Public Debt Accumulation and Fiscal Consolidation

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Abstract

In this study, we analyze the relationship between the interest rates of government bonds (GB) and the fiscal consolidation rule by using an overlapping generation model with endogenous and stochastic growth settings.

Our key findings are summarized as follows. First, the interest rates of GB may be declining as public debt accumulates relative to private capital, as opposed to the conventional view that the buildup of public debt accompanies a rise in interest rates. Second, the fiscal consolidation rule plays a key role in determining interest rates in equilibrium. Third, the economy may exhibit discrete changes with divergent interest rates, implying that our observation of relatively low GB interest rates does not ensure the continuation of that trend in the future. Fourth, a preventive tax increase to contain public debt at sustainable levels will not gain the political support of existing generations, whose life-span is limited. Citizens prefer to shift the ultimate burden of public debt to future generations.

JEL classifications: E17; H30; H5; H60; E62; H63

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1. Introduction

Public debt as a percentage of gross domestic product (GDP) has recently been increasing in developed countries. Japan's gross public debt–GDP ratio was especially high in 2011, compared to those of other developed countries. The International Monetary Fund (2009) estimates that Japan's gross public debt could reach 277% of GDP by 2016. In addition, Japan's net public debt–GDP ratio was also higher than that of other developed countries in 2011.[1] In such circumstances, the interest rates of government bonds (GB) theoretically rise as a reflection of default risk, as shown by Manganelli and Wolswijk (2009). Codogno et al. (2003), Bernoth et al. (2004), and Akitobi and Stratmann (2008) have each also found the existence of spreads that may be interpreted as risk premiums.

On the other hand, the interest rates of Japanese Government Bonds (JGB) have been lower than those of other developed countries' government bonds. In addition, we can observe that the interest rates of JGB are currently declining, even though Japanese public debt continues to increase (see Figure 1). Although Reinhart et al. (2012) found that, in 11 of the 26 high-debt overhang cases in advanced economies, interest rates on government bonds were either lower or about the same as during the years of lower public debt–GDP ratios, the driving mechanism is unclear. A relevant model and mechanism are required to illustrate a seemingly paradoxical confluence of trends. The following possibilities are considered as facets of the mechanism: (1) the reflection of default risk for JGB is weak because 95% of JGB are held by domestic investors; (2) domestic investors may believe that the Japanese government will not default on its debt obligations, because there are several fiscal reform opportunities (e.g., consumption tax increases) that could help maintain fiscal sustainability;[2] and (3) domestic and foreign investors believe that the interest rates on JGB are low because economic growth while the country's population is both aging and declining is also expected to be low.

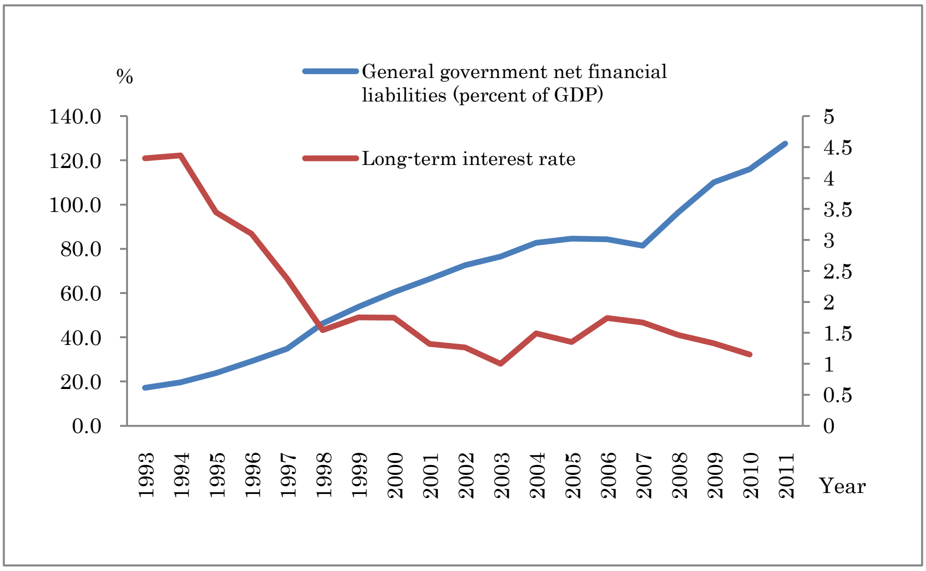
Despite these possible drivers, the mechanism behind the current decline in interest rates on JGB remains unclear, and there is no model that explains it. One complication is that GB interest rates also depend on fiscal policy: in particular, the fiscal consolidation rule (which involves e.g., tax increases, expenditure cuts, and defaulting on bonds) is important, as governments cannot always

[1] According to the OECD Economic Outlook 90 database, the general government net financial liabilities–GDP ratio of Japan in 2011 was 127.6%; that of the United States, 73.8%; United Kingdom, 61.7%; Australia, 4.9%; Canada, 33.6%; France, 62.7%; Germany, 51.5%; Italy, 100.2%; and Greece, 133.1%.

[2] Braun and Nakajima (2012) examine the case of public bond bubble in which different investors possess different prospects for debt crisis with imperfect capital market. They establish that bond price does not immediately respond to the possibility of the crisis and thus its interest rate remains low. Their model is based upon exchange economy however, abstracting production.

roll over public debt to future administrations and generations. Gale and Orszag (2002) and Laubach (2009) have pointed out that the response of GB interest rates to fiscal policy depends on expectations about the future course of fiscal policy. Perotti (2007) and Favero and Giavazzi (2007) also have found evidence of a change in the relationship between macro-level variables and fiscal policy; this change has been interpreted as evidence of a change in reactions of fiscal policy to a stabilization of the public debt–GDP ratio. Uribe (2006) and Juessen et al. (2009) have analyzed government default risk and its reflection on GB interest rates, each by using a quantitative macroeconomic model. However, Uribe (2006) focuses on external debt within an open economy. In the case of external debt, defaulting is considered a deliberate strategic decision of the government that reflects the outcome of an optimization problem (e.g., Eaton and Gersovitz, 1981; Arellano, 2008). Although Juessen et al. (2009) focus on internal debt using a model and a closed economy, these theoretical studies never examine the potential effects of the fiscal consolidation rule on interest rates. Therefore, we provide a macroeconomic model to explain the importance of the fiscal consolidation rule, and this model clarifies the relationship between decreasing GB interest rates and increasing public debt.

Figure 1. Japan government bond interest rates vs. public debt, 1993–2011



Source: OECD Stats Extracts

There are a number of assumptions inherent in this study. We consider overlapping generation models that use endogenous and stochastic growth settings. Production technology contains spillover associated with private capital and productivity shock. Each generation comprises a representative household that lives for two periods; we also account for demographic changes in the economy. Population growth is assumed to be known, but can vary over time. In the young period, the household supplies labor in an elastic manner. Taxes are levied on wages, and part of after-tax wage income is saved. We also assume that there are two types of assets that are tax-free: private capital and GB. The former yields uncertain returns due to the productivity shock of the subsequent period, and the latter promises a fixed return, but also a risk of default. Unlike Juessen et al. (2009), we consider that default may be partial. In the old period, the household is retired and receives returns on private capital and GBs.

The key findings of this study can be summarized as follows. The interest rate of GB may decline as public debt accumulates relative to private capital—as found by Reinhart et al. (2012)—so the former may "crowd out" the latter; this scenario contrasts with the conventional view that an accumulation of public debt accompanies an increase in interest rate. The prospect of future tax increases due to the fact that the fiscal consolidation rule serves to lower expected returns on private capital, which in turn decreases through arbitrage interest rates charged on GB. We establish that he fiscal consolidation rule plays a key role in determining equilibrium interest rates. In addition, the economy may exhibit discrete changes and divergent interest rates, implying that a trend wherein there is a decline in GB interest rates may not continue as public debt continues to accumulate. We also show that any preventive tax increases to contain public debt will not gain the political support of existing generations, whose life-span—and therefore their period of self-interest—is limited; instead, citizens prefer to shift the ultimate burden of public debt to future generations who cannot currently vote. This finding aligns with the commonly held belief that public debt is exploitative of future generations.

The rest of this paper is organized as follows. Section 2 describes the model, and Section 3 considers the fiscal consolidation rule and establishes equilibrium GB interest rates. Section 4 uses the equilibrium in the analysis of comparative statics. In Section 5, we clarify our theoretical argument by simulation and discuss the implications of the results, and Section 6 concludes.

2. Model Settings

2.1 Basic setting

In this study, we employ a stochastic overlapping generation model with endogenous growth setting. Following the literature, we consider both human and physical capital as driving force for economic growth. The former takes form of educational expense that enhances effective labor but is distorted by wage taxation, whereas the latter generating spillover is crowded out by public debt. The model is highly stylized and does not intend to replicate the realities of economy. Rather, it is constructed in a way to highlight the essence of fiscal consolidation risk. In this model, each generation contains a representative household that lives for two periods; in each period, a single good is produced by labor and capital, and the production is stochastic due to technology shock.

In the present model, production shock features economic states that in turn affect the accumulation of public debt. We suppose that the states are largely divided into normal and high-performance regimes. The latter is an unlikely event, but it could resolve debt overhangs, as the government can raise sufficient tax revenue to meet debt liability without overly relying on a new debt issuance. It incorporates the optimistic view that a fiscal consolidation will not be required once high economic growth can be achieved. Related to this, Imrohoroglu and Sudo (2011) estimate that unprecedentedly high growth of total factor productivity at an average of 6% for one decade must be in place for Japan to eliminate the outstanding debt. Such optimistic view for growth miracle seems to be implicit in the argument against tax hike and massive expenditure cut to restructure public finance often capturing popularity among people even when accumulating public debt is clearly unsustainable.

Each period is divided into several stages. At Stage 1, production shock is revealed. The household of the young generation supplies (effective) labor at Stage 2. Then, output is realized at Stage 3, whereupon a wage is paid to the young and a return on capital is distributed to the old. The government collects taxes and repays public debt at Stage 4. At Stage 5, the young and the old households consume, while also saving and choosing a portfolio. Public debt and private capital are then carried forward to the next period.

In the following, to clarify, our analysis follows two steps. First, we establish an intra-period or static equilibrium, given public debt and capital carried over from the previous period. We then turn to dynamics. Economic growth is endogenous and stochastic, due to productivity shock.

2.2 Production

Y_t denotes the aggregate output at period t that is produced by a representative private firm. The production function of the economy is given as

$$(1)\quad Y_t = (A\varepsilon_t)^{\eta} K_t^{\mu} k_t^{\alpha} (n_t e_t)^{1-\alpha}, \quad a_t \equiv (A\varepsilon_t)^{\eta} K_t^{\mu}$$

where the dynamic of the technological progress is represented as a_t, and A (>0) is constant, $\mu > 0$

and 0 < α < 1. ε_t denotes productivity shock. η is a constant parameter that is defined later. k_t represents private (physical) capital that is invested in the previous period, and e_t represents human effective labor supply per worker at period *t*. The population of generation *t* is denoted by n_t (t = 1, 2,…). Then $E_t = n_t e_t$ gives the total size of labor as an effective term. K_t refers to the average capital investment and represents the external effect of capital accumulation; following the literature on endogenous growth, it may be interpreted as knowledge spillover that serves as pure public, generating an economy of scale (Romer, 1986). In the equilibrium, we have $k_t = K_t$.

We assume that the shock is distributed according to the distribution function $F(\varepsilon_t) = \int_{\underline{\varepsilon}}^{\varepsilon_t} f(\varepsilon)d\varepsilon$ over $[\underline{\varepsilon}, \bar{\varepsilon}]$, with $E_t \varepsilon_t = 1$. Its support can be divided into two ranges.

(*Assumption 1*) $[\underline{\varepsilon}, \bar{\varepsilon}] = [\underline{\varepsilon}, \breve{\varepsilon}] \cup \{\bar{\varepsilon} - \upsilon, \bar{\varepsilon}\}$ with $\bar{\varepsilon}$ being sufficiently large and υ is small with $\int_{\bar{\varepsilon}-\nu}^{\bar{\varepsilon}} \varepsilon f(\varepsilon)d\varepsilon$ being infinitesimal

$[\underline{\varepsilon}, \breve{\varepsilon}]$ refers to the normal regime of the economy, whereas $\{\bar{\varepsilon} - \upsilon, \bar{\varepsilon}\}$ corresponds to the unlikely event of high performance; in the latter, the fiscal consolidation risk is removed.

Suppose that production is perfectly competitive. The price of output being normalized to unity, we can write the wage and return on capital as

(2) $w_t = (1-\alpha)Y_t / E_t$; $r_t = \alpha Y_t / k_t$.

As is customary in the literature, the market transaction fails to account for the spillover effect in determining the return on capital.

