Budget Deficits and Economic Growth*

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Abstract

This paper investigates the sustainability and welfare effects of government budget deficits by using a simple endogenous growth model with overlapping generations. It is shown that, if the initial volume of government debt and the ratio of primary budget deficits to GDP are not large, then there can exist two steady-growth equilibria, one of which is associated with a higher growth rate and the other of which is associated with a lower growth rate. It is also shown that changes in government spending cannot be Pareto improving although they affect the long-run growth rate and each generation's utility.

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1 Introduction

In the 80s, several countries such as the United States and the United Kingdom have experienced large government budget deficits, and in the last decade Japan has run huge budget deficits. These events have prompted many researchers to investigate the sustainability of government budget deficits and the economic implications of the government intertemporal budget constraint.[1] Most early theoretical studies on this subject employ models without capital accumulation and derive conditions under which government deficits are sustainable (see Domer (1944), Turnovsky (1977) and Christ (1979)).[2] In these models, the interaction between budget deficits and economic growth cannot be analyzed, and thus several authors reexamine this issue with explicit consideration of capital accumulation and transitional processes. Ihori (1988) employs an overlapping generations model á la Diamond (1965), while Nielsen (1992) adopts a continuous-time overlapping generations model developed by Weil (1989).[3] These studies show that, if the growth rate of an economy is high, the subjective discount rate is low and the size of the public sector is modest, then there exist economically meaningful steady-growth equilibria with deficits, which can be financed by rolling over public debts forever. Introducing capital income taxation into an overlapping generations model, Uhlig (1996) shows that, if the tax rate is sufficiently high, then permanent budget deficits are feasible.[4] Moreover, Chalk (2000) and Rankin and Roffia (2002) investigate the sustainability of government debt, rather than that of government deficits, in an overlapping generations

[1] See Sargent and Wallace (1981), Darby (1984), McCallum (1984), Miller and Sargent (1984), Weil (1987) and Bhattacharya and Kudo (2002) for the economic implications of the government intertemporal budget constraint.

[2] See, for empirical studies on the issue, Hamilton and Flavin (1986), Wilcox (1989), Trehan and Walsh (1991) and Bohn (1998). Chalk and Hemming (2000) give a comprehensive survey on empirical methods in this literature.

[3] See also Okuno (1983).

[4] Azariadis (1993) and De La Croix and Michel (2002) survey this line of research.

model and show that the deficits and the initial stock of debt must not be too large for the debt to be sustainable.[5]

These studies use exogenous growth models in which the long-run growth rate is exogenously given as a parameter value. However, recent developments in endogenous growth theory show that we can analyze effects of fiscal policies in a more satisfactory way by employing a model where the long-run growth rate is determined endogenously.[6] In such an endogenous growth model, the long-run growth rate would change depending on the level of budget deficits, and hence the interaction between budget deficits and economic growth can be analyzed. This paper aims to examine this interaction theoretically by constructing a simple endogenous growth model.

There are a few closely related papers to ours. Saint-Paul (1992) investigates the effects of public debts on the long-run growth rate by using a model similar to ours. He shows that the higher the level of public debts becomes, the lower the growth rate is. However, his analysis is restricted to balanced growth paths where the government purchases nothing. In this paper, we take government consumption into account and assume that the government keeps the budget deficits - GDP ratio constant.[7] Under this setting, we show that the possibility of multiple balanced growth paths and examine the stability of these balanced growth paths. Azariadis and Reichlin (1996) adopt an overlapping generations model á la Diamond (1965) with production externalities and public debts, and investigate the stability property of the model. In their model, there exist two types of steady-growth equilibrium: one is an exogenous growth equilibrium with a positive level of

[5]Ganelli (2002) investigates a related issue in an overlapping generations model with endogenous labor supply, though the model lacks capital accumulation. Moreover, theoretical analyses on the sustainability under a stochastic environment are provided by Bohn (1995) and Blanchard and Weil (2001).

[6]See, for example, Barro (1990) and Futagami, Morita and Shibata (1993).

[7]Alogoskoufis and van der Ploeg (1991) investigate the sutainability of budget deficits under the assumption that the tax rate changes over time.

public debts and the other an endogenous growth equilibrium.

This paper is organized as follows. Section 2 sets up the basic model involved in this paper. In section 3, it is shown that there exist at most two steady-growth equilibria with permanent budget deficits, if the ratio of budget deficits to GDP are not so large. It is also shown that, when there exist two steady-growth equilibria, one steady-growth equilibrium associated with a higher growth rate is saddle-point stable and the other equilibrium with a lower growth rate is unstable. Section 4 characterizes the steady-growth equilibria and examines effects of an increase in budget deficits on the long-run growth rate. In section 5, the welfare effects of the budget deficits are examined. The final section concludes the paper.

2 The Model

This section describes the basic structure of our model.[8] At each point in time a new generation is born at the rate n, and thus the total population at time t is given by $N_t = N_0 e^{nt}$, where N_0 is the size of population at time 0. The goods and factor markets are perfectly competitive. Each agent has perfect foresight and supplies one unit of labor inelastically.

2.1 Production

There are N symmetric firms.[9] The production technology of firm j is represented by

$$y(j,t) = Ak(j,t)^{\alpha}[l(j,t)E_t]^{1-\alpha}, \tag{1}$$

where k, l and E stand for capital employed by firm j, labor employed by firm j and an external effect which raises the efficiency of labor. Although

[8]Similar models are employed by Alogoskoufis and van der Ploeg (1991), Saint-Paul (1992), Futagami and Shibata (1999, 2000) and Mino and Shibata (2000).

[9]The number of firms is assumed to equal that of households.

E would depend on several factors, we assume here that

$$E = \bar{K},$$

where $\bar{K}$ is the average capital stock in the economy. This formulation represents a kind of capital deepening effects *à la* Romer (1986).

Because of perfect competition the optimal conditions are given by:

$$w_t = (1-\alpha)Ak(j,t)^{\alpha}l(j,t)^{-\alpha}\bar{K}^{1-\alpha}, \tag{2}$$

$$r_t = \alpha Ak(j,t)^{\alpha-1}[l(j,t)\bar{K}]^{1-\alpha}, \tag{3}$$

where w and r are the wage rate and the real interest rate.

Since all firms are symmetric, we have, in equilibrium,

$$\bar{K}_t = k(j,t), \;\; l(j,t) = \frac{N}{N} = 1. \tag{4}$$

Substituting (4) into (2) and (3), we obtain the following relations:

$$w_t = (1-\alpha)Ak(j,t), \;\; r_t = \alpha A. \tag{5}$$

Equation (5) shows that the real interest is constant over time. Moreover, by use of (4) the reduced form production function is represented as

$$y_t = Ak(j,t). \tag{6}$$

2.2 Households

The household behavior is the same as that in Weil (1987, 1989).[10] There is an initial cohort endowed with non-human wealth at time 0 and it is called generation 0^{-}. A newly born generation receives no bequests from its

[10] Alternatively, we can employ the Blanchard (1985) model with declining relative labor income. See Buiter (1988) for the relation of the two models.

predecessor. At time t, the representative cohort born at s $(0 \leq s \leq t)$ solves the following maximization problem:

$$\max \int_t^{\infty} \ln c(s,u) e^{-\rho(u-t)} du \tag{7}$$

$$s.t. \; \frac{da(s,t)}{dt} = rv(s,t) + w_t - \tau_t^h - c(s,t),$$

where c, v and τ^h stand for consumption, asset holding and lump-sum tax.[11] In order to assure economically meaningful solutions we assume that $\tau^h < w$.

The intertemporal optimal condition is given by

$$\frac{dc(s,t)}{dt} = (r - \rho)c(s,t). \tag{8}$$

This condition and the no-Ponzi game condition, $\lim_{t\to\infty} v(s,t)e^{-rt} = 0$, are sufficient for optimality.

2.3 Aggregation and Dynamic Equations

Let us define the aggregate consumption as follows:

$$C_t = c(0^-, t)N(0) + \int_0^t c(u,t)N(0)e^{nu} du. \tag{9}$$

Applying similar definitions to other variables gives the following aggregate dynamics of consumption and financial asset holdings:

$$\dot{C} = (r - \rho + n)C - n\rho V, \tag{10}$$

$$\dot{V} = rV + W - T - C. \tag{11}$$

Moreover, aggregating (6) in the same way, we have

$$Y_t = AK_t. \tag{12}$$

[11] It is quite easy to extend this log-linear utility function to the CES class since the real interest rate is constant over time in this model.

Equation (12) shows that national income is proportional to the aggregate capital stock, as in the standard AK model.

