^I\right) - \frac{1}{8\left(\sigma-1\right)}\left(\theta N F^R\right)^{\frac{1}{2}}\left(n^*\right)^{\frac{1}{2(1-\sigma)}-1}\left(\gamma_0\right)^{-\frac{1}{2}}\right]. \tag{A10}$$

In (A8), we omit a subscript $\gamma = \gamma_0$. The first term in the right hand side of (A8) is a direct effect of public capital accumulation on the social welfare, while the second term is an indirect effect through the change of the number of the intermediate goods. From (20) and (A7), we find that

$$n^*\left[-f'\left(n^*\right)\right] = \left(\sigma F^I\right) - \frac{1}{2\left(\sigma-1\right)}\left(\theta N F^R\right)^{\frac{1}{2}}\left(n^*\right)^{\frac{1}{2(1-\sigma)}-1}\left(\gamma_0\right)^{-\frac{1}{2}} > 0. \tag{A11}$$

Therefore, from (A10) and (A11) we observe that

$$\begin{aligned}
\frac{\partial V}{\partial n^*} &= (n^*)^{\frac{1}{\sigma-1}} \left[\frac{\sigma}{\sigma-1} \left(\sigma F^I\right) - \frac{1}{8(\sigma-1)} \left(\theta N F^R\right)^{\frac{1}{2}} (n^*)^{\frac{1}{2(1-\sigma)}-1} (\gamma_0)^{-\frac{1}{2}} \right] \\
&> (n^*)^{\frac{1}{\sigma-1}} \left[\left(\sigma F^I\right) - \frac{1}{2(\sigma-1)} \left(\theta N F^R\right)^{\frac{1}{2}} (n^*)^{\frac{1}{2(1-\sigma)}-1} (\gamma_0)^{-\frac{1}{2}} \right] \\
&= (n^*)^{\frac{\sigma}{\sigma-1}} \left[-f'(n^*)\right] > 0. \qquad \text{(A12)}
\end{aligned}$$

Finally, substitute (A6) into (A8) and rearrange it, we obtain

$$\frac{dV}{d\gamma} = \frac{\partial V}{\partial n^*} \frac{1}{n^* \left[-f'(n^*)\right]} \left[(1+\eta) \left(-\frac{\partial m^*}{\partial \gamma} \right) - a^G \right],$$

where

$$\eta = \frac{1}{4} \frac{(n^*)^{\frac{\sigma}{\sigma-1}} \left[-f'(n^*)\right]}{\partial V / \partial n^*}.$$

From (A12), we observe that $0 < \eta < 1/4$.

References

Asako, Kazumi, Jun Tsuneki, Shin-ich Fukuda, Hiroshi Teruyama, Takashi Tsukamoto, and Masanori Sugiura. "Productivity of Government Capital and the Welfare Evaluation of Government Investment Policy" (in Japanese) *Economic Analysis*, no. 135, (1994), the Economic and Social Research Institute, Japan.

Aschauer, David A. "Is Public Expenditure Productive?" *Journal of Monetary Economics*, 23 (1989a): 177-200.

Aschauer, David A. "Does Public Capital Crowd Out Private Capital?" *Journal of Monetary Economics*, 24 (1989b): 171-188.

Barro, Robert J. "Government Spending in a Simple Model of Endogenous Growth" *Journal of Political Economy*, 98 (1990): S103-S125.

Baxter, Marianne, and Robert G. King. "Fiscal Policy in General Equilibrium" *American Economic Review* 83 (1993): 315-334.

Chandra, Amitabh, and Eric Thompson. "Does Public Infrastructure Affect Economic activity? Evidence from the Rural Interstate Highway System" *Regional Science and Urban Economics* 30 (2000): 457-490.

Devarajan, Shantayanan, Danyang Xie, and Heng-fu Zou. "Should Public Capital be Subsidized or Provided?" *Journal of Monetary Economics*, 41 (1998): 319-331.

Dixit, Avinash K., and Joseph E. Stiglitz. "Monopolistic Competition and Optimum Product Diversity" *American Economic Review*, 67 (1977): 297-308.

Ethier, Wilfred J. "National and International Returns to Scale in the Modern Theory of International Trade" *American Economic Review*, 72 (1982): 389-405.

Fisher, Walter H., and Stephen J. Turnovsky. "Public Investment, Congestion, and Private Capital Accumulation" *Economic Journal*, 108 (1998): 399-413.

Ford, Robert, and Pierre Poret. "Infrastructure and Private-Sector Productivity" *OECD Economic Studies*, no. 17 (1991): 63-89.

Futagami, Koichi, Yuichi Morita, and Akihisa Shibata. "Dynamic Analysis of an Endogenous Growth Model with Public Capital" *Scandinavian Journal of Economics* 95 (1993): 607-625.

Glomm, Gerhard, and B. Ravikumar. "Productive Government Expenditures and Long-Run Growth" *Journal of Economic Dynamics and Control* , 21 (1997): 183-204.

Gramlich, Edward M. "Infrastructure Investment: A Review Essay" *Journal of Economic Literature* 32 (1994): 1176-1196.

Holtz-Eakin, Douglas. "Public-Sector Capital and the Productivity Puzzle" *Review of Economics and Statistics* 76 (1994): 12-21.

Holtz-Eakin, Douglas, and Mary E. Lovely. "Scale Economies, Returns to Variety, and the Productivity of Public Infrastructure" *Regional Science and Urban Economics* 26 (1996): 105-123.

Hotelling, Harold. "Stability in Competition" *Economic Journal* 39 (1929): 41-57.