2.3 The household problem

Consider the household problem. The household living for two periods choose consumption (and thus saving) as well as effective labor supply. In the present context, we assume that the effective labor is enhanced by educational expense. Its budget constraints at the young and the old periods are respectively given by

(3.1) $c_t^y + s_t = \omega_t e_t - \dfrac{e_t^{1+1/\delta}}{1+1/\delta}$,

(3.2) $\tilde{c}_{t+1}^o = s_t \left(q_t (1-\tilde{\zeta}_{t+1}) R_{t+1} + (1-q_t)\tilde{r}_{t+1} \right)$,

where τ_t is wage income tax, $\omega_t = (1-\tau_t)w_t$ is after tax wages, R_t is the GB's (one plus) interest rate, and q_t represents the share of the GB in total savings. ζ_{t+1} represents the default rate as a value between 0 and unity. The variables with tilde address unknown quantities when saving at period *t*. R_t is determined at period *t* with default risk; the net return on GB is not certain.[3] The

[3] In Section 3.2, we abstract idiosyncratic risk, including the bankruptcy of private capital. This presumes that the household can fully diversify such risk and that only aggregate shock will remain.

household is confronted with risks of both productivity shock and government default in the subsequent period when making saving decisions.

The second term in (3.1) designates the cost of education, which serves to increase effective labor supply e_t while labor hours are taken as unity. $\delta > 0$ captures the decreasing marginal return on education. We suppose that such a cost takes the form of pecuniary expenses and for the sake of simplicity, it is contemporary expense. That is, educational investment occurs in the same period as labor being supplied. Admittedly this abstracts dynamic characteristics of human capital investment but is convenient way to formulate the essence. See for instance Bovenberg and Jacobs (2005). In addition, we allow e_t to be unbounded. The effective labor supply is decided only after ε_t becomes known and enters the budget constraint, so that it responds to after-tax wage, thus abstracting income effect. This simplification aligns with the literature on optimal income taxation. Given that each generation works only in one period, the human capital is not accumulated and carried over to the next period.

Let us turn to the household's lifetime utility. It is assumed to take the following form:

(4) $U_t = (c_t^y)^\theta c_{t+1}^o,$

where θ > 0. c_t^y denotes the young period of consumption, whereas c_t^o is the older period. Eq. (4) implies that its preference is neutral to these risks. Given the Cobb–Douglas form of the utility function, however, the inter-temporal elasticity of substitution turns out to be unity. One may find it odd that risk and time preferences are defined separately; our specification deviates from the standard setting that assumes that lifetime utility is additive over periods and over different economic states. In the present context, inter-temporal elasticity is not tied to the inverse of risk aversion. Admittedly *ad hoc*, Eq. (4) helps isolate the household's portfolio choice between private capital and GB from the decision on total savings, s_t.

Let us substitute (3) into the utility function. In the young period, the household decides effective labor supply e_t, saves s_t, and chooses portfolio q_t to maximize returns:

$$E_t U_t = (\omega_t e_t - s_t - \frac{e_t^{1+1/\delta}}{1+1/\eta})^\theta s_t E\left(q_t(1-\tilde{\zeta}_{t+1})R_{t+1} + (1-q_t)\tilde{r}_{t+1}\right).$$

where the expectation is calculated over ζ_{t+1} and r_{t+1}. The household's optimization yields the following:

(5.1) $e_t^* = \omega_t^\delta$

(5.2) $s_t^* = \dfrac{z_t}{1+\theta}$

and

(6) $R_{t+1}E_t(1-\tilde{\zeta}_{t+1}) = E_t\tilde{r}_{t+1}$,

where

$$z_t = \omega_t e_t - \frac{e_t^{1+1/\delta}}{1+1/\delta} = \left(1-\frac{\delta}{1+\delta}\right)\omega_t^{1+\delta} \equiv \frac{1}{1+\delta}\omega_t^{1+\delta}.$$

By (5.1), the wage elasticity of effective labor supply is constant at δ. Wage taxation becomes distortive as elasticity increases. Admittedly, the endogenous education expense captures only a part of the tax-induced inefficiency. In a more general context, the distortion will be attributable to choices vis-à-vis risk-taking, occupational choice, and labor hours, as well as tax-avoidance activities that divert some resources from production. δ may be then regarded as a parameter that represents a stylized notion of the perverse effect of tax on economic growth.

Due to the Cobb–Douglas specification, savings is a fixed share of the wage income, net of the labor disutility z_t, with the income effect and substitution effect offsetting one another, as given in (5.2).

Finally, (6) gives the arbitrage condition between private capital and the GB. Given that the household is risk-neutral, arbitrage leads to the equation of expected return of both assets, which should be intuitive.

2.4 Market equilibrium

This subsection considers market equilibrium, given fiscal policy. At every period, both the labor and capital markets are cleared. Given ε_t and k_t, the equilibrium values of wage and return on private capital at period *t* are determined by substituting (5.1) into (2), such that

(7.1) $\omega_t \equiv (1-\tau_t)w_t = \left((1-\alpha)(1-\tau_t)(\varepsilon_t A)^\eta K_t^{\ \mu}(k_t/n_t)^\alpha\right)^{1/(1+\alpha\delta)}$

(7.2) $r_t = \alpha((\varepsilon_t A)^\eta K_t^{\ \mu})^{(1+\delta)/(1+\alpha\delta)}(n_t/k_t)^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)(1-\tau_t)\right)^{\delta(1-\alpha)/(1+\alpha\delta)}$

The output at period *t* becomes

(8) $Y_t = \left((1-\alpha)(1-\tau_t)\right)^{\delta(1-\alpha)/(1+\alpha\delta)}(A\varepsilon_t)^{\eta(1+\delta)/(1+\alpha\delta)}\left(K_t\right)^{\mu(1+\delta)/(1+\alpha\delta)}n_t^{\ (1-\alpha)/(1+\alpha\delta)}(k_t)^{\alpha(1+\delta)/(1+\alpha\delta)}$.

Consider the external effect: in the equilibrium, we have $K_t = k_t$, with the capital investment in market being exactly equal to the average in the economy. In addition, we set the parameter associated with externality, such that the equilibrium output is proportional to private capital. The following assumption is imposed:

(*Assumption 2*) $\mu = \dfrac{1-\alpha}{1+\delta}$

Note that $\mu = 1-\alpha$ if δ = 0 or if the effective labor supply is completely inelastic, as assumed by

Romer (1986). We also let $\eta=(1+\alpha\delta)/(1+\delta)$, for expositional convenience. Then, (8) turns out to be

$$(8') \quad Y_t=\left((1-\alpha)(1-\tau_t)\right)^{\delta(1-\alpha)/(1+\alpha\delta)} A\varepsilon_t n_t^{(1-\alpha)/(1+\alpha\delta)} k_t \, .$$

The above is familiar in the endogenous growth model, which yields constant growth rates as a function of policy parameters. The wage rate is linear with respect to k_t as well, whereas the return on private capital turns out to be independent of k_t:

$$(7.1') \quad \omega_t \equiv (1-\tau_t)w_t=\left((1-\alpha)(1-\tau_t)n_t^{-\alpha}\right)^{1/(1+\alpha\delta)}\left(A\varepsilon_t\right)^{1/(1+\delta)} k_t$$

$$(7.2') \quad r_t=\alpha\varepsilon_t A(n_t)^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)(1-\tau_t)\right)^{\delta(1-\alpha)/(1+\alpha\delta)}$$

Finally, we turn to the capital market. Because of the closed economy, household savings must meet the demand of private firms and the government. b_{t+1} denotes GBs issued at period *t* and repaid at *t* + 1.[4] Given that total savings at period *t* is $n_t s_t$ as allocated between k_{t+1} and b_{t+1}, the equilibrium condition is expressed by

$$(9) \quad k_{t+1}+b_{t+1}=n_t s_t^*=\frac{n_t^{(1-\alpha)/(1+\alpha\delta)}}{(1+\theta)(1+\delta)} A\varepsilon_t\left((1-\alpha)(1-\tau_t)\right)^{(1+\delta)/(1+\alpha\delta)} k_t \, .$$

Manipulating the above establishes the dynamics of private capital accumulation as the following:

$$(9') \quad \left(1+\frac{b_{t+1}}{k_{t+1}}\right)\frac{k_{t+1}}{k_t}=\frac{n_t^{(1-\alpha)/(1+\alpha\delta)}}{(1+\theta)(1+\delta)} A\varepsilon_t\left((1-\alpha)(1-\tau_t)\right)^{(1+\delta)/(1+\alpha\delta)} .$$

Now let us consider the growth rate. This economy grows at the rate of χ_{t+1}, which is stochastic and defined as

$$(10) \quad \tilde{\chi}_{t+1} \equiv \frac{\tilde{Y}_{t+1}}{Y_t}=\left(\frac{1-\tilde{\tau}_{t+1}}{1-\tau_t}\right)^{\delta(1-\alpha)/(1+\alpha\delta)}\left(\frac{\tilde{\varepsilon}_{t+1}}{\varepsilon_t}\right)\left(\frac{n_{t+1}}{n_t}\right)^{(1-\alpha)/(1+\alpha\delta)}\frac{k_{t+1}}{k_t} \, .$$

Compared to the growth model containing agents bearing an infinite life, the OLG may exhibit dynamic inefficiency, in which the growth rate becomes more than the interest rate. The following lemma yields the condition that the economy remains dynamically efficient:

Lemma 1: The economy is dynamic efficient, or $\chi_{t+1} \le r_{t+1}$ if $\dfrac{1-\alpha}{(1+\theta)(1+\delta)} \le \alpha$

Proof of this is shown in Appendix A. Throughout the paper, we assume that the above inequality holds.

2.5 Government budget

The government raises revenue by issuing GB and taxing wage income; t then spends this revenue on debt repayment and public expenditure, the latter of which is denoted by G_t. G_t is assumed not

[4] We consider only a single-period bond, and thus we abstract issues of bond maturity composition.

to contribute to production (1) or directly enter into household utility (3). This assumption is motivated to simplify our analysis, but may be plausible when government spending comprises political rents or so-called pork diverted to special interest groups. The funding flow of the government budget at period *t* is written as

(11) $b_{t+1} = R_t b_t - \{T_t - G_t\}$,

where $T_t = \tau_t w_t L_t = (1-\alpha)\tau_t Y_t$ and Y_t is given in (8′). b_{t+1} is public debt carried over to the next period. The government may hold financial assets such as pension funds. Eq. (11) incorporates such a circumstance by interpreting b_t in terms of net debt. For the latter use, the following lemma establishes the revenue-maximizing tax rate that determines the upper bounds for tax rates, in the case of fiscal consolidation:

Lemma 2: Tax revenue is maximized at $\tau_{Max} = \dfrac{1+\alpha\delta}{1+\delta}$

At this point, we distinguish fiscal rule between pre-fiscal consolidation and fiscal consolidation regimes. This is denoted by $\Omega_t \equiv \{\tau_t, \lambda_t, \zeta_t\}$, which contains tax rate τ_t, government expenditure ratio λ_t, and default rate ζ_t, and it may be state-contingent for consolidation regimes. Fiscal rules are assumed to be public information; this assumption implies that these rules are incorporated in the pricing of GB, as discussed below. The present model does not suppose optimization behavior on the part of the government in the pursuit of social welfare. We instead take the pragmatic view that government policy is largely politically constrained, as opposed to being designed on the basis of economic rationale.

Let $\Omega^0 \equiv \{\tau, \lambda, \zeta\}$, with $\zeta = 0$. In the pre-consolidation regime, the government taxes wage income at the rate of $\tau_t = \tau$. It spends a given portion of potential output, $\lambda_t = \lambda$, that calculates Y_t at the mean of ε_t; that is, $\varepsilon_t = 1$ and $\tau_t = \tau$, such that $G_t = \lambda \bar{Y}_t$, where

(12) $\bar{Y}_t = \left((1-\alpha)(1-\tau)\right)^{\delta(1-\alpha)/(1+\alpha\delta)} A n_t^{(1-\alpha)/(1+\alpha\delta)} k_t$.

This may capture the stickiness of government spending; it is neither pro nor counter-cyclical. We can then interpret G_t as neutral to economic fluctuation. For analytical convenience, we suppose that the expenditure remains proportional to $\bar{Y}_t$ in the consolidation regime as well, although λ_t may be lowered. With (11) and (12), the primary surplus at period *t* is defined by

(13) $PS_t = T_t - G_t = k_t\left((1-\alpha)\right)^{(1+\delta)/(1+\alpha\delta)} A n_t^{(1-\alpha)/(1+\alpha\delta)} \Delta(\tau, \lambda, \varepsilon_t)$,

where

$$\Delta(\tau_t, \lambda_t, \varepsilon_t) \equiv \left(\tau_t (1-\tau_t)^{\delta(1-\alpha)/(1+\alpha\delta)} \varepsilon_t - \frac{\lambda_t}{1-\alpha} (1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)} \right).$$

Substituting (13) into (11) and manipulating it establishes the dynamics of public debt over multiple periods; we arrive at

$$\text{(14)} \quad \frac{b_{t+1}}{k_{t+1}} \frac{k_{t+1}}{k_t} = R_t \frac{b_t}{k_t} - \left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)} n_t^{(1-\alpha)/(1+\alpha\delta)} \Delta(\tau, \lambda, \varepsilon_t),$$

where k_{t+1}/k_t is as given in (9′).