Next the government sector is considered. Denote government spending by G. Then, the government budget deficit is $G-T$ and this must be financed by issuing government bonds B, that is,

$$\frac{dB}{dt} = rB + G - T. \tag{13}$$

The goods and asset market equilibrium conditions are

$$Y = C + \dot{K} + G, \tag{14}$$

$$V = B + K. \tag{15}$$

Following convention in the endogenous growth literature, we normalize the variables by capital, that is, we define $c = C/K$, $w = W/K$, $b = B/K$, $g = G/K$ and $\tau = T/K$.[12] From (5), (10) , (11), (14) and (15) we can derive the following equations:

$$\dot{c} = (r - \rho + n - \gamma)c - n\rho v, \tag{16}$$

$$\dot{v} = (r - \gamma)v + w - \tau - c, \tag{17}$$

$$A = c + g + \gamma, \tag{18}$$

$$v = b + 1, \tag{19}$$

$$w = (1 - \alpha)A, \tag{20}$$

where γ denotes the growth rate of the economy defined as:

$$\gamma = \frac{\dot{Y}}{Y} = \frac{\dot{K}}{K}.$$

Substituting (18), (19) and (20) into (16) and (17) yields:

$$\dot{c} = [c + g - \rho + n - (1 - \alpha)A]c - n\rho(b + 1), \tag{21}$$

$$\dot{b} = [c + g - (1 - \alpha)A]b + g - \tau, \tag{22}$$

which describe the equilibrium dynamics of the economy.

[12] Here do not confuse the normalized aggregate variables with variables for each generation, for example, $c(s, t)$.

3 Steady-Growth Equilibria and Equilibrium Dynamics

In this section, we examine the existence and stability of steady-growth equilibria in our model. For simplicity, we assume that the government keeps the ratios of government spending and tax to the capital stock (or GDP) constant over time, that is, g and τ are kept constant.

3.1 Steady-Growth Equilibria

Let us first derive steady growth equilibria. Setting $\dot{c} = 0$ and $\dot{b} = 0$ in (21) and (22), we have the following two relations:

$$b = \frac{1}{n\rho}\left[c^2 + \{g - \rho + n - (1-\alpha)A\}\, c - n\rho\right], \tag{23}$$

$$b\,[(1-\alpha)A - g - c] = g - \tau. \tag{24}$$

Therefore, if a steady-growth equilibrium exists, from (23) and (24) it must satisfy the following cubic equation:

$$-\frac{1}{n\rho}\left[c^2 + \{g - \rho + n - (1-\alpha)A\}\, c - n\rho\right][c + g - (1-\alpha)A] = g - \tau. \tag{25}$$

Denote the left-hand side of this equation by $f(c)$ since it contains only one endogenous variable, c. Then, $f(c)$ intersects the vertical axis at $g-(1-\alpha)A$, as is easily checked. This means that, when the graphs of both sides of (25) are depicted in the same figure, the right-hand side of (25), $g - \tau$, is always located above the intersection of $f(c)$ and the vertical axis because $g - \tau$ is always larger than $g - (1-\alpha)A$.[13] Moreover, the sign of the slope of $f(c)$ at the vertical axis, $f'(0)$, changes depending on the value of g, as Appendix 1 shows.

[13]In order for consumption to be positive net wage must be positive, that is, $\tau < (1-\alpha)A = w$.

Let us consider the case where $g < (1-\alpha)A$. Obviously, $f(c) = 0$ has one negative and two positive roots.[14] Figures 1a and 1b depict this case. From the figures, it is easily seen that (25) has one negative and at most two positive roots and that this result does not depend on the sign of $f'(0)$. Negative consumption is trivially meaningless and thus it is ruled out. Figures 1a and 1b show that, if $g - \tau$ belongs to a certain range, that is, if $g - \tau < H$ (see Figures 1a and 1b for the definition of H), there exist two steady-growth equilibria, while, if $g - \tau > H$, there is no steady-growth equilibrium.[15]

Let us next consider the case where $(1-\alpha)A < g$. In this case $f(c) = 0$ has one positive root and two negative roots. Figure 2a depicts the case where $(1-\alpha)A < g < (1-\alpha)A + \rho$. In this case $f'(0) > 0$ as is proved in Appendix 1. Thus, there exist at most two steady-growth equilibria when $g - \tau$ is moderate, while there exists no steady-growth equilibrium when $g - \tau$ is larger than the critical value, H, as is seen from Figure 2a. Figure 2b depicts the case where $g > (1-\alpha)A + \rho$, that is, $f'(0) < 0$. In this case there exists no steady-growth equilibrium, as Figure 2b shows.

3.2 The Sustainability of Government Budget Deficits and the Stability of Steady-Growth Equilibria

Next, the sustainability of government budget deficits is analyzed. First, we examine how the ratio of government bonds to GDP is determined in a steady-growth equilibrium, and, then, we investigate the stability of steady-growth equilibria.[16]

Let us here depict phase diagrams for all cases. The phase diagrams are

[14] One solution is $(1-\alpha)A - g$. The other two solutions are derived from $c^2 + \{g - \rho + n - (1-\alpha)A\}c - n\rho = 0$. Because the last term is negative, there exist two solutions: one is negative and the other is positive.

[15] If $g - \tau = H$, then there exists just one steady-growth equilibrium.

[16] Saint-Paul (1992) examines the effects of fiscal policy in a similar model to ours, but his analysis is restricted to a steady-growth equilibrium.

quite useful in examining not only the stability of steady-growth equilibria but also the determinants of steady state level of government debts. First, suppose that $g < (1-\alpha)A$ and $g > \tau$. Then, there are two possible cases. In case 1, when $c = (1-\alpha)A - g$, the $\dot{c} = 0$ locus passes a point located above the horizontal axis, and in case 2, the locus passes a point below the horizontal axis when $c = (1-\alpha)A - g$. That is, if

$$(n-\rho)(1-\alpha)A - (n-\rho)g - n\rho > 0 \tag{26}$$

holds, then we have case 1. In this case, there can exist two steady growth equilibria with positive steady state values of government debts, b^*. In fact, for small values of $g - \tau$ (> 0 by assumption) there exist two steady-growth equilibria where $b^* > 0$ (Figure 3a), while there exists no such steady-growth equilibrium for larger values of $g - \tau$ than a critical value (Figure 3b). When (26) does not hold, there exists no steady-growth equilibrium with a positive government debt for any value of $g - \tau > 0$ although there can exist steady-growth equilibria where $b^* < 0$. Figures 3c and 3d correspond to case 2. Figure 3c depicts the case where there is no steady-growth equilibrium and Figure 3d depicts a steady-growth equilibrium with $b^* < 0$.

Next suppose that $g < (1-\alpha)A$ and $g < \tau$. Then it is easily checked that

$$b(0) \mid_{\dot{b}=0} = -\frac{g-\tau}{g-(1-\alpha)A} > -1,$$
$$b(0) \mid_{\dot{c}=0} = -1.$$

Thus, we can depict the phase diagram in this case (case 3) as Figure 4, which shows that there exist two steady-growth equilibria. In one equilibrium $b^* > 0$ while in the other equilibrium $b^* < 0$. In the case where $g = \tau$ (balanced budget), the phase diagram is depicted as Figure 5. We call this situation case 4.

Finally we consider the case where $(1-\alpha)A < g$ holds (case 5). This situation automatically means $\tau < g$ because τ must be smaller than $(1-\alpha)A$.

We can depict the phase diagram as Figures 6a and 6b. As the figures represent, if g or $g - \tau$ is large enough, then there exists no steady-growth equilibrium.

3.2.1 The Stability

Cases 1 and 2 When there exist two steady-growth equilibria (Figure 3a), steady-growth equilibrium E_1 is saddle-point stable and steady-growth equilibrium E_2 is unstable (see Appendix 2). The ratio of government bonds to GDP is a predetermined variable while consumption per GDP is not predetermined, and hence, if the initial ratio of government bonds to GDP is not too large, then the economy either converges to steady-growth equilibrium E_1 along saddle path SS_I or stays in steady-growth equilibrium E_2. Starting from point A in Figure 3a, the ratios of consumption and government bonds to GDP are decreasing over time along SS_I, and thus the growth rate is increasing over time.[17] When there exists no steady-growth equilibrium (Figures 3b and 3c), the economy evolves along either P_1 , P_2 or P_3, depending on the initial level of government bonds. However, all of these paths are infeasible: along path P_1 the consumption-capital ratio becomes zero in a finite time, and along P_2 and P_3 the consumption capital ratio becomes infinite, which is obviously infeasible. In the case of Figure 3d, there can exist two steady-growth equilibria. However, $b^* < 0$ in these equilibria. Because our concern lies in analyzing the sustainability of government budget deficits, we ignore this case in the following analysis .

Cases 3 and 4 Figures 4 depicts case 3 and Figure 5 depicts case 4. As is easily seen from these figures, the stability property in each case is essentially the same, and thus we need not distinguish them. Here, let us see Figure 4. It is apparent that E_3 is saddle-point stable and E_4 is unstable as is similar

[17] We can have a path on which the economy exhibits non-monotonic movements around E_2. However, the economy eventually follows SS_I in Figure 3a.

to case 1. For a relatively small initial amount of government bonds, the economy either converges to E_3 along saddle path SS_{II} or stays in E_4.

Case 5 As Figures 6a and 6b show, the steady state levels of government bonds always become negative if they exist, that is, in case 5 the government lends to private sectors. Because our concern lies in analyzing the sustainability of government budget deficits, we ignore case 5 in the following analysis.

