Evolution of Standards and Innovation *

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Abstract

We develop a framework to examine how a standard evolves when a standard consortium or firm (incumbent) innovates either to improve the standard or to strengthen the installed base, which increases switching costs. Both investments make it more difficult for another firm (entrant) to introduce a standard by investing in technology improvement. Our analysis shows that that incumbent's strategy depends on whether the technology is in its infancy or has matured, and that entrants cannot supplant the existing standard. A standard consortium brings dynamic benefits by preventing replacement by an entrant. When the technology is in its infancy, the incumbent deters entry, but when the technology is mature, entry and the coexistence of two standards are tolerated. The dominance of a single standard, even for well-established technologies, suggests that incumbents have market power. Our results also suggest that having superior technology is not enough to enable entrants to supplant an existing standard.

Key Words: standards, innovation, technology, , upgrades, standardization, replacement effect

1 Introduction

In this paper, we consider a situation in which a standard is established and examine the incentive of a firm (or other entity) with a stake in the current standard to invest in technology to improve the standard or the installed base. Having a stake in a standard includes owning patents for the standard or manufacturing products under the standard. In order to maintain the standard, a firm either can improve the technology and upgrade the standard or may invest in the installed base to take advantage of inertia (Farrell and Saloner, 1985). Upgrading a standard maintains its attractiveness to consumers, while investing in the installed base increases consumer costs of switching to the new standard.

We develop a two-stage game, in the first stage of which the incumbent invests in upgrading and the installed base, and the entrant invests to improve its potential standard technology. These investments determine the qualities of the respective products and the switching costs incurred by consumers who buy from the entrant. In the second stage, firms simultaneously choose prices; i.e., they engage in Bertrand competition.

We adopt the approach used by Laffont, Rey, and Tirole (1998) to modeling differentiated products with elastic demand in the presence of heterogeneous consumers. Thus, our model is particularly applicable to a market such as the smartphone market, in which there are competing platforms, with each vendor being identified with a platform. Because consumers pay a fixed cost and per-unit fee, there is a cost of switching to a different provider. Incumbents and entrants

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also represent patent pools or a standard consortium, and consumers can be interpreted as manufactures that pay licensing royalties. Another applicable market is that for game consoles, considering the indirect payments that consumers make to the console manufacturer through games. Part of the price paid for a game goes to the console manufacturer in licensing fees.[1] The market analysis of stage two is a special case of models of nonlinear price competition (Calem and Spulber, 1984; Oren, Smith, and Wilson, 1983) in the absence of switching costs. However, we provide a more complete characterization of price determination and welfare implications.

Bertrand competition in the second stage results in one of four outcomes according to the configuration of technology and the switching costs chosen in stage one: (I) only firm 0; (II) only firm 1; (III) coexistence (unique equilibrium); and (IV) coexistence (multiple equilibria). "Only firm 0" in regime (I) means that the incumbent deters entry through upgrading or creating inertia in the installed base. "Only firm 1" in regime (II) means that the entrant's quality is so good that it drives the incumbent out of the market and the existing standard is replaced.

We characterize the subgame perfect Nash equilibrium (SPNE) of the whole game. Only regimes (I) and (III) constitute a potential SPNE. Regime (I) occurs when technology improvement is not costly. In this case, the incumbent invests in technology improvement or the installed base to deter entry. The existing standard is upgraded. If technology improvement is costly, incumbent and entrant quality

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[1]Both console and software are produced by a single firm, or at least production is coordinated. We do not model a two-sided market.

are sufficiently similar for both firms to coexist in the market. Regime (II) never occurs in equilibrium. This is because by investing slightly more in stage one, firm 0 avoids being priced out of the market. In this case, the overall payoff is negative because profit is zero but investment is sunk. Given that regime (II) never occurs, in our framework, an existing standard is never replaced.

Given decreasing returns to technology investment, innovation costs are low when technology is in its infancy. In this case, incumbents can deter entry by upgrading or increasing switching costs. By contrast, if the technology is mature so that innovation costs are high, different standards coexist. Technology improvement increases the consumer surplus, whereas increased switching costs decrease it, even when both firms are in the market.

Farrell and Saloner (1985) examined a situation in which firms can either adopt a technologically superior standard or rely on inertia. They showed that firms choose not to improve the standard when there is incomplete information. In their framework, technological superiority of the standard is exogenous to firms, and the choice of standard is a coordination problem. Coordination is relevant because the standard is based on the network effect. In the context of a consumer's optimization decision, we use switching costs to represent the cost of moving from one network to another. We then endogenize technological improvement as an investment choice of firms, as well as the switching cost.