Note that in the present economy, b_{t+1}/k_{t+1} and k_{t+1} serve as state variables that are determined at period t and carried over to period $t+1$. They then affect the risk of fiscal consolidation at $t+1$, as discussed in the next section.

3. Equilibrium

3.1 Fiscal sustainability

The fiscal rule $\Omega^0 \equiv \{\tau, \lambda, \zeta\}$ in the pre-consolidation regime does not ensure that public debt remains at fiscally sustainable levels. Tax rates may be too low and/or expenditure ratio may be too high to structurally generate a primary deficit; that is, $\Delta(\tau, \lambda, \varepsilon_t) < 0$ for most of ε_t. The public debt may reach a level at which the status quo fiscal rule cannot be sustained. We do not suppose that the government undertakes precautionary measures to prevent such circumstances. Political economy considerations on such measure will be discussed later.

The literature of fiscal sustainability discusses the transversality condition of the present value of the primary fiscal surplus in the infinite future. Indeed, Juessen et al. (2009), using infinitely living agents, considers that the government is forced to default on its debt once the condition fails to hold. In the OLG setting, however, the capital market may not work to discipline government financing, because the finite-life household may not be concerned with long-term fiscal sustainability. The household is willing to purchase GB in its young period as long as the arbitrage condition (6) is fulfilled. The consolidation risk is compensated with a higher GB interest rate. [5]

In the present context, therefore, the transversality condition does not necessarily guide the threshold

[5] It is arguable that transversality condition does not represent rational decision making but is rather normative criterion to assure fiscal discipline. Such discipline must be enforced by investors. In the OLG setting, no investors are motivated in such a way.

that triggers the fiscal consolidation or maximum b_{t+1}/k_{t+1} under which the government can sustain status quo. Such a threshold may be determined in terms of the "tolerance" or confidence of the market in the government's capacity to remain solvent, which in turn could be dependent on its historical record of fiscal consolidation and/or investors' beliefs; however, it may instead emerge as a sort of optimal default strategy set by the government. (On a related note, we illustrate in section 6 the political economy of deciding upon fiscal consolidation.) We leave determinants of the consolidation threshold to future research, but do define it as exogenous.

Moreover, we allow the scope of government solvency to be maximal. To be specific, we assume that the government can access credit insofar as the GB level does not exceed domestic savings. [6] Indeed, the arbitrage condition (6) is fulfilled even for large b_{t+1}/k_{t+1} since there exists small probability that the high economic performance with $\varepsilon_{t+1}=\bar{\varepsilon}$ eliminates public debt

In the following, consider the government solvency at period t + 1. Suppose that the economy reaches $b_{t+2}=n_{t+1}s_{t+1}$—that is, the domestic savings at t + 1 is fully absorbed by government borrowing, given that the economy is closed and no private investment can take place, which implies that there is no production in the subsequent period, or $Y_{t+2}=0$ for all ε_{t+2}. Once this occurs, the government can find no resource for repayment, whereupon it must then default on the debt so that ζ_{t+2} =1 is certain—and therefore, there will be no return on GB.[7] This in turn implies that households cease to lend to the government. The government is then forced to undertake fiscal consolidation with no further borrowing; this entails tax increases, expenditure cuts, and further defaulting on GB.

Lemma 3: Full default (ζ_{t+2} =1) is inevitable at period t + 2, irrespective of ε_{t+2}, when $b_{t+2}=n_{t+1}s_{t+1}$ at t + 1

With $b_{t+2}=n_{t+1}s_{t+1}$ or $k_{t+2}=0$, we have $b_{t+2}/k_{t+2}=\infty$ at period t + 1. Obviously the transversality condition is not fulfilled (See Appendix B). In the following, we suppose that $b_{t+2}/k_{t+2}=\infty$ triggers the fiscal consolidation, and we present the equilibrium compatible with it.

3.2 The threshold

6 This may represent the notion that risk of fiscal crisis remains low when domestic investors are dominant in purchasing GB as is the case in Japan although we model closed economy abstracting international transactions.

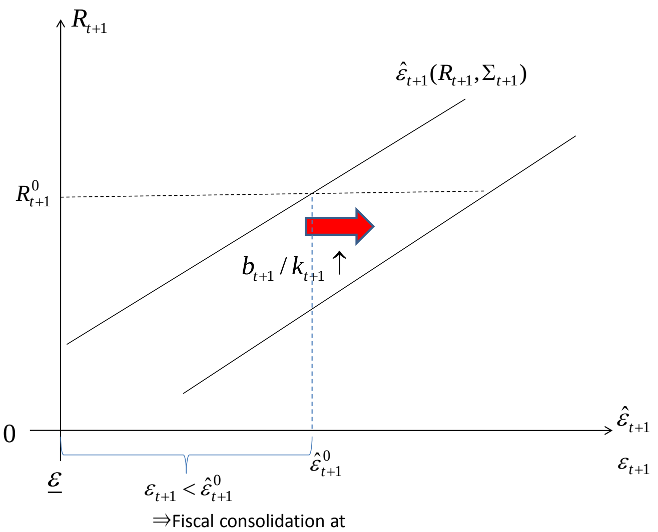
7 On the other hand, return on private capital remains positive, with the revenue-maximizing tax rate being bounded by less than 100%.

Inserting $b_{t+2} / k_{t+2} = \infty$ into (14) and manipulating it yields the following condition of the threshold level of ε_{t+1}, below which regime change occurs:[8]

$$(15)\quad R_{t+1} \frac{b_{t+1}}{k_{t+1}} = An_t^{(1-\alpha)/(1+\alpha\delta)} \left((1-\alpha)(1-\tau)\right)^{(1+\delta)/(1+\alpha\delta)} \left(\frac{\hat{\varepsilon}_{t+1}}{(1+\theta)(1+\delta)} + \frac{1}{1-\tau} \left(\tau\hat{\varepsilon}_{t+1} - \frac{\lambda}{1-\alpha} \right) \right)$$

This defines the threshold $\hat{\varepsilon}_{t+1}$ as the function of the interest rate charged on b_{t+1}, as well as the debt–capital ratio and demography: $\hat{\varepsilon}_{t+1} = \hat{\varepsilon}_{t+1}(R_{t+1}, \Sigma_{t+1})$ where $\Sigma_{t+1} = (b_{t+1} / k_{t+1}, n_{t+1})$. With R_{t+1} and b_{t+1} / k_{t+1}, $\hat{\varepsilon}_{t+1}$ increases such that fiscal consolidation is more likely to be in place, whereas it is lowered with n_{t+1}.

Figure 2



Lemma 4: Fiscal consolidation must occur at period *t* +1 when $\varepsilon_{t+1} \leq \hat{\varepsilon}_{t+1}$

Fiscal consolidation involves tax increases, expenditure cuts, and defaulting on GB, with no new debt issuance. In the present context, we use the term "default" rather loosely: the default may be partial or refer to a "restructuring" that recontracts the conditions of repayment. The state of the economy at period *t*+1 is denoted by $\{\varepsilon_{t+1}, \Sigma_{t+1}\}$. The fiscal rule is then expressed as $\Omega_{t+1} \equiv \Omega(\varepsilon_{t+1}, \Sigma_{t+1})$, with $b_{t+2} = 0$, which contains

[8] In a more general context, if we denote the debt tolerance by $b_{t+2} / k_{t+2} = \gamma$, the threshold (15) can be re-expressed as

$$R_{t+1} \frac{b_{t+1}}{k_{t+1}} = An_t^{(1-\alpha)/(1+\alpha\delta)} \left((1-\alpha)(1-\tau)\right)^{(1+\delta)/(1+\alpha\delta)} \left(\frac{\hat{\varepsilon}_{t+1}\gamma}{(1+\theta)(1+\delta)(1+\vartheta)} + \frac{1}{1-\tau} \left(\tau\hat{\varepsilon}_{t+1} - \frac{\lambda}{1-\alpha} \right) \right).$$

Except that parameter γ is added, the feature of the threshold does not change.

$$\tau \le \tau_{t+1} = \tau(\varepsilon_{t+1}, \Sigma_{t+1}) \le \tau_{Max}\,, \lambda \ge \lambda_{t+1} = \lambda(\varepsilon_{t+1}, \Sigma_{t+1}) \quad 0 \le \zeta_{t+1} = \zeta(\varepsilon_{t+1}, \Sigma_{t+1}) \le 1\,,$$

where the default rate ζ_{t+1} fulfills[9]

$$(16) \quad (1-\zeta_{t+1})R_{t+1}\frac{b_{t+1}}{k_{t+1}} = \left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)} n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} \Delta(\tau_{t+1}, \lambda_{t+1}, \varepsilon_{t+1})\,,$$

The government cannot fully meet its obligation, but repays its outstanding debt as much as possible out of the primary surplus, as illustrated in (16). Under the consolidation rule, either the tax rate, expenditure ratio, or default rate deviates from initial levels. The fiscal rule can take a general form, but may be plausibly levied according to the following restrictions:

(*Assumption 3*)

$$(i)\partial\tau_{t+1}/\partial(b_{t+1}/k_{t+1}) \ge 0 \quad (ii)\partial\zeta_{t+1}/\partial(b_{t+1}/k_{t+1}) \ge 0 \quad (iii)\partial\lambda_{t+1}/\partial(b_{t+1}/k_{t+1}) \le 0$$

In the simulation within section 5, we specify the fiscal consolidation rule. Note that it takes only one period to restructure government finance. Given that no GB is issued, the economy will return to the initial regime in the next period, without debt liability being carried over.

3.3 Interest rate

Let us turn to the GB interest rate R_{t+1}, which is settled at period *t* and accounts for fiscal consolidation in the event of $\varepsilon_{t+1} \le \hat{\varepsilon}_{t+1}$. Recall the arbitrage condition (6), which equates return on GB with capital in the expected term. Manipulating it with the use of (6) and (16) establishes the following:

$$(17) \quad \begin{aligned} &(1-F(Min[\hat{\varepsilon}_{t+1}, \varepsilon^0_{t+1}]))R_{t+1} \\ &= n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}(1-\alpha)^{(1+\delta)/(1+\alpha\delta)} A\left\{\frac{\alpha}{1-\alpha}\Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1}) - \frac{k_{t+1}}{b_{t+1}}\int^{Min[\hat{\varepsilon}_{t+1}, \varepsilon^0_{t+1}]} \Delta(\tilde{\tau}_{t+1}, \tilde{\lambda}_{t+1}, \tilde{\varepsilon}_{t+1}) dF(\tilde{\varepsilon}_{t+1})\right\} \end{aligned},$$

where

$$\Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1}) = \int^{\hat{\varepsilon}_{t+1}} (1-\tau_{t+1})^{\delta(1-\alpha)/(1+\alpha\delta)} \left(\varepsilon_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)} dF(\varepsilon_{t+1}) + (1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)} \int_{\hat{\varepsilon}_{t+1}} \left(\varepsilon_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)} dF(\varepsilon_{t+1})$$

and T_{t+1} in $\Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1})$ refers to the vector of tax rates τ_{t+1}. Note that $\Phi(\hat{\varepsilon}_{t+1}, \tau_{t+1})$ reflects the expected return on private capital.[10] We can clearly see that it is nonincreasing with the threshold level, given that $\tau \le \tau_{t+1}$. This represents the perverse effect of wage tax increases under fiscal consolidation that would discourage an effective labor supply and in turn lower the productivity of private capital.

[9] The consolidation rule can be interpreted in a reduced form that incorporates the dependence of the equilibrium values of R_{t+1} and $\hat{\varepsilon}_{t+1}$ on $\Sigma_{t+1} = (b_{t+1}/k_{t+1}, n_t)$.

[10] $E_{t+1}r_{t+1} = \dfrac{\alpha}{1-\alpha}(n_t)^{(1-\alpha)/(1+\alpha\delta)}(A(1-\alpha))^{(1+\delta)/(1+\alpha\delta)}\Phi(\hat{\varepsilon}_{t+1}, \tau_{t+1})$

In (17), $\hat{\varepsilon}_{t+1}$ is treated as a control variable. For a sufficiently large $\hat{\varepsilon}_{t+1}$, the default rate according to (16) may turn out to be negative, which cannot occur. To restrict ζ_{t+2} to be nonnegative, we cut the threshold at $\varepsilon^0_{t+1} = \varepsilon^0_{t+1}(R_{t+1}, \Sigma_{t+1})$, which is implicitly defined by:

$$(16') \quad R_{t+1}\frac{b_{t+1}}{k_{t+1}} = ((1-\alpha))^{(1+\delta)/(1+\alpha\delta)} An_{t+1}^{(1-\alpha)/(1+\alpha\delta)} \Delta(\tau_{t+1}(\varepsilon^0_{t+1}, \Sigma_{t+1}), \lambda_{t+1}(\bullet), \varepsilon^0_{t+1})$$

Note that for $\varepsilon^0_{t+1} < \varepsilon_{t+1} \le \hat{\varepsilon}_{t+1}$, the fiscal consolidation does not involve default.