3.2.2 The Sustainability of Government Budget Deficits

If the long-run level of government budget deficits is finite and positive and if the steady-growth equilibrium is attainable, we call the government budget deficits *sustainable*. As is easily understood from our discussion above, we can have sustainable budget deficits only in the cases of Figures 3a and 5, where τ 5 $g < (1-\alpha)A$ is satisfied. In this situation, a necessary condition for sustainability is given by (26), as we already showed. This condition directly means that

$$n > \rho. \tag{27}$$

Noting (27), we can rewrite (26) as

$$A - g > r + \frac{n\rho}{n-\rho} > r. \tag{28}$$

This inequality implies that the real interest rate and the ratio of government spending to GDP must be not too large and that the subjective discount rate must be sufficiently small (remember that $n > \rho$). Moreover, (28) shows that a higher population growth rate makes it easier for the government to run permanent budget deficits.[18]

[18]This corresponds to Weil's (1989) intergenerational effect.

4 Characterization of Steady-Growth Equilibria

As is apparent from our argument presented above, only the cases of Figures 3a, 4 and 5 have steady-growth equilibria with positive b^*. Moreover, the properties of equilibria are essentially the same among Figures 3a and 5, and hence we classify these two cases into one category. In this category, we denote one equilibrium with lower b^* and c^* by E_1 and the other equilibrium with higher b^* and c^* by E_2. This section derives the long-run growth rate in each steady-growth equilibrium and analyzes the effects of budget deficits on the long-run growth rate.

4.1 Growth Rates

Comparing steady-growth equilibrium E_1 (E_3) with E_2 (E_4) reveals that the ratio of consumption to GDP is smaller in E_1 (E_3) than in E_2 (E_4) (see Figures 1a and 1b). Therefore, from (18), the long-run growth rate in E_1, γ^{1*}, is higher than that in E_2, γ^{2*}. Similarly, we can easily see that $\gamma^{3*} > \gamma^{4*}$. Moreover, the long-run level of government bonds is smaller in E_1 (E_3) than in E_2 (E_4), and hence the investment-GDP ratio is larger in the former steady-growth equilibrium.

4.2 Comparative Statics

Let us next see the effects of a change in g on c_i^* and b_i^*. Totally differentiating (23) and (24), we have

$$\frac{dc_i^*}{dg} = -\frac{c_i^*[c_i^* + g - (1-\alpha)A] + n\rho(b_i^* + 1)]}{\Delta_i} = -\frac{(n-\rho)c_i^*}{\Delta_i},\ i = 1, 2, 3, 4, \tag{29}$$

$$\frac{db_i^*}{dg} = -\frac{b_i^* + 1}{\Delta_i}\left((1-\alpha)A - g - (n-\rho) - \frac{b_i^* + 2}{b_i^* + 1}c_i^*\right),\ i = 1, 2, 3, 4, \tag{30}$$

where $\Delta_i \equiv [2c_i^* + \{g - \rho + n - (1-\alpha)A\}][c_i^* + g - (1-\alpha)A\}] + n\rho b_i^*$. As is shown in Appendix 2, the sign of Δ is negative in E_1 and E_3 while it is positive in E_2 and E_4. From (29), (30) and the result on the sign of Δ we can derive the long-run effects of budget deficits on the ratios of consumption and government bonds to GDP. In the case of Figure 3a, since n is greater than ρ, the sign of dc_i^*/dg is opposite to that of Δ_i. Therefore an increase in g raises (reduces) consumption per GDP in steady-growth equilibrium E_1 (E_2).[19]

When $(1-\alpha)A > g$, the sign of the bracket in equation (30) is negative since c_i^* $(i = 1, 2)$ is larger than $(1-\alpha)A - g - (n - \rho)$. Hence, the sign of db_i^*/dg is opposite to that of Δ_i: an increase in budget deficits raises the government bonds - GDP ratio in E_1 and E_3 while it reduces the ratio in E_2 and E_4.

Let us next examine the effects of budget deficits on the long-run growth rate. From (18) it is easily seen that

$$\frac{d\gamma_i^*}{dg} = -\frac{d(c_i^* + g)}{dg}. \tag{31}$$

Utilizing (29), we can calculate (31) as

$$\begin{aligned}\frac{d\gamma_i^*}{dg} &= -\frac{[c_i^* + g - (1-\alpha)A][c_i^* + \{g - \rho + n - (1-\alpha)A\}] - n\rho}{\Delta_i} \\ &= \frac{(1-\alpha)A - \tau}{\Delta_i c_i^*} n\rho. \end{aligned} \tag{32}$$

Thus, an increase in government budget deficits is harmful to long-run growth in E_1 and E_3 while it is beneficial to long-run growth in E_2 and E_4.

These findings seem to contradict to Sait-Paul's (1992) result, which indicates a negative correlation between long-run growth and government budget deficits. However, even in our analysis, if we pick up one steady-growth equilibrium and see the relationship between public debts and long-run growth

[19] In the case of Figure 3b, n is smaller than ρ, and hence the sign of dc_i^*/dg is the same as that of Δ_i.

in that equilibrium, the direction of changes in the government bonds - GDP ratio is opposite to that of the long-run growth rate, that is, the higher the ratio of public debts to GDP is, the lower the long-run growth rate is. This is consistent with Saint-Paul's result.

In steady-growth equilibria E_1 and E_3, the usual crowding out phenomena are observed: an increase in government consumption reduces private investment. In E_2 and E_4, however, an increase in government consumption stimulates private investment, that is, we observe crowding in effects of government spending in E_2 and E_4.

5 Long-Run Welfare Analysis

Assuming that the economy is always in a steady-growth equilibrium, this section investigates welfare effects of government budget deficits.

5.1 Formulae for Welfare Evaluation

We first derive formulae which represent the total utility of generation $s \in [0^-, \infty)$.[20]

From (7), (8) and the No-Ponzi game condition, the consumption function at time t of the generation born at s is derived as

$$c(s,t) = \rho[v(s,t) + h(s,t)]. \tag{33}$$

In (33) $h(s,t)$ is the human wealth of this generation defined as

$$h(s,t) = \int_t^{\infty} [w_u^h - \tau_u^h] e^{-r(u-t)} du. \tag{34}$$

Combining (8) and (33), we have

$$c(s,u) = \rho[v(s,t) + h(s,t)] e^{(r-\rho)(u-t)},\ u \geq t. \tag{35}$$

[20]Similar formulae are derived by Futagami and Shibata (1999) and Saint-Paul (1992).

Substituting (35) into the utility function yields the lifetime utility of generation s, $U(s,s)$:

$$U(s,s) = \frac{r-\rho}{\rho^2} + \frac{ln\rho}{\rho} + \frac{1}{\rho}\ln[h(s,s)] \text{ for } s \in [0^+, \infty), \tag{36}$$

since $v(s,s) = 0$ for $s \in [0^+, \infty)$.

In a similar way, the lifetime utility of generation 0^- is given by

$$U(0^-,0^-) = \frac{r-\rho}{\rho^2} + \frac{ln\rho}{\rho} + \frac{1}{\rho}\ln[v(0^-,0^-) + h(0^-,0^-)], \tag{37}$$

where $v(0^-,0^-) > 0$ and

$$v(0^-,0^-) = \frac{K_0 + B_0}{N_0} = (1 + b_i^*)\frac{K_0}{N_0} \tag{38}$$

The value of human wealth of generation 0^- in steady-growth equilibrium E_i is easily obtained as.

$$\begin{aligned} h^0(s,s) &= \int_s^\infty \left(w_u - \tau_u^h\right) e^{-r(u-s)} du = \int_s^\infty \left((1-\alpha)\frac{AK_u}{N_u} - \frac{T_u}{N_u}\right) e^{-r(u-s)} du \\ &= [(1-\alpha)A - \tau] \frac{e^{(\gamma^0 - n)s}}{(n + r - \gamma_i^*)} \frac{K_0}{N_0}[21]. \end{aligned} \tag{39}$$

Substituting (38) and (39) into (36) and (37), we obtain the following expressions:

$$U_i(s,s) = \frac{r-\rho}{\rho^2} + \frac{ln\rho}{\rho} + \frac{1}{\rho}\left\{\ln\left[(1-\alpha)A - \tau\right]\frac{K_0}{N_0} + (\gamma_i^* - n)s - \ln(n + r - \gamma_i^*)\right\}, \tag{40}$$

$$U_i(0,0) = \frac{r-\rho}{\rho^2} + \frac{ln\rho}{\rho} + \frac{1}{\rho}\left\{\ln\left[(1+b_i^*) + \frac{(1-\alpha)A - \tau}{n + r - \gamma_i^*}\right]\frac{K_0}{N_0}\right\}. \tag{41}$$

Thus, lifetime utility of an agent depends on its *birth date* (s) and the *economic growth rate* (γ_i^*). The first term in the brace of (40) represents the *income effect*, which comes from income after tax payments at time 0. The second term in the brace is the *initial human wealth effect* because this depends on the generation's birth date. The third term in the brace is the *growth rate effect*, which reflects the fact that a rise in the economic growth rate raises the growth rate of wage income. Similarly, the first term in the brace of (41) is the *initial non-human wealth effect* because only generation 0^- has initial non-human wealth. The second term in the brace of (41) represents a mixture of the *income effect* and *growth rate effect.*

5.2 Welfare Effects of Budget Deficits

In this subsection we assume that the government budget constraint is balanced initially (the case depicted in figure 5) and derive the welfare effects of a marginal increase in government spending.