Iwamoto, Yasushi, Satoshi Oh-uchi, Satoshi Takesita, and Tadashi Bessyo. "Productivity of Public Capital and Interregional Allocation of Public Investment" (in Japanese) *Financial Review*, no. 41, (1996): 242-250. Policy Research Institute, Ministry of Finance, Japan

Krugman, Paul. "Increasing Returns and Economic Geography" *Journal of Political Economy* 99 (1991): 483-99.

Matsuyama, Kiminori."Complementarities and Cumulative Processes in Models of Monopolistic Competition" *Journal of Economic Literature* 33 (1995): 701-29.

Mitsui, Kiyoshi, Yasuko Takezawa, and Shigeru Kawachi. "Interregional Distribution of Social Capital – Estimates Based on Production Function and Cost Function" (in Japanese) *IPTP Review* no. 6 (1995):205-61, Institute for Posts and Telecommunications Policy, Ministry of Posts and Telecommunications, Japan.

Munnell, Alicia H. "Why Has Productivity Growth Declined? Productivity and Public Investment" *New England Economic Review*, Jan/Feb, (1990): 3-22.

Neill, Jon R. "Fueling the Engine of Growth with Investment in Infrastructure: A Lesson from Neoclassical Growth Theory" *Journal of Macroeconomics* 18 (1996): 521-529.

Salop, S. C. "Monopolistic Competition with Outside Goods" *Bell Journal of Economics* 10 (1979): 483-498

Solow, Robert M. "Monopolistic Competition and the Multiplier" ch.12 in W. P. Heller, R. M. Starr, and D. A. Starrett (eds.) *Equilibrium Analysis: Essays in Honor of Kenneth J. Arrow*, vol. 2, (1986): 301-315

Strum, Jan-Egbert, Jan Jacobs, and Peter Groote. "Output Effects of Infrastructure Investment in the Netherlands, 1853-1913" *Journal of Macroeconomics* 21 (1999): 355-380.

von Ungern-Sternberg, T. "Monopolistic Competition and General Purpose Products" *Review of Economic Studies* 55 (1988): 231-246

Weitzman, Martin L. "Increasing Returns and the Foundations of Unemployment Theory" *Economic Journal*, 92 (1982): 787-804

Weitzman, Martin L. "Monopolistic Competition with Endogenous Specialization" *Review of Economic Studies*, 61 (1994): 45-56.

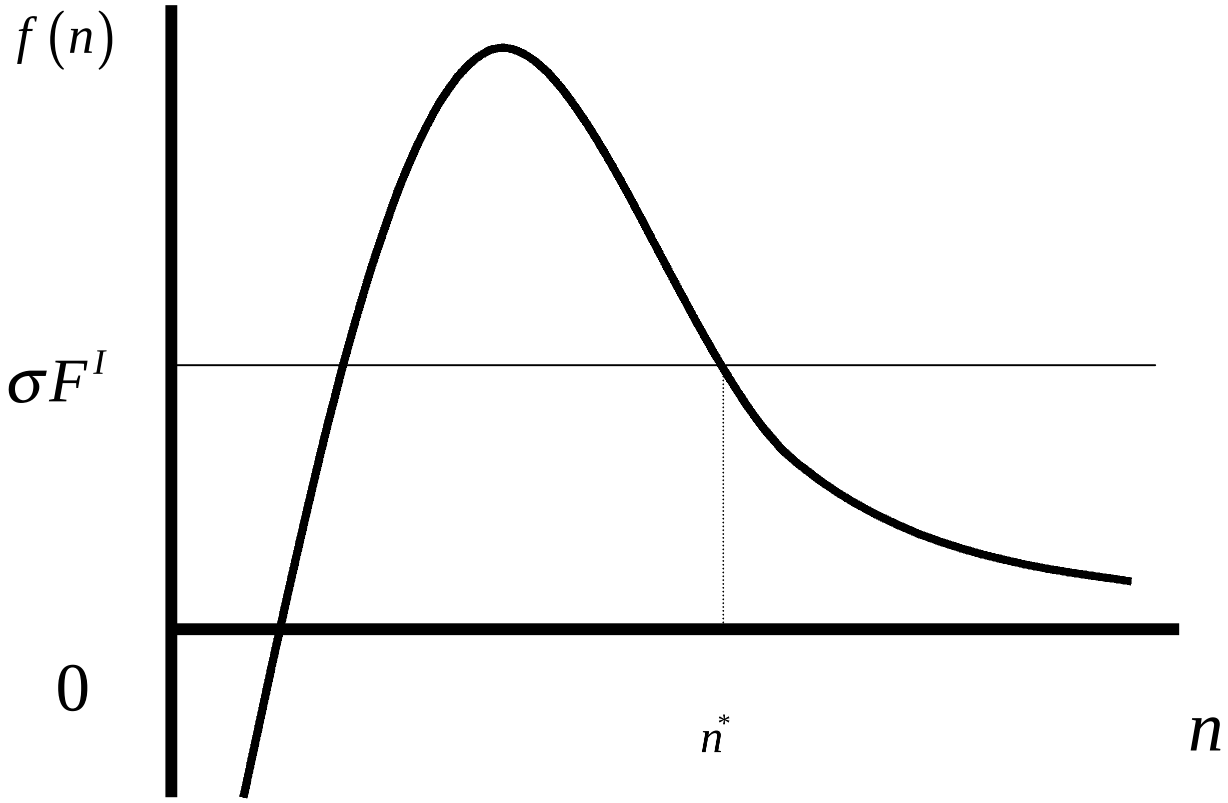

figure 1