(17) yields the GB interest rate as a function of the threshold, the debt–capital ratio, and the population: $R_{t+1} = R_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$. Consider the comparative statics: R_{t+1} is increasing with n_{t+1}, since the productivity of private capital is enhanced as effective labor supply expands. k_{t+1}/b_{t+1}, directly appearing in (16), serves to raise R_{t+1}, whereas according to Assumption 2, the induced tax increase under consolidation works in the opposite direction, given that $\partial\tau_{t+1}/\partial(b_{t+1}/k_{t+1}) \ge 0$.

To further address features of the GB gross interest rate, differentiate $R_{t+1} = R_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ with respect to the threshold that establishes:

(18)

$$\frac{\partial}{\partial\hat{\varepsilon}_{t+1}} R_{t+1} = \frac{f(\hat{\varepsilon}_{t+1})}{1 - F(Min[\hat{\varepsilon}_{t+1}, \varepsilon^0_{t+1}])} \times \left\{ R_{t+1}(\hat{\varepsilon}_{t+1}, Z_{t+1}) d_{t+1} - \underbrace{n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} (1-\alpha)^{(1+\delta)/(1+\alpha\delta)} A\left(\frac{\alpha}{1-\alpha} \Lambda(\hat{\tau}_{t+1}) \hat{\varepsilon}_{t+1} + d_{t+1} \frac{k_{t+1}}{b_{t+1}} \Delta(\hat{\tau}_{t+1}, \hat{\lambda}_{t+1}, \hat{\varepsilon}_{t+1})) \right)}_{M_{t+1} = M(\varepsilon_{t+1}, \Sigma_{t+1})} \right\}$$

where $d_{t+1} = d(\hat{\varepsilon}_{t+1}, \varepsilon^0_{t+1})$ takes a value of 1 if $\hat{\varepsilon}_{t+1} < \varepsilon^0_{t+1}$, and 0 otherwise. Then, the effect of $\hat{\varepsilon}_{t+1}$ is described in the following lemma:

Lemma 5:

Denoting $\hat{\tau}_{t+1} = \tau_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ and $\hat{\zeta}_{t+1} = \hat{\zeta}_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ where $\Sigma_{t+1} = (b_{t+1}/k_{t+1}, n_t)$, we have

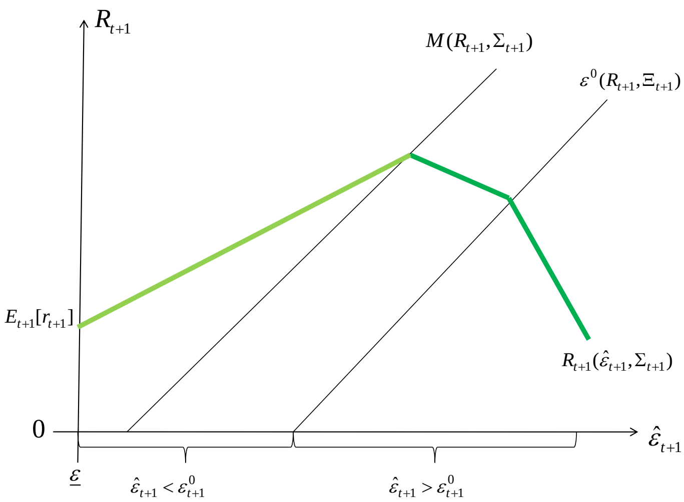
$$\frac{\partial}{\partial\hat{\varepsilon}_{t+1}} R_{t+1} \underset{<}{\overset{>}{}} 0 \Leftrightarrow R_{t+1}\hat{\zeta}_{t+1} \underset{<}{\overset{>}{}} \alpha n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} A(1-\alpha)^{\delta(1-\alpha)/(1+\alpha\delta)} \hat{\varepsilon}_{t+1} \Lambda(\hat{\tau}_{t+1}),$$

where $\Lambda(\hat{\tau}_{t+1}) = (1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)} - (1-\hat{\tau}_{t+1})^{\delta(1-\alpha)/(1+\alpha\delta)}$.

This lemma implies that the interest rate is increasing (resp. decreasing) in $\hat{\varepsilon}_{t+1}$ when fiscal consolidation entails no tax increase (resp. no default), and thus revenue loss is compensated for by defaulting on outstanding debt (resp. by raising tax) at the threshold level. It may be counterintuitive for R_{t+1} to be lowered as $\hat{\varepsilon}_{t+1}$ increases. To see this point, note that in the present model, fiscal consolidation involves both the default and the tax increase. The former adds the risk premium of GB relative to private capital, thereby raising its interest rate; the latter, on the other hand, reduces the return on private capital, which works to lower the GB interest rate through arbitrage. We then have the case of $\partial R_{t+1}/\partial\hat{\varepsilon}_{t+1}<0$ when tax increases dominate the default risk.

Figure 3 depicts the shape of interest as the threshold changes. M_{t+1} refers to the last two terms on the large bracket on the right-hand side of (18). Given that $1-\tau>1-\hat{\tau}$, it locates the upper bound of $\varepsilon_{t+1}^{0}=\varepsilon_{t+1}^{0}(R_{t+1},Z_{t+1})$ that corresponds to the last term in the above equation. When $\hat{\varepsilon}_{t+1}$ is at a low level, (18) becomes positive with R_{t+1} exceeding M_{t+1}. At the point where R_{t+1} intersects with M_{t+1}, the sign of (18) is reversed and R_{t+1} declines with $\hat{\varepsilon}_{t+1}$. Once $\hat{\varepsilon}_{t+1}$ reaches ε_{t+1}^{0}, d_{t+1} turns out to be 0, thus enhancing the absolute value of $\partial R_{t+1}/\partial\hat{\varepsilon}_{t+1}$.

Figure 3.



To further clarify the feature of the GB gross interest rate, we consider different rules of fiscal consolidation.

No Tax Increase: Let $\tau_{t+1}=\tau$ for all ε_{t+1}, such that there is no need for a tax increase. Consolidation entails defaulting on outstanding debt as well as cutting government expenses. The default rate fulfills

$$(17') \quad (1-\zeta(\varepsilon_{t+1},\Sigma_{t+1}))R_{t+1}\frac{b_{t+1}}{k_{t+1}}=\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\Delta(\tau,\lambda(\varepsilon_{t+1},\Sigma_{t+1}),\varepsilon_{t+1})\,.$$

According to Lemma 5, the function of $R_{t+1}=R_{t+1}(\hat{\varepsilon}_{t+1},\Sigma_{t+1})$ should be upward with respect to $\hat{\varepsilon}_{t+1}$.

Lemma 6

Consider fiscal rule with no tax increase; also assume that $F(\varepsilon_{t+1})$ is uniform. Then $R_{t+1}(\hat{\varepsilon}_{t+1},Z_{t+1})$ is increasing and globally convex in ε_{t+1} with $\partial R_{t+1} / \partial(b_{t+1} / k_{t+1}) \geq 0$ if

$$(19) \quad \frac{\partial}{\partial \hat{\varepsilon}_{t+1}} R_{t+1}(\underline{\varepsilon}, Z_{t+1}) > \frac{A}{2}(1-\alpha)^{(1+\delta)/(1+\alpha\delta)} n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} \frac{k_{t+1}}{b_{t+1}} \tau(1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)} .$$

Eq. (19) ensures that $R_{t+1} = R_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ is located above $M_{t+1} = M(\varepsilon_{t+1}, \Sigma_{t+1})$ in Figure 2. The left-hand side of (19) is increasing in b_{t+1} / k_{t+1}, given that $\partial R_{t+1} / \partial(b_{t+1} / k_{t+1}) \geq 0$ and $\partial M_t / \partial(b_{t+1} / k_{t+1}) > 0$ is defined in (18), whereas the right-hand side is decreasing with it. This in turn implies that the inequality holds for a sufficiently large public debt–capital ratio.

No Default: Suppose instead that no default is allowed—or $\zeta_{t+1} = 0$ for all ε_{t+1}—but the outstanding debt must be fully repaid through tax increases and expenditure cuts, with $\tau_{t+1} = \tau(\varepsilon_{t+1}, \Sigma_{t+1})$ being implicitly determined by (16) with ζ_{t+1} = 0. Then, (17) reduces to

$$(17'') \quad R_{t+1} = n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} \left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)} \frac{\alpha}{1-\alpha} \Phi(\hat{\varepsilon}_{t+1}, \tau_{t+1}) ,$$

which is decreasing with $\hat{\varepsilon}_{t+1}$.[11] R_{t+1} is also decreasing in the debt–capital ratio if τ_{t+1} is raised, which in turn reduces the right-hand side of (17”).

3.4 Interaction

There exists interaction between the threshold of the fiscal consolidation $\hat{\varepsilon}_{t+1}$ and the GB interest rate R_{t+1}, which are defined by (15) and (17), respectively. Solving these equations yields their equilibrium values. Note that these are assessed from period *t* or the *ex ante* perspective when ε_{t+1} is not known and fiscal consolidation is not yet in place.

Proposition 1: Denote by R_{t+1}^* and $\hat{\varepsilon}_{t+1}^*$ the equilibrium levels of the GB interest rate and the threshold of fiscal consolidation, conditional upon b_{t+1} / k_{t+1} and the consolidation rule. If the equilibrium values exist, these are given as solutions to the following equations:

$$R_{t+1} \frac{b_{t+1}}{k_{t+1}} = n_t^{(1-\alpha)/(1+\alpha\delta)} A\left((1-\alpha)(1-\tau)\right)^{(1+\delta)/(1+\alpha\delta)} \left(\frac{\hat{\varepsilon}_{t+1}}{(1+\theta)(1+\delta)} + \frac{1}{1-\tau}\left(\tau \hat{\varepsilon}_{t+1} - \frac{\lambda}{1-\alpha} \right) \right)$$

$$\begin{aligned} &(1-F(Min[\hat{\varepsilon}_{t+1}, \varepsilon_{t+1}^0]))R_{t+1} \\ &= n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} (1-\alpha)^{(1+\delta)/(1+\alpha\delta)} A \left\{ \frac{\alpha}{1-\alpha} \Phi(\hat{\varepsilon}_{t+1}, \mathrm{T}_{t+1}) - \frac{k_{t+1}}{b_{t+1}} \int^{Min[\hat{\varepsilon}_{t+1}, \varepsilon_{t+1}^0]} \Delta(\tilde{\tau}_{t+1}, \tilde{\lambda}_{t+1}, \tilde{\varepsilon}_{t+1}) dF(\tilde{\varepsilon}_{t+1}) \right\} . \end{aligned}$$

In the above proposition, we do not preclude the case that there arise multiple equilibria, with the

[11] The sufficient condition needed for this to exist is given by:

$$\Delta(\bar{\tau}, 0, \underline{\varepsilon}) \geq (1-\tau)^{(1+\delta)/(1+\alpha\delta)} \left(\frac{(\varepsilon)^{(1+\delta)/(1+\alpha\delta)}}{(1+\theta)(1+\delta)} + \frac{1}{1-\tau}\left(\tau(\bar{\varepsilon})^{(1+\delta)/(1+\alpha\delta)} - \frac{\lambda}{1-\alpha} \right) \right) .$$

two equations intersecting more than twice or with the equilibrium diverging—that is, $\hat{\varepsilon}^*_{t+1}$ reaching $\bar{\varepsilon}$.

In the above proposition, we do not preclude the case that there arise multiple equilibria, with the two equations intersecting more than twice or with the corner equilibrium with $\hat{\varepsilon}^*_{t+1}$ diverging. To be specific, we can establish:

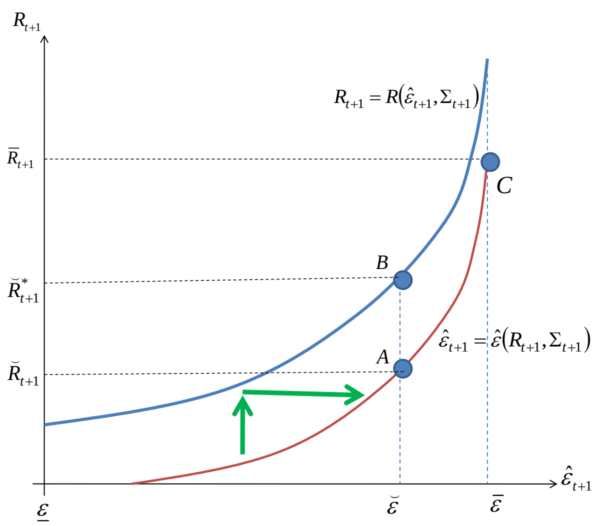
Corollary to Proposition 1:

In the corner equilibrium with $\hat{\varepsilon}^*_{t+1} = \breve{\varepsilon}$, we can define the GB interest rate at $\breve{R}^*_{t+1}$ where $\breve{R}^*_{t+1}$ is defined as (17) at $\hat{\varepsilon}_{t+1} = \breve{\varepsilon}$ if

$$(20)\quad \left(\frac{\breve{\varepsilon}}{(1+\theta)(1+\delta)} + \frac{1}{1-\tau}\left(\tau\breve{\varepsilon} - \frac{\lambda}{1-\alpha}\right)\right) < \breve{R}^*_{t+1}\frac{b_{t+1}}{k_{t+1}}\left(An_t^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)(1-\tau)\right)^{(1+\delta)/(1+\alpha\delta)}\right)^{-1} < \left(\frac{\bar{\varepsilon}}{(1+\theta)(1+\delta)} + \frac{1}{1-\tau}\left(\tau\bar{\varepsilon} - \frac{\lambda}{1-\alpha}\right)\right).$$

(20) ensures that $\breve{R}_{t+1} < \breve{R}^*_{t+1} < \bar{R}_{t+1}$ where $\breve{R}_{t+1}$ is the solution to $\breve{\varepsilon} = \hat{\varepsilon}_{t+1}(\breve{R}_{t+1}, \Sigma_{t+1})$, and $\bar{R}_{t+1}$ is calculated from $\bar{\varepsilon} = \hat{\varepsilon}_{t+1}(\bar{R}_{t+1}, \Sigma_{t+1})$. In the corner equilibrium, the GB interest rate is so high that the fiscal consolidation is inevitable unless $\varepsilon \in \{\bar{\varepsilon} - \upsilon, \bar{\varepsilon}\}$, i.e., the growth miracle occurs as illustrated in Figure 4. Recall that $\bar{\varepsilon}$ takes an extremely high value. that in turn assures the arbitrage condition (6) to hold.