The long-run growth rate in E_1 is higher than that in E_2, and thus the levels of lifetime utility of generations born at time s (≥ 0) in E_1 are higher than those in E_2. Moreover, a change in government consumption which raises the long-run growth rate is beneficial to generations born after time 0.

Let us here examine the welfare effect of government consumption on generation 0^-. Differentiating $b_i^* + \frac{(1-\alpha)A-\tau}{n+r-\gamma_i^*}$ in the brace of (41) with respect to g, we have

$$\frac{1}{\Delta_i}\left\{\frac{[(1-\alpha)A-\tau-c_i^*]\left[(1-\alpha)A-g-n+\rho-\frac{(b_i^*+2)c_i^*}{b_i^*+1}\right]}{(1-\alpha)A-g-c_i^*}\right.$$
$$\left.+\frac{[(1-\alpha)A-\tau]^2 n\rho}{[(1-\alpha)A-c_i^*-g-n]^2 c_i^*}\right\}. \quad (42)$$

As is shown in Appendix 3, the sign of the brace in (42) is negative when

$$\frac{n\rho}{n-\rho} < (1-\alpha)A - g < \rho + \frac{n}{2} \quad (43)$$

is satisfied. It is easy to confirm that when n is relatively large then (43) is likely to hold. Here we restrict our attention to the case where population growth rate is high enough so that (43) holds. Then, the sign of this derivative, (42), is opposite to that of Δ_i. Because $\Delta_1 < 0$ and $\Delta_2 > 0$, the lifetime utility of this generation in E_1 is raised by a marginal increase in g and that in E_2 it is reduced by a marginal increment in g.

We can summarize the welfare effects of marginal changes in budget deficits. In steady-growth equilibrium E_1 an expansionary fiscal policy is harmful to all generations except for generation 0^- and improves only generation 0^-'s welfare. In E_2, a marginal increase in government consumption is beneficial to all generations except for generation 0^-. Fiscal policy of this type cannot be Pareto improving.

6 Concluding Remarks

Incorporating a government sector which runs permanent budget deficits, this paper constructed a simple endogenous growth model and analyzed the relationship between government budget deficits and long-run growth. Our main results are as follows. If the size of budget deficits is modest, then at most two steady-growth equilibria exist, and that, when two steady-growth equilibria exist, the government can run permanent budget deficits by issuing bonds if the following conditions hold: (i) The ratio of budget deficits to GDP is not so large. (ii) The initial level of government bonds does not exceed a critical value. (iii) The population growth rate is relatively high and the subjective discount rate is relatively low. One of the two steady-growth equilibria is associated with a higher growth rate and the other is associated with a lower growth rate. The high growth equilibrium corresponds to a low ratio of public debts to GDP and the low growth equilibrium to a high ratio of public debts to GDP. It was also shown that an increase in government consumption reduces the long-run growth rate in the high growth equilibrium while it raises the long-run growth rate in the low growth equilibrium. In the high growth equilibrium, the lifetime utility of all generations except for the generation which already exists at time 0 is improved by decreasing government budget deficits through a reduction in government consumption. On the other hand, the contrary applies to the low growth equilibrium.

Appendices

Appendix 1

In this appendix the sign of $f'(0)$ is examined. Differentiating $f(c)$ with respect to c and evaluating it at $c = 0$ yields

$$f'(0) = -\frac{1}{n\rho}\left[-n\rho + \{g - \rho + n - (1-\alpha)A\}\{g - (1-\alpha)A\}\right].$$

This can be rewritten as:

$$f'(0) = -\frac{1}{n\rho}\left[\{g - (1-\alpha)A - \rho\}\{g - (1-\alpha)A + n\}\right].$$

Hence if the following inequalities:

$$(1-\alpha)A - n < g < (1-\alpha)A + \rho \tag{A1}$$

hold, then $f'(0) > 0$. Otherwise, $f'(0) < 0$.

Appendix 2

This appendix analyzes the stability of steady-growth equilibria. Linearizing (21) and (22) around a steady-growth equilibrium gives

$$\begin{bmatrix} \dot{c} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} 2c^* + \{g - \rho - (1-\alpha)A\} & -n\rho \\ b^* & c^* + g - (1-\alpha)A \end{bmatrix} \begin{bmatrix} c - c^* \\ b - b^* \end{bmatrix}$$

Denote the determinant of the coefficient matrix $J(c^*)$ by Δ. In order to examine the stability of this dynamic system it is necessary to check the sign of Δ. Here, denote the left-hand side of (25) by $f(c)$. Because $f'(c) = -\Delta/n\rho$, the sign of Δ equals that of $-f'(c)$. From the graphs of $f(c)$ depicted in Figures 1a, 1b, 2a and 2b, the sign of $f'(c)$ can be easily seen, and thus the sign of Δ can be checked.

In steady-growth equilibrium E_1 and E_3, $\Delta < 0$ since $f(c)$ is increasing around these equilibria as is depicted in the figures. Hence, they are saddle-point stable. On the other hand, $\Delta > 0$ in steady-growth equilibria E_2 and E_4 as the figures indicate. The trace is calculated as

$$\begin{aligned}\mathrm{trace}J(c^*) &= 3c^* - 2[(1-\alpha)A - g] + n - \rho \\ &= 2[c^* - \{(1-\alpha)A - g\}] + c^* + n - \rho.\end{aligned}$$

In this expression the sign of the trace is ambiguous because it depends on c^*. Thus let us examine the property of c^*. The $\dot{c} = 0$ locus is given by the following quadratic function:

$$b = \frac{1}{n\rho}\left[c^2 - \{(1-\alpha)A - g + n - \rho\}\, c - n\rho\right].$$

Define the right-hand side of this as $b(c)$. Then, it can be shown

$$b\left[(1-\alpha)A - g - (n-\rho)\right] = -n\rho < 0.$$

Since the analysis in this paper is restricted to the case where $b^* > 0$, this means that

$$(1-\alpha)A - g - (n-\rho) < c_2^*.$$

From this relation the following inequality is derived:

$$\begin{aligned}\mathrm{trace}J(c_2^*) &= 2[c_2^* - \{(1-\alpha)A - g\}] + c_2^* + n - \rho \\ &> 2[c_2^* - \{(1-\alpha)A - g\}] + [(1-\alpha)A - g - (n-\rho) + n - \rho] \\ &= 2c_2^* - [(1-\alpha)A - g]. \qquad \text{(A2)}\end{aligned}$$

Substituting $[(1-\alpha)A - g]/2$ into $b(c)$ gives

$$b\left(\frac{(1-\alpha)A - g}{2}\right) = \frac{1}{n\rho}\left[(n-\rho)\left(g - (1-\alpha)A\right) - n\rho\right] < 0. \qquad \text{(A3)}$$

Because it is assumed that $n > \rho$ and $(1-\alpha)A > g$, (A3) and figures 3-7 imply that $[(1-\alpha)A - g]/2$ is smaller than c_2^*. Hence, from (44) the sign of trace $J(c_2^*)$ is positive. Since both the determinant and trace are positive, steady-growth equilibrium E_2 is unstable. A similar argument applies to E_4, and hence E_4 is also a source.

Appendix 3

This appendix shows that the sign of (42) is negative under some condition. First let us consider the sign of (42) in E_1. In this case we have $b_1^* = 0$, and thus the brace of (42) becomes

$$[(1-\alpha)\,A - g - n + \rho - 2c_1^*] + \frac{[(1-\alpha)\,A - g]^2\,n\rho}{[(1-\alpha)A - g - c_1^* - n]^2\,c_1^*}. \tag{A4}$$

From (23) and $b_1^* = 0$ we have

$$c_1^{*2} + \{g - \rho + n - (1-\alpha)A\}\,c_1^* - n\rho = 0.$$

Substituting this into (A4) yields

$$\begin{aligned}
& -\frac{1}{c_1^*}\left\{c_1^{*2} + n\rho - \frac{[(1-\alpha)\,A - g]^2\,n\rho}{[(1-\alpha)A - g - c_1^* - n]^2}\right\} \\
= \;& -\frac{1}{c_1^*}\left\{c_1^{*2} + n\rho\frac{\left([(1-\alpha)A - g - c_1^* - n]^2 - [(1-\alpha)\,A - g]^2\right)}{[(1-\alpha)A - g - c_1^* - n]^2}\right\} \\
= \;& -\frac{1}{c_1^*}\left\{c_1^{*2} + n\rho\frac{(c_1^* + n)\,[c_1^* + n - 2\,\{(1-\alpha)\,A - g\}]}{[(1-\alpha)A - g - c_1^* - n]^2}\right\} \\
\equiv \;& -\frac{1}{c_1^*}\left\{c_1^{*2} + n\rho\frac{1}{[(1-\alpha)A - g - c_1^* - n]^2}\Omega\right\},
\end{aligned}$$

where c_1^{*2} is positive and the sign of Ω is determined as

$$\begin{aligned}
\Omega \;&\leqq\; 0 \text{ when } 0 \leqq c_1^* \leqq 2\,\{(1-\alpha)\,A - g\} - n \\
\Omega \;&>\; 0 \text{ when } 2\,\{(1-\alpha)\,A - g\} - n < c_1^*.
\end{aligned}$$

To examine the sign of Ω in more detail we define a function ϕ:

$$\phi(c_1) \equiv c_1^2 + \{g - \rho + n - (1-\alpha)A\}\, c_1 - n\rho.$$

Then c_1^* is given by $\phi(c_1^*) = 0$. Figure 7 depicts the function. From the figure it is easily seen that

$$\phi\left(2\left\{(1-\alpha)\,A - g\right\} - n\right) \lesseqgtr 0 \leftrightarrow c_1^* \gtreqless 2\left\{(1-\alpha)\,A - g\right\} - n.$$

Because

$$\phi\left(2\left\{(1-\alpha)\,A - g\right\} - n\right) = \left\{(1-\alpha)\,A - g\right\}\left[2\left\{(1-\alpha)\,A - g\right\} - n - 2\rho\right],$$

if

$$(1-\alpha)\,A - g < \rho + \frac{n}{2}, \tag{A5}$$

then $\phi\left(2\left\{(1-\alpha)\,A - g\right\} - n\right)$ is negative and $c_1^* > 2\left\{(1-\alpha)\,A - g\right\} - n$. This implies $\Omega > 0$. It should be noted here that, in the case of Figure 5, conditions (27) and (28) must be satisfied (see subsection 3.2.2). Combining (A5), (27) and (28), we have

$$\frac{n\rho}{n-\rho} < (1-\alpha)\,A - g < \rho + \frac{n}{2},$$

which is a necessary and sufficient condition under which Ω is positive. This condition is likely to hold when n is relatively large. In other words, for any value of n larger than 2ρ, we can choose some g which satisfies the condition.

Second, we examine the sign of (42) in E_2. Substituting $c_2^* = (1-\alpha)A - g$ and $b_2^* = [(n-\rho)/(n\rho)]\,[(1-\alpha)A - g] - 1$ into (42), we obtain

$$-\left[(1-\alpha)A - g\right]\left[\frac{(n-\rho)^2}{n\rho} + \frac{n-\rho}{n}\right] < 0.$$

Thus the sign of (42) is definitely negative.

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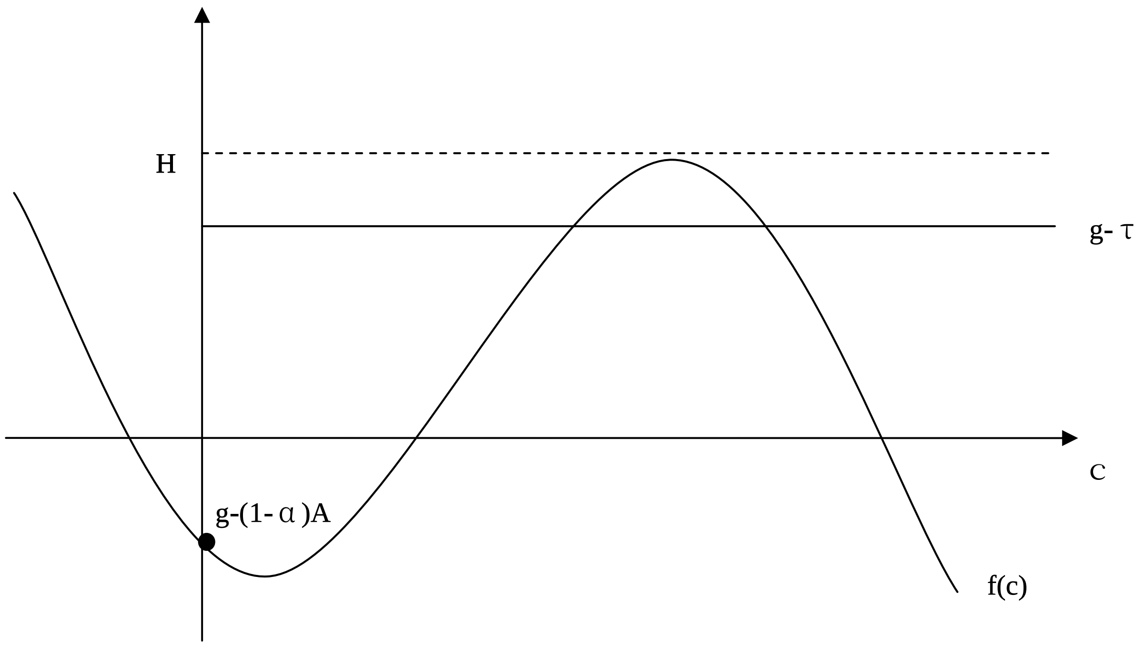