Figure 4



4. Comparative Statics

4.1 Debt accumulation

Let us focus on the interior solution. Regarding comparative statics, fully differentiating equation (15) of threshold $\hat{\varepsilon}_{t+1} = \hat{\varepsilon}_{t+1}(R_{t+1}, \Sigma_{t+1})$ and equation (17) of the GB interest rate $R_{t+1} = R_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ establishes

$$(21)\quad \begin{bmatrix} 1 & -\partial\hat{\varepsilon}_{t+1}/\partial R_{t+1} \\ & (-) \\ -\partial R_{t+1}/\partial\hat{\varepsilon}_{t+1} & 1 \end{bmatrix} \begin{pmatrix} d\hat{\varepsilon}^*_{t+1} \\ \\ dR^*_{t+1} \end{pmatrix} = \begin{bmatrix} \partial\hat{\varepsilon}_{t+1}/\partial(b_{t+1}/k_{t+1}) \\ (+) \\ \partial R_{t+1}/\partial(b_{t+1}/k_{t+1}) \end{bmatrix} d\left(\frac{b_{t+1}}{k_{t+1}}\right) + \begin{bmatrix} \partial\hat{\varepsilon}_{t+1}/\partial n_{t+1} \\ (-) \\ \partial R_{t+1}/\partial n_{t+1} \\ (+) \end{bmatrix} dn_{t+1}.$$

Meanwhile, we focus on the interior and stable equilibrium where the equilibrium interest rate is finite, with $H \equiv 1-(\partial R_{t+1}/\partial\hat{\varepsilon}_{t+1})(\partial\hat{\varepsilon}_{t+1}/\partial R_{t+1})$ > 0. Then we obtain the following proposition in the case of there being no demographic change.

Proposition 2: Debt–capital ratio

(i) $\hat{\varepsilon}^*_{t+1}$ is increasing if $\partial R_{t+1}/\partial(b_{t+1}/k_{t+1}) \geq 0$

(ii) R^*_{t+1} is increasing if $\partial R_{t+1}/\partial\varepsilon_{t+1} > 0$ and $\partial R_{t+1}/\partial(b_{t+1}/k_{t+1}) \geq 0$

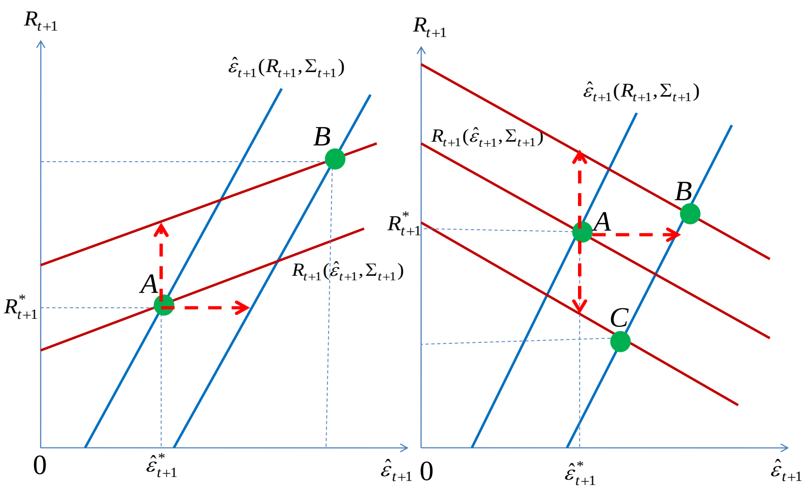
(iii) R^*_{t+1} is decreasing if $\partial R_{t+1}/\partial\varepsilon_{t+1} < 0$ and $\partial R_{t+1}/\partial(b_{t+1}/k_{t+1}) \leq 0$

Figure 5 depicts some possible scenarios. Figure 5(a) gives the case of $\partial R_{t+1}/\partial\varepsilon_{t+1} > 0$ and $\partial R_{t+1}/\partial(b_{t+1}/k_{t+1}) \geq 0$. This applies to the consolidation rule where there is no tax increase. The initial equilibrium is located at point A. Increasing the debt–capital ratio moves $\hat{\varepsilon}_{t+1} = \hat{\varepsilon}_{t+1}(R_{t+1}, \Sigma_{t+1})$ rightward and $R_{t+1} = R_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ upward. The intersection of the two functions is then shifted up and to the right, such that both the interest rate and the threshold are raised.

In Figure 5(b), $R_{t+1} = R_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ slopes downward, which is so if the consolidation rule does not allow default or if it is located below $M_{t+1} = M(\varepsilon_{t+1}, \Sigma_{t+1})$, as in Figure 2. The initial equilibrium is again given by point A. $\hat{\varepsilon}_{t+1} = \hat{\varepsilon}_{t+1}(R_{t+1}, \Sigma_{t+1})$ moves in the same way as Figure 5(a), with b_{t+1}/k_{t+1}. Suppose $\partial R_{t+1}/\partial(b_{t+1}/k_{t+1}) > 0$. The threshold is raised, moving the new equilibrium to point B. The change in interest rate is not certain. Let $\partial R_{t+1}/\partial(b_{t+1}/k_{t+1}) \leq 0$. In a more general context, a tax increase induced by a fiscal restructuring that in turn lowers $\Delta(\tilde{\tau}_{t+1}, \tilde{\lambda}_{t+1}, \tilde{\varepsilon}_{t+1})$ in (17) may be dominant. R^*_{t+1} is then lowered to point C, whereas change in $\hat{\varepsilon}^*_{t+1}$ is ambiguous.
As a consequence, we have the circumstance in which R^*_{t+1} decreases as debt accumulates relative to private capital, as depicted in Figure 5(b). This is likely to occur when the fiscal consolidation rule includes large tax increases while keeping the default rate low at most of ε_{t+1}, as discussed earlier. (In contrast, Figure 5(a) implies that the default risk is significant in the fiscal rule.)

It is often said that as a consequence of crowding out private investment, a buildup of public debt leads to surges in interest rates. The rationale behind such an argument involves the diminishing marginal returns on investment. Decreased private capital then enhances its productivity on margin, which in turn raises the market interest rate. The endogenous growth setting, however, does not translate in a crowding out into such increases in the marginal product of k_{t+1}, as it is fixed and depends upon the wage tax rate and other elements of the economic environment. Rather, expectations of future tax increases reduce the expected return on private capital, which is in turn reflected in lower levels of R_{t+1}. To state it differently, a lower R_{t+1} signals a cautionary view of future government financing, in the present context.

Figure 5(a) Figure 5(b)



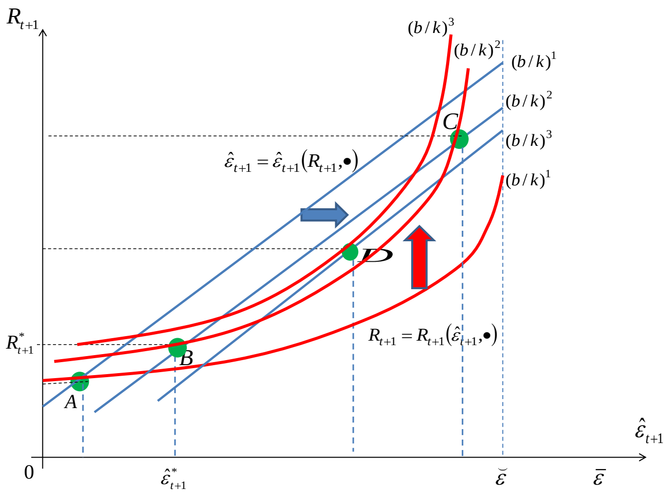
At this point, it is worth pointing out that the consolidation rule plays a key role in determining equilibrium. If consolidation is done mostly by defaulting on public debt in the extreme, the case of Figure 5(a) becomes likely, increasing R^*_{t+1} as public finances deteriorate. The contrasting trend—shown in Figure 5(b)—is observed when consolidation includes a significant tax increase while respecting the debt obligation. It can be seen that R^*_{t+1} is lower under the latter fiscal rule than under the former, given that b_{t+1}/k_{t+1}. The different fiscal rules are compared in the simulation as well.

4.2 Discrete change

In Figure 5, we assume a unique and interior solution to equations (15) and (17). However, we may have multiple equilibria, or the interior equilibrium solution may cease to exit. As an example, consider the consolidation rule with no tax increase. Suppose that $F(\varepsilon_{t+1})$ is uniform in the normal regime; also assume that (19) holds, which in turn yields $\partial^2 R_{t+1} / \partial\varepsilon_{t+1}{}^2 > 0$ with $\partial^2 R_{t+1} / \partial\varepsilon_{t+1}{}^2 > 0$ and $\partial R_{t+1} / \partial(b_{t+1} / k_{t+1}) \geq 0$

Figure 6 illustrates such to be the case. That figure depicts three different levels to the ratio, with $(b/k)^1 < (b/k)^2 < (b/k)^3$. At $(b/k)^1$, the GB interest rate and the threshold functions intersect only once; at point A; this yields a unique equilibrium. For the middle level of the ratio, the two functions turn to cross twice, at points B and C; point B provides a stable equilibrium, whereas C is unstable. In the case of multiple equilibria, the outcome depends upon the beliefs or perceptions of the household that purchases GB. In a more general context in which households of one generation are heterogeneous, some coordination of beliefs is needed to determine which equilibrium is achieved. Note that further increasing b_{t+1} / k_{t+1} shifts the two curves so that points B and C are too close; they touch at point D in the case of $(b/k)^3$, beyond which the interior solution disappears. Then the economy jumps to the corner solution with $\hat{\varepsilon}^*_{t+1} = \breve{\varepsilon}$ and $\bar{R}_{t+1} < \breve{R}^*_{t+1} < \breve{R}_{t+1}$, as illustrated in the Corollary to Proposition 1, thus yielding higher interest rates and a higher threshold. (For expositional simplicity, the corner solution is not depicted in Figure 5.) Generally speaking, the consolidation risk becomes maximal when the GB interest rate takes an extremely high level. In the present context, a high interest rate enhances the probability of fiscal consolidation, the latter serving to increase GB interest rate. It should be noted that the case in Figure 6 can apply to a wider range of consolidation rule scenarios, insofar as they are confirmed in the simulation.

Figure 6



In summary, we have addressed the circumstances in which a buildup of public debt might lead to lower GB interest rates, owing to the prospect of future tax increases that relate to fiscal consolidation—as illustrated in Figure 5(b)—and in which there might thus be a sudden rise of interest rates (i.e., as seen in Figure 6), thus increasing the fiscal consolidation risk. Therefore, the observation that the GB interest rate has remained relatively low does not ensure that the same trend will continue in the future as public debt accumulates. We confirm these scenarios in the simulation in Section 5.

4.3 Demographic impact

Next, we examine the effects of demographic changes. Note that in the endogenous growth model, the population or labor force is a key driving force that enhances productivity. When the population size decreases, so too does productivity, and thus the expected return on capital is diminished; this in turn works to reduce R_{t+1}. The threshold of the regime $\hat{\varepsilon}_{t+1}$ switch is increased, on the other hand, because the primary surplus is lowered. As these elements interact, the net impact is as stated in the following proposition:

Proposition 3: Consider a reduction in n_{t+1}. Then

(i) R^*_{t+1} decreases if $\partial R_{t+1} / \partial \varepsilon_{t+1} < 0$

(ii) $\hat{\varepsilon}^*_{t+1}$ is enhanced if R^*_{t+1} is increasing

In general, the equilibrium effects of the demographic change on R^*_{t+1} and $\hat{\varepsilon}^*_{t+1}$ are ambiguous. In Section 5, we conduct a simulation to compare the scenarios of different demographic changes.