Figure 1-a

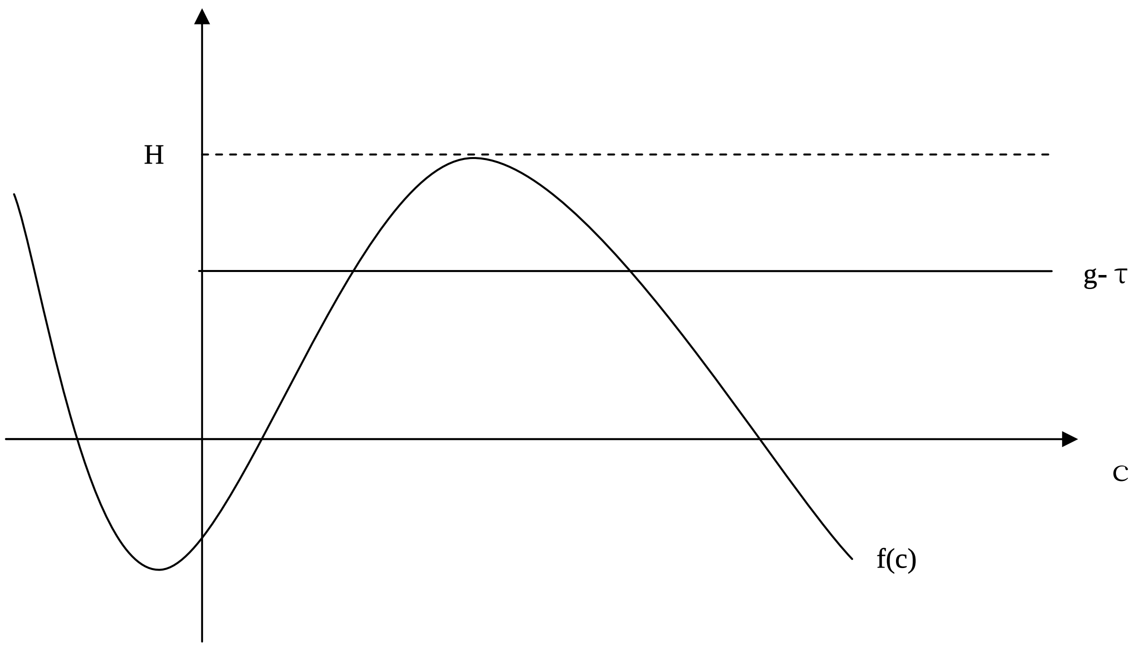

Figure 1b

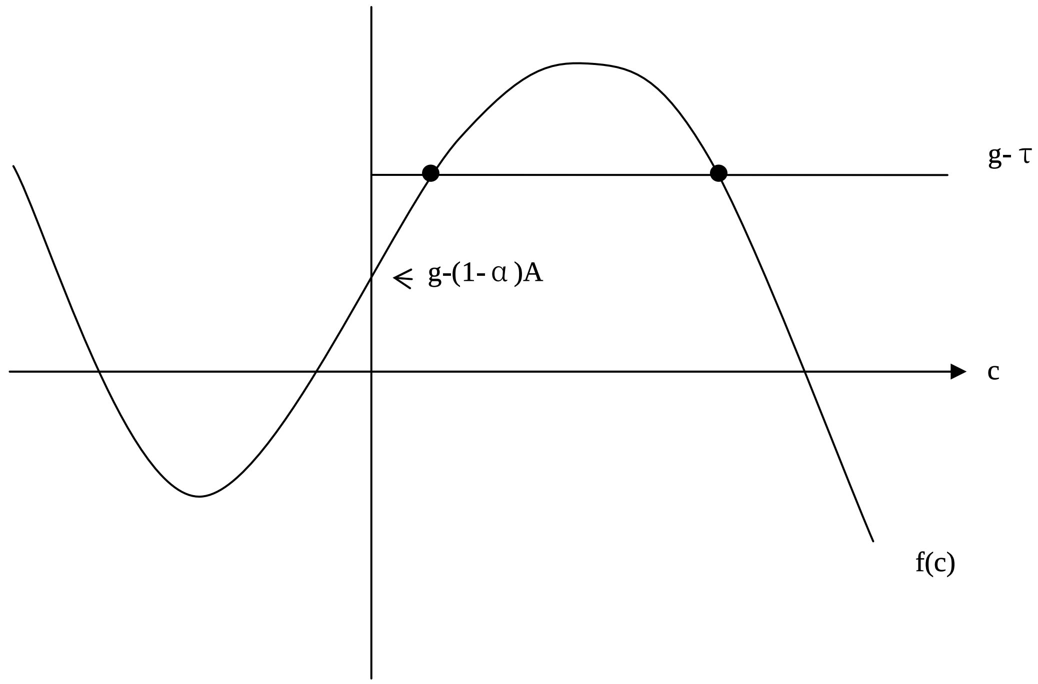

Figure 2a

(1- α)A<g(1- α)A+ ρ

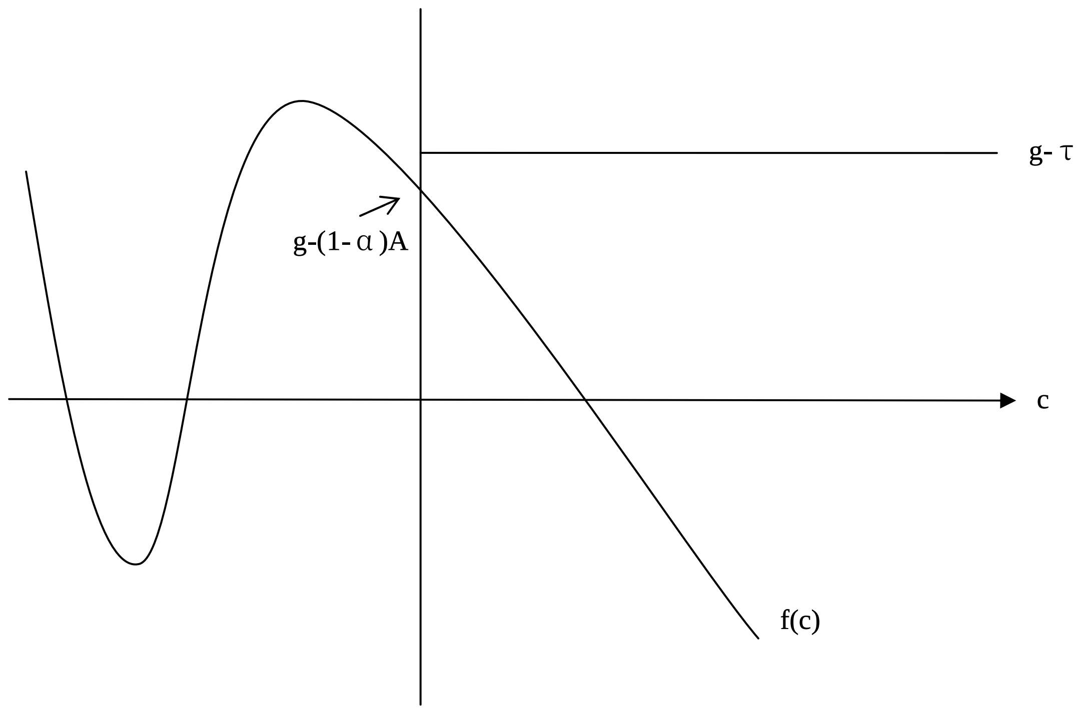

Figure2b

(1- α)A+ ρ <g

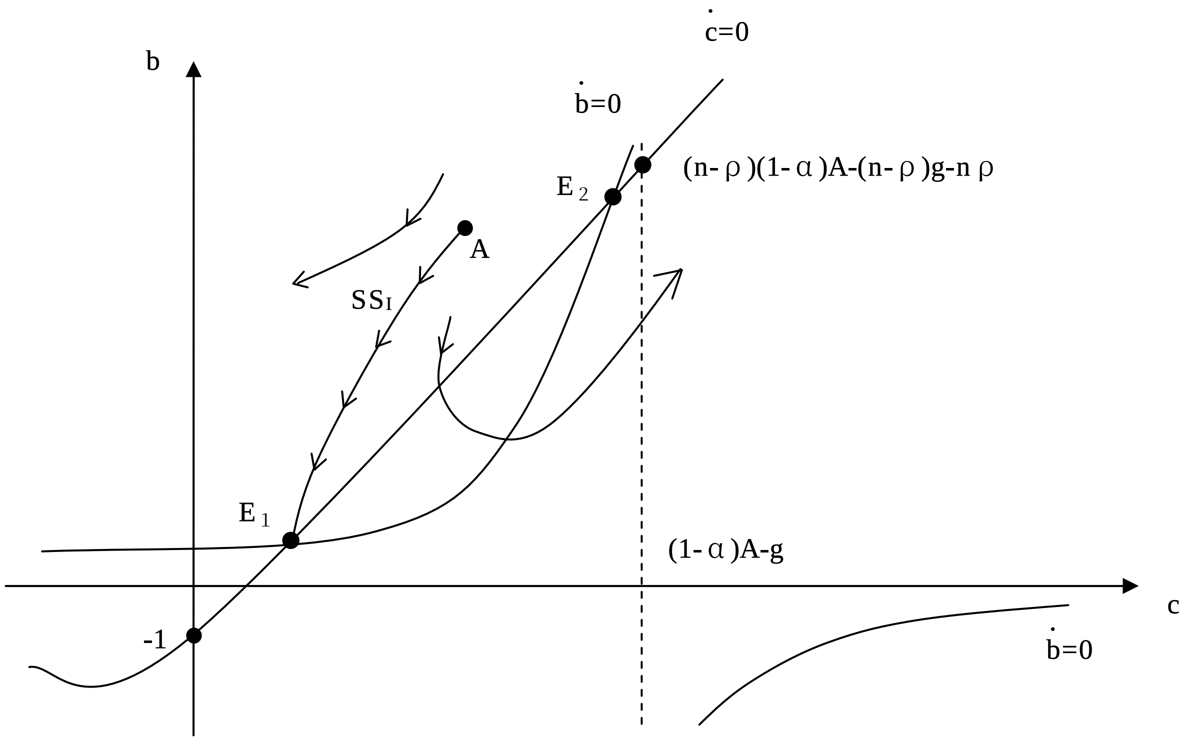

Figure 3a

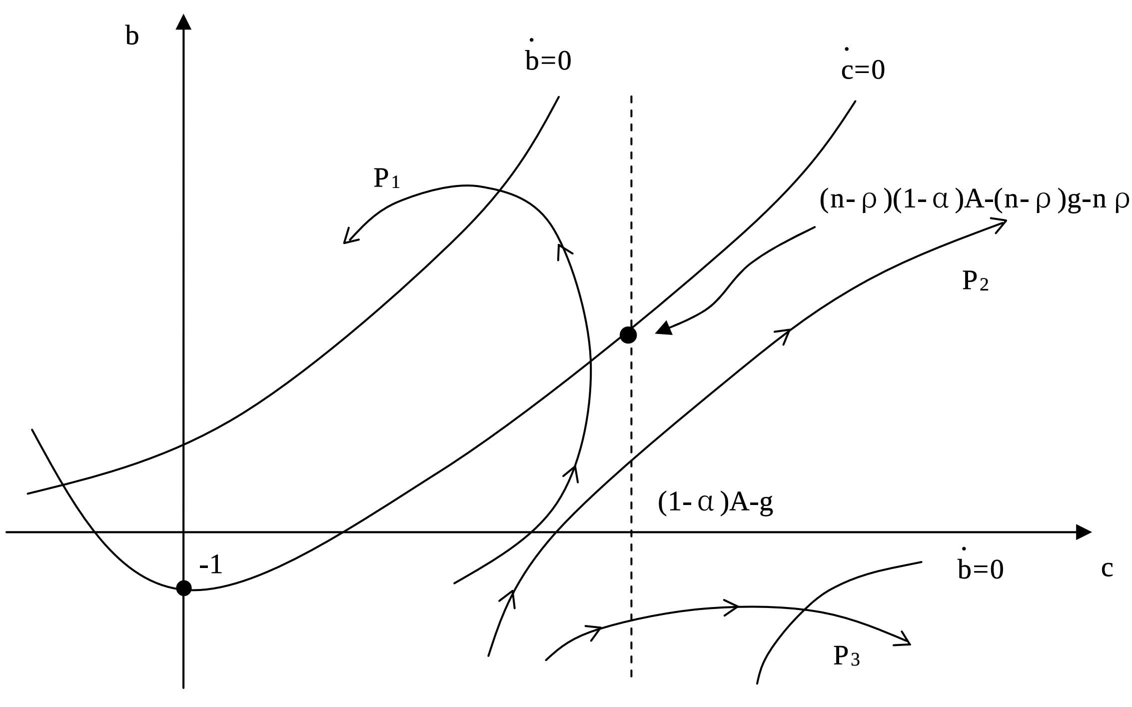

Figure 3b

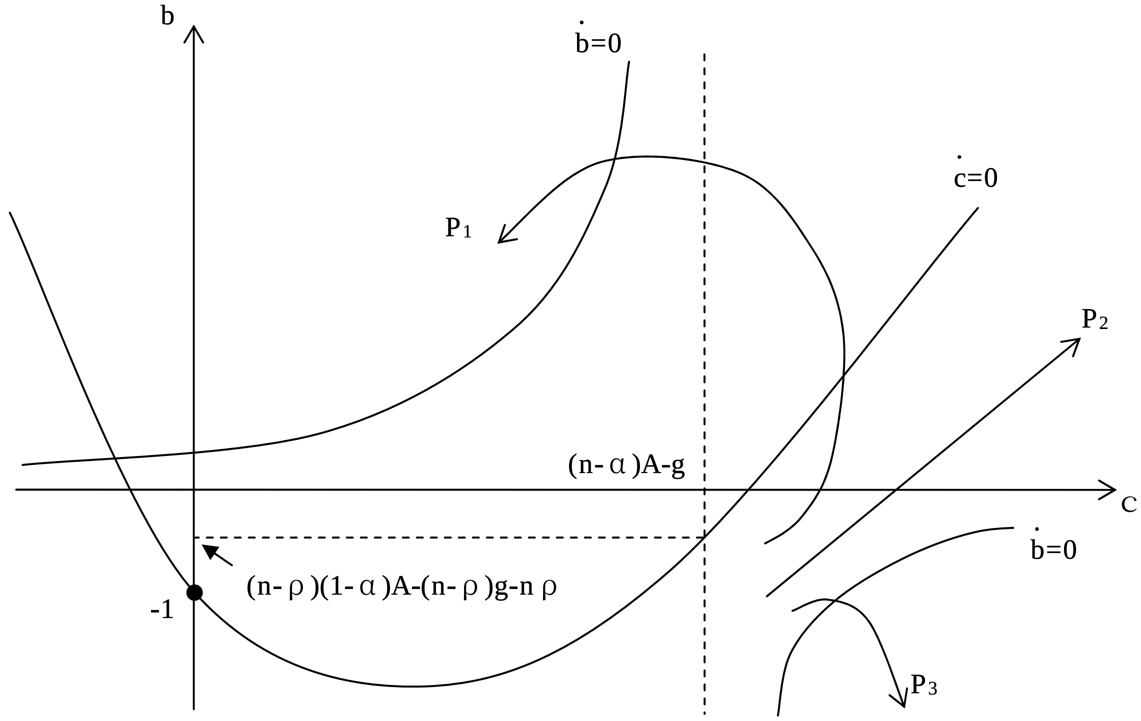

Figure 3c

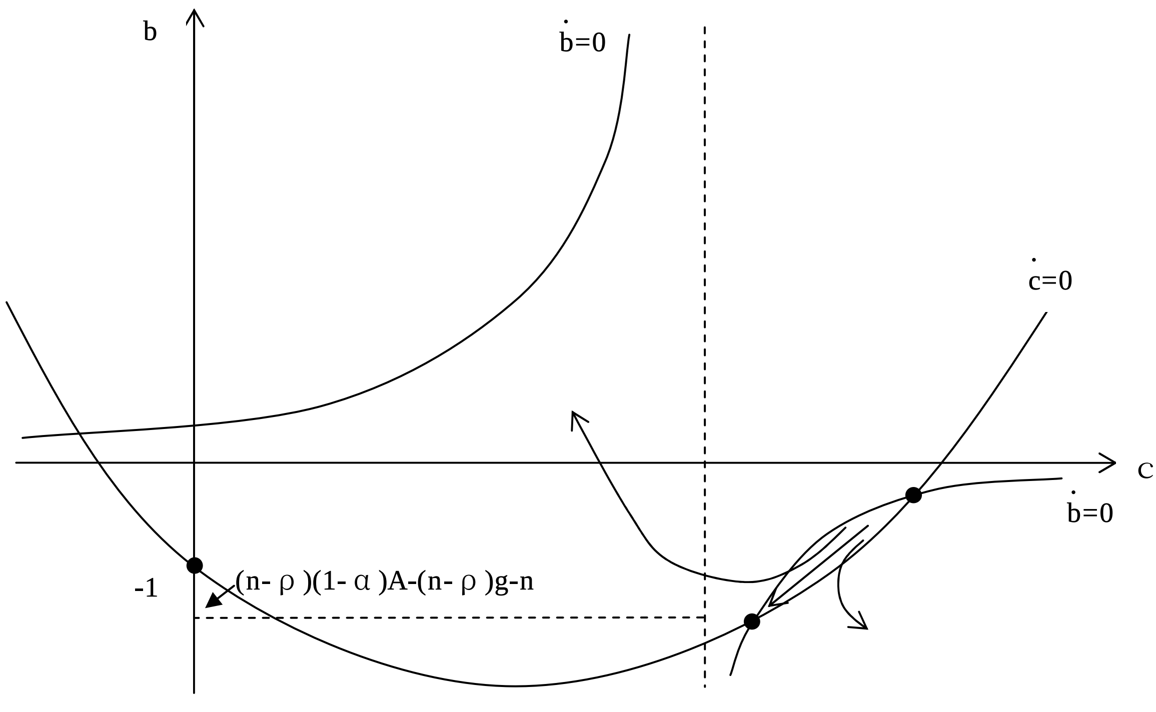

Figure 3d

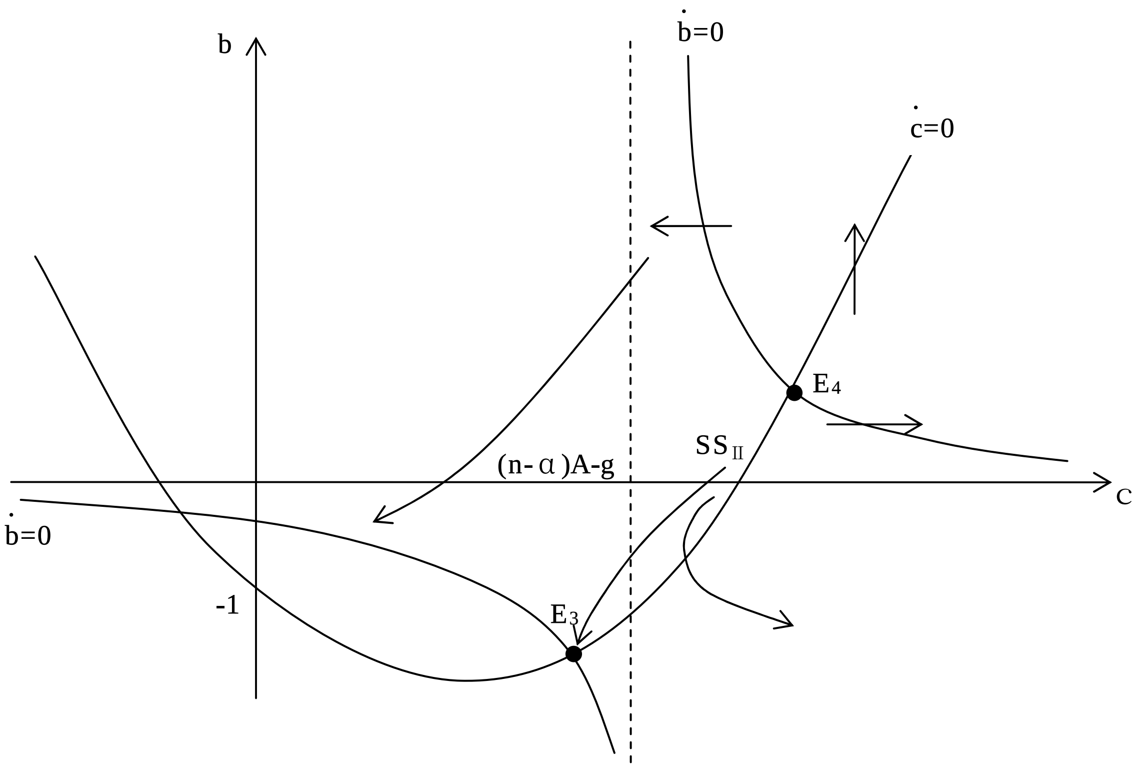