4.4 Debt accumulation

In the previous section, b_{t+1}/k_{t+1} was taken as fixed and the equilibrium was established as conditional upon it. We now consider the accumulation of public debt, which is stochastic as it relies on the realization of productivity shock. Combining (9′) and (14) and advancing the period by one, we obtain the transition process of the debt–capital ratio as follows:

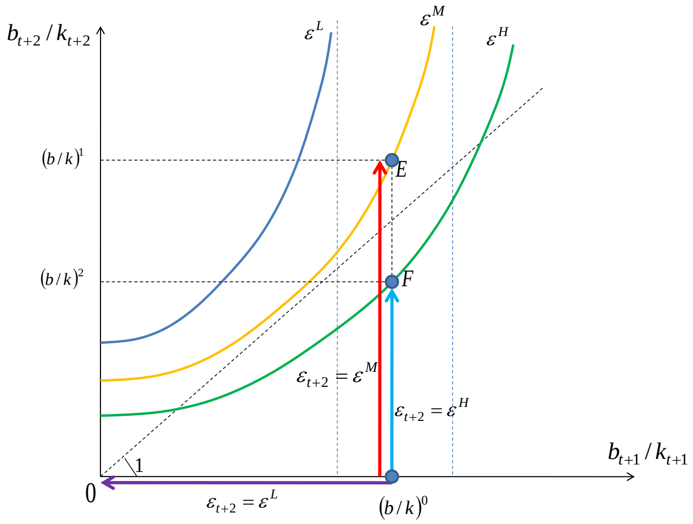
$$\frac{b_{t+2}/k_{t+2}}{1+b_{t+2}/k_{t+2}} = \frac{(1+\theta)(1+\delta)}{(1-\alpha)(1-\tau)} \times \left\{ \frac{\hat{R}^*_{t+1} b_{t+1}/k_{t+1}}{\left((1-\alpha)(1-\tau)\right)^{\delta(1-\alpha)/(1+\alpha\delta)} n_{t+1}{}^{(1-\alpha)/(1+\alpha\delta)} \left(A\tilde{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}} + \left(\frac{\lambda}{\left(\tilde{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}} - \tau(1-\alpha) \right) \right\}, \tag{22}$$

where $\tilde{\varepsilon}_{t+1}$ states that its value is uncertain at period *t*.

The immediately higher b_{t+1}/k_{t+1} is transited to a higher b_{t+2}/k_{t+2} in the next period, given that ε_{t+1} accounts for the dependency of R^*_{t+1} on b_{t+1}/k_{t+1}. According to Proposition 2, this implies that the risk of fiscal consolidation at period $t + 1$ increases along with the current debt–capital ratio if $\partial R_{t+1} / \partial(b_{t+1} / k_{t+1}) \geq 0$.

Figure 7 depicts the shape of the transition function (22) with the constant n_{t+1} at n. ε_t has three levels: low, middle, and high. Note that a smaller value of ε_{t+1} shifts (22) upward. Also note that the curve approaches infinity if b_{t+1} / k_{t+1} goes to the critical level, such that $\varepsilon_{t+1} = \hat{\varepsilon}_{t+1t}(R^*_{t+1}, b/k, n)$ —that is, ε_{t+1} becomes coincident with the threshold of fiscal consolidation. Suppose that $b_{t+1}/k_{t+1} = (b/k)^0$. By (22), the debt–capital ratio carried over to the next period is given by $b_{t+1}/k_{t+1} = (b/k)^1$, being located at point E if $\varepsilon_{t+1} = \varepsilon^M$. In the figure, we have $\varepsilon^L < \hat{\varepsilon}_{t+1}\left(R^*_{t+1}, (b/k)^1\right) < \varepsilon^M$, which implies that there arises a regime change at period $t+1$ if $\varepsilon_{t+1} = \varepsilon^L$, whereas government financing is sustainable when $\varepsilon_{t+1} = \varepsilon^H$. Figure 7 shows that b_{t+2}/k_{t+2} approaches infinity at $\varepsilon_{t+1} = \varepsilon^L$ without consolidation. In the event $\varepsilon_{t+1} = \varepsilon^L$, no public debt—that is, $b_{t+2} = 0$—is issued under the consolidation rule, and thus the economy moves back to its origin. On the other hand, the ratio is lowered to $b_{t+2}/k_{t+2} = (b/k)^2$ or point F when $\varepsilon_{t+1} = \varepsilon^H$.

Figure 7



5. Numerical Example

5.1 Specification

This section aims to provide numerical examples of the comparative statics developed in Section 3. Specifically, we examine the theoretical hypotheses that public debt accumulation may lead to lower interest rates and that the equilibrium interest rate may exhibit a discrete change from a relatively low to an extremely high level. In addition, we confirm whether the equilibrium in the preconsolidation regime is affected by the fiscal consolidation rule and demographic change. Moreover, we calculate the threshold at which consolidation occurs, given b_t / k_t at period *t*. Needless to say, our quantitative analysis does not intend to replicate any real-world economic practice; rather, it supplements our theoretical model, resolving the ambiguity of its results and clarifying its policy implications.

The parameters are specified in Table 1. ε_t distributes over $[0.5,1.5]$ according to the inverse U-shaped density function with a mean of 1.[12] We set the tax rate at a relatively low 10%, and we also set the expenditure rate to 10% of potential output. This implies that the primary deficit is likely to result unless ε_t is larger than the mean of 1; thus, there exists the possibility that public debt is accumulated, with consolidation risk thus being enhanced. The fiscal consolidation rule commands the expenditure to be cut in half to $\lambda_{t+1} = 0.05$. The wage tax rate under consolidation is increasing in ε_{t+1}, whereas it increases with b_{t+1} / k_{t+1}, as imposed by Assumption 2. This is specified in Table 1. Consolidation relies on more tax increases to mitigate a large debt–capital ratio, whereas the default rate is raised when ε_{t+1} is small and the economy is therefore depressed. Such a presumption should be plausible.

The parameter *g* in the tax function refers to the extent of the required tax increase. The simulation set three values for *g* (i.e., 0.5, 0.75, and 1.0). A higher *g* implies a larger tax increase in the consolidation, which in turn implies a lower default rate ξ_{t+1} that is defined as residual by (15): the above tax function is constrained so that ξ_{t+1} takes an interior value. By comparing the results of different values of *g*, we can assess the effect of the fiscal rule on R^*_{t+1} and $\hat{\varepsilon}^*_{t+1}$, as well as changes in the debt–capital ratio. To examine the demographic change, we consider the case that the population remains constant over time, and also the case that it is declining; in the latter, we assume that it annually decreases by 0.3%. Taking one period to stand for 30 years, we let $n_{t+1} = (1-0.03)^{30} n_t$. Distinct in terms of the parameter *g* and demographics, four scenarios are presented as summarized in Table 2. Scenario 1 is taken as a benchmark in the following table.

Table 1: Parameters

δ	0.5
α	0.3
θ	1.333

[12] More correctly, form assumption 1, the range of ε_t is $[\underline{\varepsilon}, \bar{\varepsilon}] = [0.5,1.5] \cup \{\bar{\varepsilon} - \upsilon, \bar{\varepsilon}\}$. Therefore, in our simulation, we also considers the effect of $\{\bar{\varepsilon} - \upsilon, \bar{\varepsilon}\}$ as the possibility of $\{\bar{\varepsilon} - \upsilon, \bar{\varepsilon}\} = 10^{-5}$, $\bar{\varepsilon} = 100$ and $\upsilon = 10^{-5}$.

A	7.0
$\Omega^0 \equiv \{\tau, \lambda, \zeta\}$	τ = 0.1, λ = 0.1
$\Omega_{t+1} \equiv \Omega(\Xi_{t+1})$	$\tau(\varepsilon_{t+1}, b_{t+1}/k_{t+1}) = Min\left(g \times \tau_{Max}, \quad \tau + 3.8\frac{b_{t+1}}{k_{t+1}}\varepsilon_{t+1}\right)$ $\lambda(\varepsilon_{t+1}, b_{t+1}/k_{t+1}) = 0.1$

Table 2: Scenarios

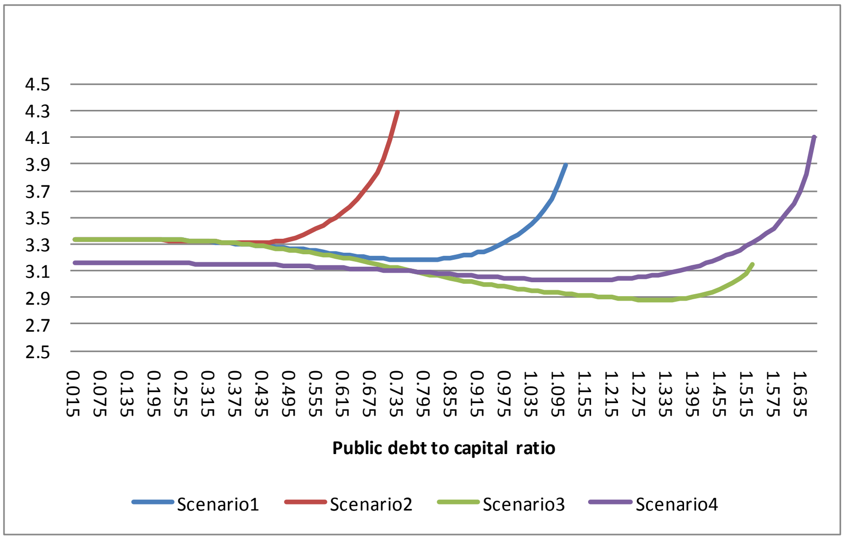
Scenario 1 (Benchmark)	g = 0.75	
Scenario 2	g = 0.5	$n_{t+1} = 1$
Scenario 3	g = 1.0	
Scenario 4	g = 0.75	$n_{t+1} = n_t(1-0.003)^{30}$

5.2 Results

In the simulation, we focus only on the interior equilibrium. The GB interest rates for different scenarios are shown in Figure 8, where b_{t+1}/k_{t+1} is treated parametrically and taken on the horizontal axis. In all scenarios, there exists a range in which R^*_{t+1} exhibits a downward slope, confirming our theoretical hypothesis. Take the benchmark scenario (Scenario 1). There, the interest rate is initially declining, with b_{t+1}/k_{t+1}. Its moderate downward trend continues until b_{t+1}/k_{t+1} = 0.78, where R^*_{t+1} takes the minimum value. The slope is then reversed, further increasing the debt–capital ratio and rapidly raising the interest rate. At b_{t+1}/k_{t+1} = 1.11, the stable interior level of R^*_{t+1} disappears, diverging to the corner equilibrium (which is not explicitly treated here). This is consistent with what is seen in Figure 6.

The benchmark scenario is compared to Scenarios 2 and 3, to assess the impacts of a tax increase. R^*_{t+1} in Scenario 2 barely differs from the benchmark for low levels of b_{t+1}/k_{t+1}. After b_{t+1}/k_{t+1} = 0.3, however, the former begins to exceed the latter, and the difference between them widens quickly. Once the ratio exceeds 0.735, Scenario 2 loses its interior equilibrium, whereas it remains in the benchmark scenario. In the former scenario, with g = 0.5, the tax increase is less significant than in the latter scenario when fiscal consolidation is implemented. Given that both scenarios impose $\lambda_{t+1} = 0.05$ in the event of the consolidation, this implies that Scenario 2 experiences a higher default rate and consequently adds a risk premium to GB. Now let us consider Scenario 3, with g = 1. Again, its interest rate moves about the same as the benchmark when the public debt–capital ratio is not high. For b_{t+1}/k_{t+1} > 0.5, the disparity turns out to be prominent, with R^*_{t+1} in Scenario 3 staying lower than that in the benchmark. The former can then sustain the interior equilibrium for a larger b_{t+1}/k_{t+1} than the latter. It can then be concluded that consolidation involving a greater tax increase leads to lower R^*_{t+1}, sustaining the interior equilibrium.[13]

Figure 8: GB interest rate



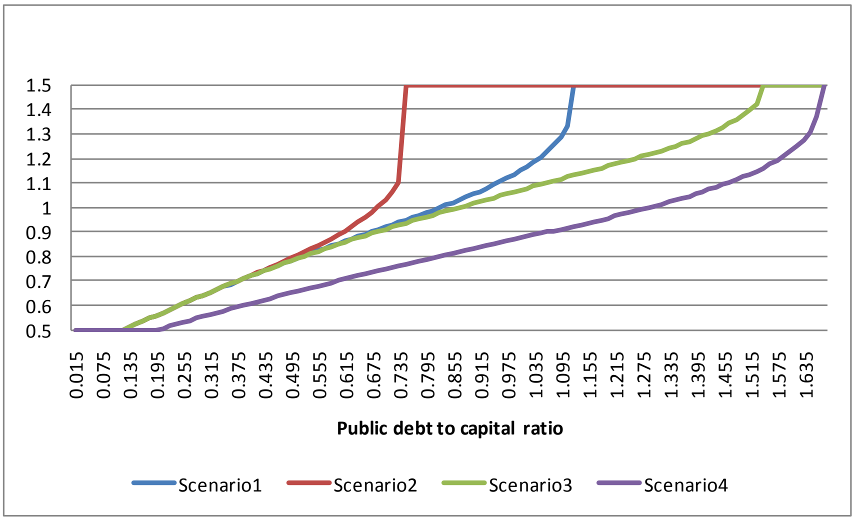
Consider the threshold of regime change, $\hat{\varepsilon}^*_{t+1}$. In all scenarios, it monotonically increases in b_{t+1}/k_{t+1}, as in Figure 9. In comparing different scenarios with different consolidation rules, $\hat{\varepsilon}^*_{t+1}$ stays lower when the tax increase is larger—that is, g is high, reflecting a lower interest rate. The prospect of large tax increases in the event of a fiscal restructuring that contributes to a lower default rate serves to mitigate consolidation risk, which should be intuitive. The risk is reflected in a GB premium that is defined as the difference between the GB interest rate and the expected return on capital. The premium remains negligible when risk is low: according to the consolidation risk, the revenue deficiency is largely filled by tax increases and expenditure cuts. The default rate in the event of consolidation is raised as the debt–capital ratio increases, which in turn augments the premium.

To see the effect of the demography, we compare the benchmark scenario with Scenario 4. The figure reveals that overall, the declining population works to diminish equilibrium interest rates. The gap in interest rates between the two scenarios first declines with b_{t+1}/k_{t+1} until the ratio reaches 0.72, whereupon it starts to increase sharply. In Scenario 4, the interior equilibrium is sustained up to b_{t+1}/k_{t+1} = 1.67, and thus the fiscal consolidation risk is reduced relative to the benchmark case. Recall that in general, the demographic impact on $\hat{\varepsilon}^*_{t+1}$ was ambiguous. The simulation establishes that the threshold is lowered in the case of smaller population—that is, the downward shift of $R_{t+1}(\hat{\varepsilon}_{t+1}, \Sigma_{t+1})$ due to a decrease in n_{t+1} dominates the upward movement of $\hat{\varepsilon}_{t+1}(R_{t+1}, \Sigma_{t+1})$, with

[13] The simulation also reveals that an unstable equilibrium appears when the debt–capital ratio is very close to the critical level of the public debt–capital ratio, in which the stable interior solution disappears.

the primary balance deteriorating as in Figure 5.

Figure 9: Threshold



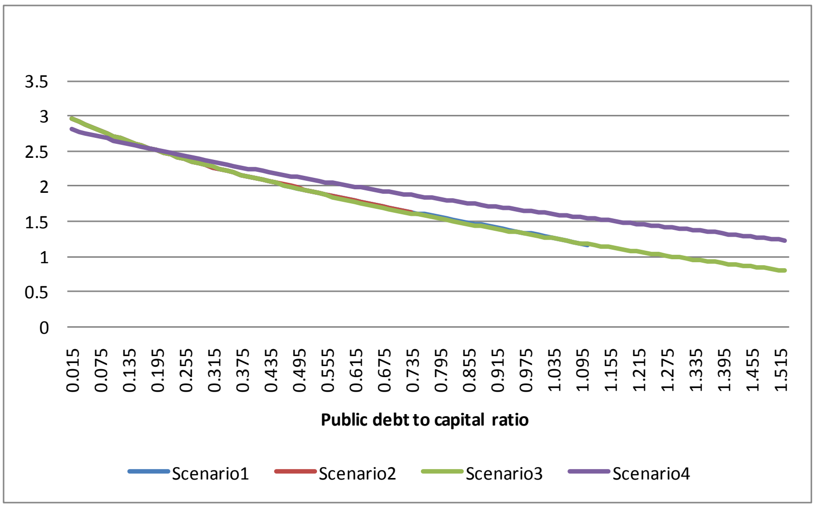
Consider the economic growth that is calculated in the expected term as

$$(10') \quad E_t \tilde{\chi}_{t+1} \equiv E_t \left[\left(\frac{1-\tilde{\tau}_{t+1}}{1-\tau_t} \right)^{\delta(1-\alpha)/(1+\alpha\delta)} \left(\frac{\tilde{\varepsilon}_{t+1}}{\varepsilon_t} \right) \right] \frac{k_{t+1}}{k_t}.$$

Growth decreases as private capital is crowded out by public debt, which decreases k_{t+1}/k_t given (9′). Figure 10 gives the expected growth rate from the perspective of period t + 1. Tax increases (i.e., a higher g) in the consolidation regime exert two opposing impacts on growth. As stated above, it serves to lower R^*_{t+1}, which increases k_{t+1}/k_t as accumulated at period t. The tax burden, on the other hand, reduces the output in the event of consolidation at period t + 1, and this is reflected in the bracket of the expectation in (10′). In comparing Scenarios 1 and 2, the two yield almost the same growth rate for a lower b_{t+1}/k_{t+1}, and the latter experiences slightly higher growth after $b_{t+1}/k_{t+1} = 0.4$ than the former, until the critical ratio in which the interior solution disappears in Scenario 2. The same can be seen when Scenario 3 is compared to Scenario 1, but the difference therein is negligible.

Demography makes a considerable difference. The expected growth rate seen in Scenario 4 is initially lower than the benchmark scenario, but as the debt-to-capital ratio increases, the relationship is reversed and the difference expands as public debt is built up relative to capital. This chain of events may be counter-intuitive, but it occurs because the lower risk of fiscal consolidation serves to decrease the expected tax rate from the perspective of period t. In addition, as the GB interest rate is lowered, private capital accumulation is less crowded in Scenario 4, thus enhancing k_{t+1}/k_t.

Figure 10: Expected economic growth rate (%)



5.3 Debt accumulation

Consider the dynamics of the public debt–capital ratio, which was treated as exogenous in the previous subsection. We divide ε_{t+1}, which is realized at the t + 1 period in four classes and for each quartered group, and the (conditional) expected level of b_{t+2}/k_{t+2} is calculated given a b_{t+1}/k_{t+1} that is determined at period t. Note that the expectation is taken from the perspective of period t. Table 3 shows the results in the benchmark scenario.

Table 3: Changes in debt–capital ratio

b_{t+1}/k_{t+1} \ ε_{t+1}	0.5˜0.75	0.75˜1.00	1.00˜1.25	1.25˜1.50
0.03	0.85	0.29	0.10	0.02
0.06	1.10	0.36	0.14	0.04
0.12		0.54	0.22	0.10
0.18		0.77	0.32	0.16
0.21		0.91	0.37	0.19
0.24		1.08	0.43	0.22
0.3			0.56	0.29
0.405			0.85	0.44
0.42			0.90	0.46
0.45			1.01	0.51
0.465			1.07	0.54
0.51				0.62
0.6				0.80
0.69				1.03
0.705				1.08

The expected ratio of b_{t+2}/k_{t+2} increases with b_{t+1}/k_{t+1} and is higher for the lower quarter of ε_{t+1}.

At the lowest quarter—that is, $\varepsilon_{t+1} \in [0.5, 0.75]$—the average of b_{t+2}/k_{t+2} exceeds the critical ratio, b_{t+2}/k_{t+2} = 1.1, at which interior equilibrium ceases to exist according to Figure 6, when the debt–capital ratio b_{t+1}/k_{t+1} is more than 0.06; this implies that fiscal consolidation is inevitable at t + 2 or $\hat{\varepsilon}_{t+2} = \bar{\varepsilon}$. In higher-quartered groups of ε_{t+1}, the expected ratio remains at a sustainable level for larger b_{t+1}/k_{t+1}. For instance, at the highest quarter—that is, $\varepsilon_{t+1} \in [1.25, 1.5]$—the interior equilibrium survives at period t + 1 for a b_{t+1}/k_{t+1} value of less than 0.72.

6. Political Economy

We have thus far assumed the process of public debt accumulation to be exogenous. One possible objection is that there should exist a self-correcting mechanism to contain public debt at sustainable levels by raising taxes and/or cutting expenditures. Indeed, rational voters may not allow the risk of fiscal crisis to deepen over time, but may undertake preventive measures to curtail such risk. In our OLG setting, however, the representative household of each generation—which naturally has a limited life-span—may not act in such a manner. To see this in effect, consider simple majority voting with respect to tax increases in a preconsolidation regime, so as to decrease the public deficit. Both the young and the old generations in each period exercise their voting rights.

What are voters' preferences with respect to such preventive measures? The consumption of the old voters under the status quo is equal to

$$(3.2') \quad c_{t-1}^{o} = \bar{s}_{t-1}\left(\bar{q}_t(1-\xi_{t+1})\bar{R}_{t+1} + (1-\bar{q}_t)r_{t+1}\right),$$

where the bar implies that their choices are sunk at the beginning of period t. Given that the return on private capital declines with wage taxes, the older group opposes any tax increase.

Let us now consider the young voters. Their lifetime expected utility in the equilibrium realization of ε_t is expressed by

$$(23) \quad E_t U_t = (\Psi_t - s_t)^{\theta} s_t E_t(\tilde{r}_{t+1}) = \psi_t^{\theta+1}\left(1 - \frac{1}{1+\theta}\right)^{\theta} \frac{E_t(\tilde{r}_{t+1})}{1+\theta} = \psi_t^{\theta+1}\left(\frac{\theta}{1+\theta}\right)^{\theta} \frac{E_t(\tilde{r}_{t+1})}{1+\theta},$$

where

$$\Psi_t = \frac{1}{1+\delta}\omega_t^{1+\delta} = \frac{1}{1+\delta}\left((1-\alpha)(1-\tau_t)A\varepsilon_t\right)^{(1+\delta)/(1+\alpha\delta)} k_t$$

and $\tau_t = \tau$ without a preventive tax increase. Differentiating (23) with respect to the tax rate yields

$$(24) \quad \frac{d}{d\tau_t} E_t U_t \propto -(1+\theta)\frac{1+\delta}{1+\alpha\delta}\frac{1}{1-\tau_t} + \frac{1}{E_t(\tilde{r}_{t+1})}\frac{dE_t(\tilde{r}_{t+1})}{d\hat{\varepsilon}_{t+1}^{*}}\frac{d\hat{\varepsilon}_{t+1}^{*}}{d\tau_t}.$$

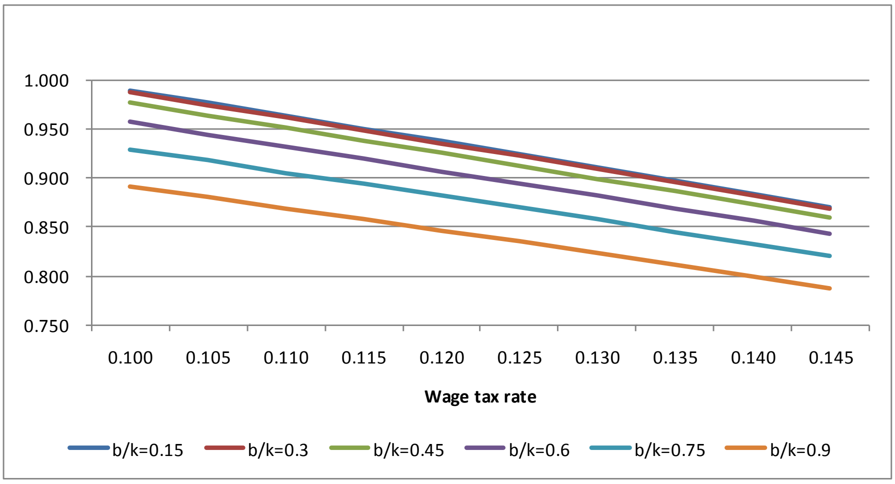
The first term represents the direct effect of raising τ_t. It lowers Ψ_t, the after-tax wage income that the young generation earns at period *t*; this decreases utility. The enhanced tax revenue, on the other hand, improves the current primary balance, which reduces the risk of the fiscal consolidation. That is,

$$\frac{d}{d\tau_t}\hat{\varepsilon}^*_{t+1} = \frac{d\hat{\varepsilon}^*_{t+1}}{d(b_{t+1}/k_{t+1})}\frac{d}{d\tau_t}\left(\frac{b_{t+1}}{k_{t+1}}\right) < 0 .$$

The above, in turn, enhances the expected return on capital. This, which is captured in the second term of (24), serves to raise the young generation's lifetime utility.

Therefore, the combined effect of such tax increases is not certain. To resolve this ambiguity, we rely on simulation. In Figure 11, we take the benchmark scenario and depict changes in the logarithm of the expected utility as the wage tax rate increases from the initial level τ = 10%. Different log EU_t values correspond to different levels of b_t / k_t that are predetermined at period *t*. b_t / k_t takes a smaller value as the ratio increases, reflecting the crowding effect that, in turn, lowers receiving wages. For all b_{t+1} / k_{t+1}, utility decreases with the tax rate; this reveals that the perverse effect of decreasing disposable wage income (i.e., by imposing higher tax rates) dominates the gain derived from augmenting the expected return on capital while the consolidation risk is reduced. Interestingly, the utility loss derived from increasing the wage tax is exacerbated, with the public debt–capital ratio being raised as a result. Thus, it becomes increasingly difficult to raise taxes as government finances worsen. We obtain mostly the same results with all the other scenarios. In short, the young household will vote against tax increases in the preconsolidation regime.