Figure 4

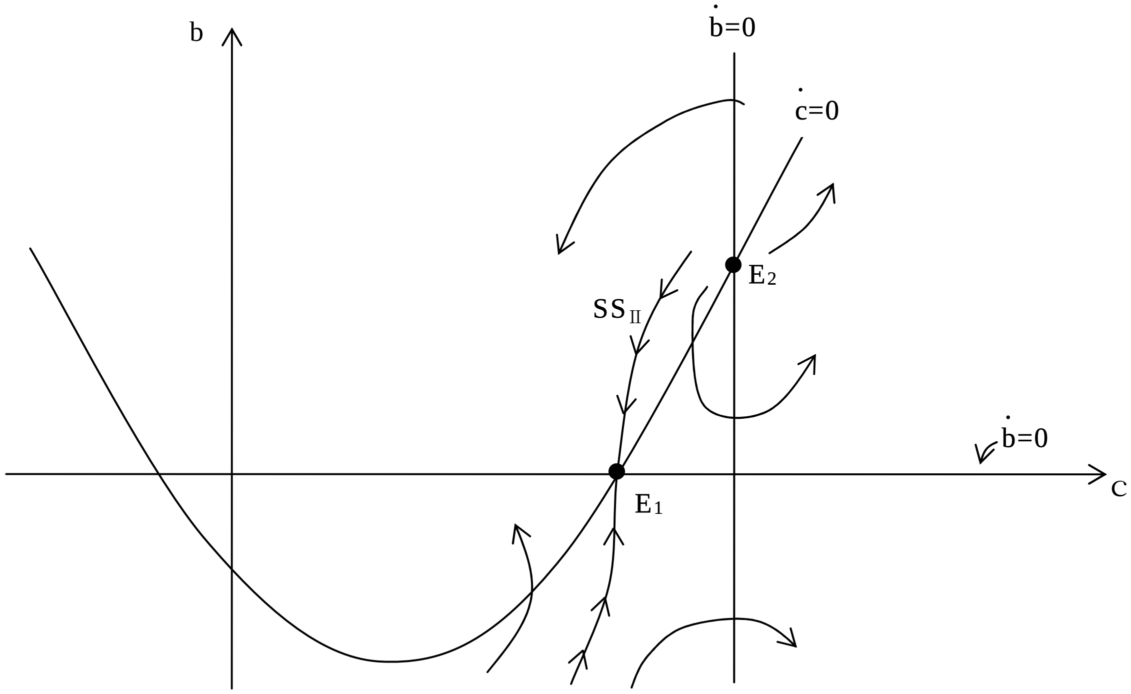

Figure 5

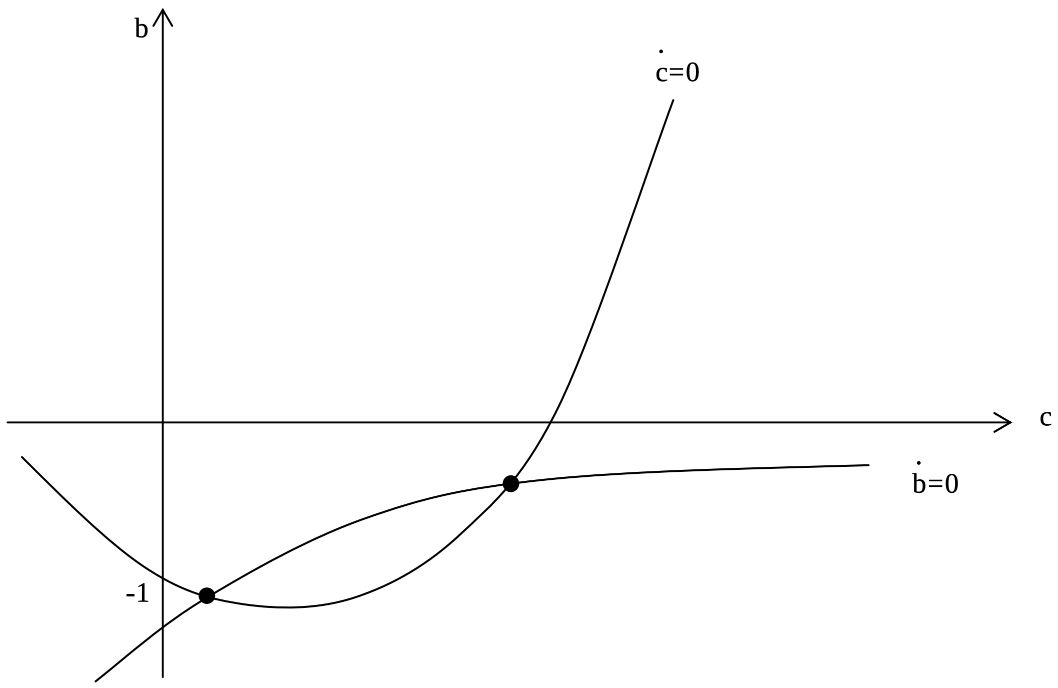

Figure 6a

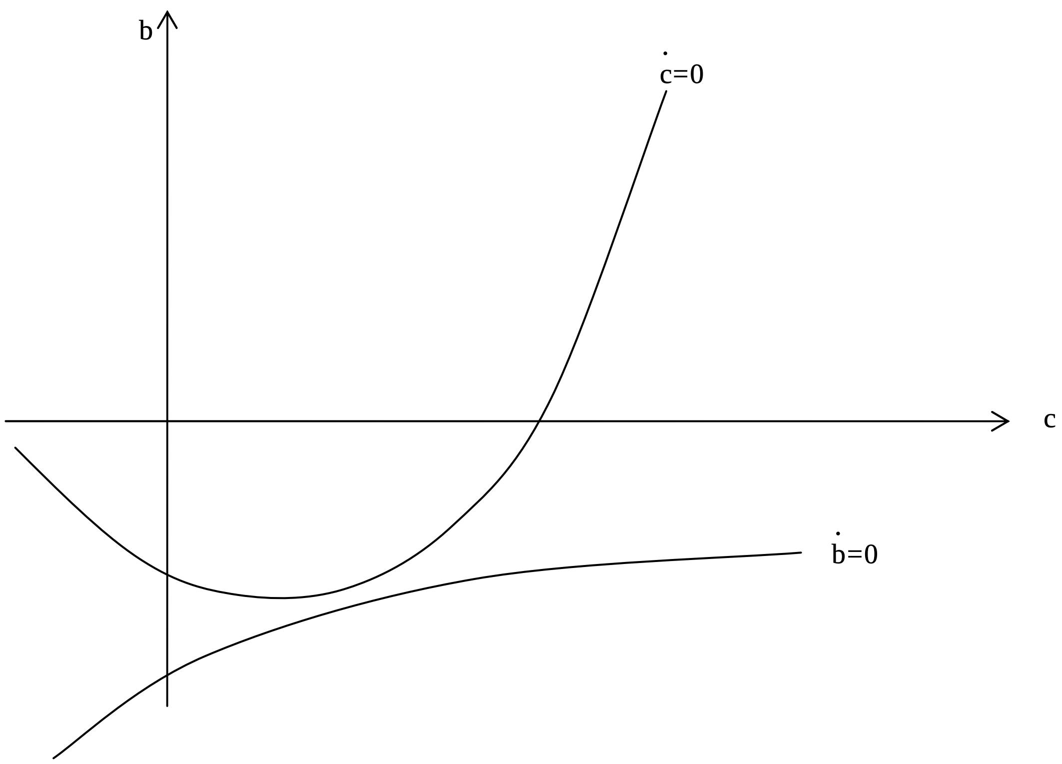

Figure 6b

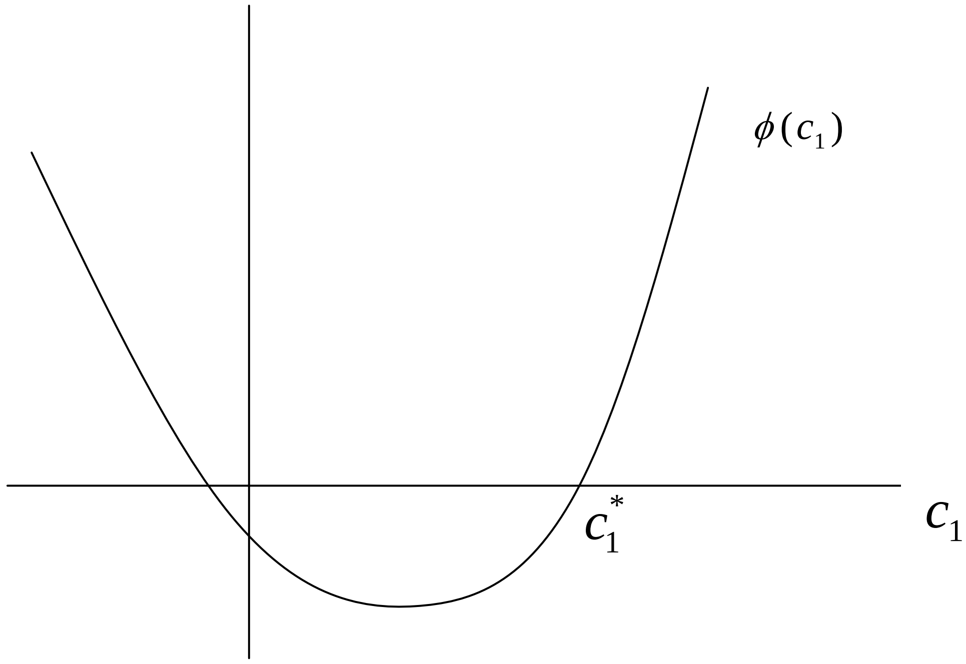

Figure 7