Figure 11: Expected utility



Thus, preventive measures vis-à-vis fiscal consolidation risk will never gain political support from existing generations. Public debt will be then left to accumulate until the regime switch becomes inevitable, when $\varepsilon_{t+1} \leq \varepsilon^*_{t+1}$. To state it differently, the normative criteria of fiscal sustainability—such as the Dormer condition or the transversality condition of a long-term government budget—do not incentivize contemporary politics to undertake fiscal restructuring. Of course, the future generation will suffer from a large public debt that lowers k_{t+1} (due to crowding out) and reduces their wages, and which can trigger wage tax increases in the event of a consolidation. Such welfare loss of the future generation is not incorporated by current voters, who are assumed to be self-interested.

The political failure to undertake a restructuring effort has been examined by Alesina and Drazen (1991), who modeled delayed stabilization as a “war of attrition” or a sort of “game of chicken” between vested interests. They address the fact that the timing of actual fiscal consolidation turns out to be too late, relative to the optimal timing that maximizes the joint payoff of stakeholders. Related studies by Velasco (2000) and Ihori and Itaya (2002) consider public debt accumulation as a consequence of a noncooperative subgame among special interest groups that freely extract resources from the government budget. In their context, fiscal restructuring is featured as a voluntary contribution for the public good that suffers from the free-riding motive. These models assume the infinite life of agents without cooperation. The present paper with its OLG setting addresses the motive of contemporary generations to shift the burden of fiscal consolidation to future generations that cannot yet vote, or even do not yet exist.

The present model does not account for the political process of determining tax and expenditure rates, or τ and λ in the preconsolidation regime. Rather, these values are taken as exogenous. However, we have established the conditions under which the initial tax and expenditure policies are not corrected and the consolidation risk is enhanced, as current generations do not agree to accept a tax hike.

6. Concluding remarks

In this paper, we analyze the relationship between GB interest rates and the fiscal consolidation rule using an overlapping generation model with endogenous and stochastic growth settings. Our key findings are summarized as follows. GB interest rates may decline as public debt accumulates relative to private capital—as suggested by the findings of Reinhart et al. (2012)—as opposed to the conventional view that a buildup of public debt accompanies a rise in interest rates. This is consistent with the seemingly paradoxical circumstances surrounding GB interest rates in Japan, where rates remain low, despite a public debt–GDP ratio that has been increasing over the last several decades. This paper also addresses the fact that fiscal consolidation rule plays a key role in determining equilibrium interest rates. Moreover, the relatively stable interior equilibrium may disappear in a discrete manner that shifts the economy to a situation in which consolidation is inevitable and GB interest is quite high. The normative standpoint suggests that preventive action should be undertaken to contain such fiscal risk. However, precautionary tax increases meant to contain public debt to sustainable levels will not win the political support of existing generations, as their life-spans and periods of self-interest are limited. Instead, voters prefer to shift the ultimate burden of public debt to future generations that cannot currently vote.

Admittedly, our model is highly stylized and abstracts some key issues that should be further examined in future research. These include (1) the search for the "real" threshold of regime change $\hat{\varepsilon}^{*}_{t+1}$ and the limits of public debt–GDP ratio, undertaken by calibrating our model to real economies (e.g., the Japanese economy), (2) the effect on our model of inflation based on the Fiscal Theory of Price Level, as illustrated by Cochrane (2010), and (3) the analysis of another voting game (e.g., between low and high income households) over tax increases in the preconsolidation regime, so as to decrease public deficit. Naturally, the results of our study would have greater bearing and import, if their generalizability could be demonstrated.

Appendix A

From Eq. (7.2′), (9′), and (10), the condition of dynamic efficiency is represented as follows:

$$\tilde{\chi}_{t+1} \le r_{t+1} \Leftrightarrow \left(\frac{1-\tilde{\tau}_{t+1}}{1-\tau_t}\right)^{\delta(1-\alpha)/(1+\alpha\delta)} \left(\frac{\tilde{\varepsilon}_{t+1}}{\varepsilon_t}\right)\left(\frac{n_{t+1}}{n_t}\right)^{(1-\alpha)/(1+\alpha\delta)}$$

$$\times \frac{n_t^{(1-\alpha)/(1+\alpha\delta)}}{(1+\theta)(1+\delta)\left(1+\dfrac{b_{t+1}}{k_{t+1}}\right)} A\varepsilon_t\big((1-\alpha)(1-\tau_t)\big)^{(1+\delta)/(1+\alpha\delta)}$$

$$\le \alpha\tilde{\varepsilon}_{t+1}A(n_{t+1})^{(1-\alpha)/(1+\alpha\delta)}\big((1-\alpha)(1-\tilde{\tau}_{t+1})\big)^{\delta(1-\alpha)/(1+\alpha\delta)}$$

$$\Leftrightarrow \frac{(1-\alpha)(1-\tau_t)}{(1+\theta)(1+\delta)\left(1+\dfrac{b_{t+1}}{k_{t+1}}\right)} \le \alpha \quad \Leftarrow \frac{1-\alpha}{(1+\theta)(1+\delta)} \le \alpha \quad .$$

Appendix B

Assume that there is no default. Then,

$$b_{t+2} = R_{t+1}b_{t+1} - \{T_{t+1}(\tau, k_{t+1}, \varepsilon_{t+1}) - G_{t+1}\} = R_{t+1}b_{t+1} - PS_{t+1} \Rightarrow \frac{b_{t+2}}{R_{t+1}} = b_{t+1} - \frac{PS_{t+1}}{R_{t+1}},$$

where

$$PS_{t+1} = T_{t+1}(\tau_{t+1}, k_{t+1}, \varepsilon_{t+1}) - G_{t+1} = k_{t+1}\big((1-\alpha)A\big)^{(1+\delta)/(1+\alpha\delta)} n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} \Delta(\tau_{t+1}, \lambda, \varepsilon_{t+1}).$$

Given that there is no consolidation risk, the transversality condition is not fulfilled if the following

holds:

$$\begin{aligned}
&Lim_{j\to\infty} E_t \frac{b_{t+2+j}}{\Pi_{j=1}^{\infty} R_{t+j}} = b_{t+1} - \sum\nolimits_{j=1}^{\infty} E_t \frac{PS_{t+j}}{\Pi_{l=1}^{j} R_{t+l}} \geq 0 \\
&\Rightarrow b_{t+1} \geq \sum\nolimits_{j=1}^{\infty} E_t \frac{PS_{t+j}}{\Pi_{l=1}^{j} R_{t+l}} \\
&\Rightarrow \frac{b_{t+1}}{k_{t+1}} \frac{k_{t+1}}{k_t} \geq \big((1-\alpha)A\big)^{(1+\delta)/(1+\alpha\delta)} \sum\nolimits_{j=1}^{\infty} E_t \frac{\Pi_{l=1}^{j} k_{t+l} / k_{t+l-1}}{\Pi_{l=1}^{j} R_{t+l}} n_{t+j}^{(1-\alpha)/(1+\alpha\delta)} \Delta(\tau_{t+j}, \lambda, \varepsilon_{t+j}) \\
&= \big((1-\alpha)\varepsilon_{t+1}A\big)^{(1+\delta)/(1+\alpha\delta)} n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} \sum\nolimits_{j=1}^{\infty} E_t \frac{1}{\Pi_{l=1}^{j} R_{t+l}} \Pi_{l=1}^{j} \chi_{t+l} \frac{\Delta(\tau_{t+j}, \lambda, \varepsilon_{t+j})}{(\varepsilon_{t+j})^{(1+\delta)/(1+\alpha\delta)}} \\
&= \big((1-\alpha)\varepsilon_{t+1}A\big)^{(1+\delta)/(1+\alpha\delta)} n_{t+1}^{(1-\alpha)/(1+\alpha\delta)} \sum\nolimits_{j=1}^{\infty} E_t \left(\Pi_{l=1}^{j} \frac{E_{t+l-1}\chi_{t+l}}{E_{t+l-1} r_{t+l}} \right) \frac{\Delta(\tau_{t+j}, \lambda, \varepsilon_{t+j})}{(\varepsilon_{t+j})^{(1+\delta)/(1+\alpha\delta)}}
\end{aligned},$$

where

$$\chi_{t+l} \equiv \frac{Y_{t+l}}{Y_{t+l-1}} = \left(\frac{\varepsilon_{t+l}}{\varepsilon_{t+l-1}}\right)^{(1+\delta)/(1+\alpha\delta)} \left(\frac{n_{t+l}}{n_{t+l-1}}\right)^{(1-\alpha)/(1+\alpha\delta)} \frac{k_{t+l}}{k_{t+l-1}}$$

By lemma 1, the right-hand side takes a finite value, whereas the left side diverges as the public debt–capital ratio rises, or b_{t+1}/k_{t+1} goes to infinity.

Appendix C

We define $\sigma \equiv n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}\left\{\frac{\alpha}{1-\alpha}\Phi(\hat{\varepsilon}_{t+1},\mathrm{T}_{t+1}) - \frac{k_{t+1}}{b_{t+1}}\int^{\hat{\varepsilon}_{t+1}}\Delta(\tilde{\tau}_{t+1},\tilde{\lambda}_{t+1},\tilde{\varepsilon}_{t+1})dF(\tilde{\varepsilon}_{t+1})\right\}$,

where $\Phi(\hat{\varepsilon}_{t+1},\mathrm{T}_{t+1}) = \int^{\hat{\varepsilon}_{t+1}}(1-\tau_{t+1})^{\delta(1-\alpha)/(1+\alpha\delta)}\left(\varepsilon_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}dF(\varepsilon_{t+1}) + (1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)}\int_{\hat{\varepsilon}_{t+1}}\left(\varepsilon_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}dF(\varepsilon_{t+1})$

$\Rightarrow$

$$\partial R_{t+1}/\partial\hat{\varepsilon}_{t+1} = \partial\left(\frac{\sigma}{1-F(\hat{\varepsilon}_{t+1})}\right)/\partial\hat{\varepsilon}_{t+1} >< 0$$

$\Leftrightarrow$

$$\frac{\partial\sigma/\partial\hat{\varepsilon}_{t+1} + f(\hat{\varepsilon}_{t+1})R_{t+1}}{1-F(\hat{\varepsilon}_{t+1})} >< 0$$

$\Leftrightarrow$

$$f(\hat{\varepsilon}_{t+1})R_{t+1} + n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}\left\{\frac{\alpha}{1-\alpha}\frac{\partial}{\partial\hat{\varepsilon}_{t+1}}\Phi(\hat{\varepsilon}_{t+1},\mathrm{T}_{t+1}) - \frac{k_{t+1}}{b_{t+1}}\Delta(\hat{\tau}_{t+1},\hat{\lambda}_{t+1},\hat{\varepsilon}_{t+1})f(\hat{\varepsilon}_{t+1})\right\} >< 0$$

(B-1)

making use of $(1-\hat{\zeta}_{t+1})R_{t+1}\frac{b_{t+1}}{k_{t+1}} = \left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\Delta(\hat{\tau}_{t+1},\hat{\lambda}_{t+1},\hat{\varepsilon}_{t+1})$

(B-1)

$\Leftrightarrow$

$$f(\hat{\varepsilon}_{t+1})R_{t+1} + n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}\frac{\alpha}{1-\alpha}\frac{\partial}{\partial\hat{\varepsilon}_{t+1}}\Phi(\hat{\varepsilon}_{t+1},\mathrm{T}_{t+1}) - (1-\hat{\zeta}_{t+1})f(\hat{\varepsilon}_{t+1})R_{t+1}$$

$$= \hat{\zeta}_{t+1}f(\hat{\varepsilon}_{t+1})R_{t+1} + n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}\frac{\alpha}{1-\alpha}\frac{\partial}{\partial\hat{\varepsilon}_{t+1}}\Phi(\hat{\varepsilon}_{t+1},\mathrm{T}_{t+1}) >< 0$$

(B-2)

making use of

$$\frac{\partial}{\partial\hat{\varepsilon}_{t+1}}\Phi(\hat{\varepsilon}_{t+1},\mathrm{T}_{t+1}) = (1-\hat{\tau}_{t+1})^{\delta(1-\alpha)/(1+\alpha\delta)}\left(\hat{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}f(\hat{\varepsilon}_{t+1}) - (1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)}\left(\hat{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}f(\hat{\varepsilon}_{t+1})$$

(B-2)

$\Leftrightarrow$

$$\hat{\zeta}_{t+1}f(\hat{\varepsilon}_{t+1})R_{t+1}$$
$$+ n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}\frac{\alpha}{1-\alpha}\left((1-\hat{\tau}_{t+1})^{\delta(1-\alpha)/(1+\alpha\delta)} - (1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)}\right)\left(\hat{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}f(\hat{\varepsilon}_{t+1})$$
$$>< 0$$

$\Leftrightarrow$

$$\hat{\zeta}_{t+1}R_{t+1} >< n_{t+1}^{(1-\alpha)/(1+\alpha\delta)}\left((1-\alpha)A\right)^{(1+\delta)/(1+\alpha\delta)}\frac{\alpha}{1-\alpha}\left((1-\tau)^{\delta(1-\alpha)/(1+\alpha\delta)} - (1-\hat{\tau}_{t+1})^{\delta(1-\alpha)/(1+\alpha\delta)}\right)\left(\hat{\varepsilon}_{t+1}\right)^{(1+\delta)/(1+\alpha\delta)}$$

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