Cabral and Salant (2010) also consider firms that invest in improving the quality of a standard. They examine how moving from the coexistence of two standards to a unified standard affects the incentive to improve the standard. However,

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they ignore the market interactions induced by quality improvement and mergers, and assume that a single standard unambiguously increases the profits of both firms because of the network effect. In the context of our framework, one can interpret a move from coexistence (or incompatibility) to a single standard (or compatibility) as an infinite reduction in switching costs. In their framework, technology improvement is a predetermined single step, whereas in ours, the degree of technology improvement is chosen. Thus, according to Cabral and Salant (2010), firms choose to reduce switching costs either before or after investing in technology. The choice is not "which" but "which first". We focus on the "which" strategy by explicitly modeling consumer behavior.

In the next section, we briefly describe the product market and characterize the Bertrand equilibrium, given the technology and switching cost. We characterize the choice of equilibrium technology and switching cost in section 3, hence characterizing the SPNE. We examine the implications for the consumer and social surpluses in Section 3.1. We discuss policy implications in Section 4. All proofs are given in the Appendix.

2   Framework

We develop a two-stage game played by two firms, firms 0 and 1. Firm 0 "owns" the current standard in the sense that it has a stake in, and controls, this standard. Firm 1 can enter the market if its technology and standard are sufficiently good. In stage one, both firms sequentially invest in the technology that determines the

level of the standard. In stage two, firms engage in Bertrand price competition, given the technology investments made in stage one. Initially, firm 0 is the only firm in the market. Hence, firm 0 and firm 1 can be characterized as incumbent and entrant, respectively. We determine the SPNE strategies, technology investment choices, and prices.

To represent the product market, we use a Hotelling model in which consumers are distributed uniformly over the interval [0,1]. Firm 0 is at point 0, and firm 1 is at 1. Each consumer purchases at most one unit of the good from one of the firms. When a consumer at $x \in [0, 1]$ purchases from firm 0 at price p_0, his or her surplus is $v_0 - p_0 - tx$, where t is the per unit transportation cost. To purchase from firm 1, because the consumer must switch to a new standard, he or she incurs a switching cost, S. The consumer's surplus is $v_1 - p_1 - S - t(1 - x)$.

The intrinsic value of the products, v_i, are determined by the technology investments made in stage one. The established standard generates a technology level of $\overline{v}$; we assume that

$$v_i \geq \overline{v} \geq 2t. \tag{M}$$

Any positive investment in stage one by firm i implies that $v_i > \overline{v}$. The second inequality implies that a monopolist selling to all consumers charges a price of $v_i - t$. Because firm 0 is such a monopolist, all consumers who buy from firm 1 incur a switching cost of S.
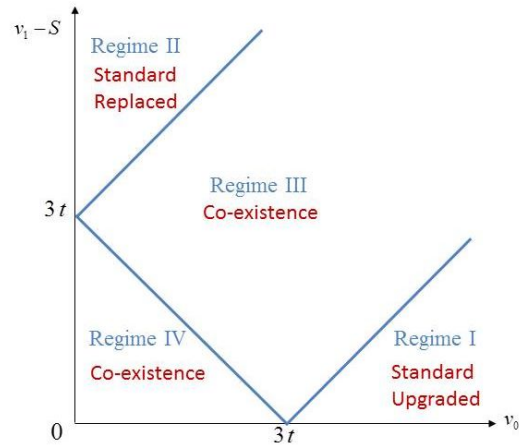
Figure 1: Stage Two (Bertrand Competition) Equilibrium

2.1 Bertrand Competition Equilibrium

The demand curve derived in the Appendix gives firm 0's profit as a function of (p_0, v_0) and (p_1, v_1). Standard analysis of the Hoteling model (outlined in the Appendix) yields the following Proposition characterizing Bertrand competition, which is illustrated in Figure 1.

Proposition 1. *In equilibrium, Bertrand price competition in stage two results in one of four regimes, which depend on the intrinsic values v_0 , v_1 and transportation cost t. Marginal consumers, characterized by $\hat{x}$, $\hat{x}_0$, and $\hat{x}_1$, are defined in the Appendix.*

Regime (I). Only firm 0 is in the market

If $v_1 - S \leq v_0 + 3t$, the equilibrium prices are

$$p_0^*(v_0, v_1, S) = v_0 - v_1 + S - t, \quad p_1^*(v_0, v_1, S) = 0.$$

In this case, all consumers buy from the incumbent. The equilibrium profits are

$$\pi_0^*(v_0, v_1, S) = v_0 - v_1 + S - t, \quad \pi_1^*(v_0, v_1, S) = 0.$$

Regime (II). Only firm 1 is in the market

If $v_1 - S \geq v_0 - 3t$, the equilibrium prices are

$$p_0^*(v_0, v_1, S) = 0, \quad p_1^*(v_0, v_1, S) = v_1 - v_0 - S - t.$$

In this case, all consumers buy from the entrant. The equilibrium profits are

$$\pi_0^*(v_0, v_1, S) = 0, \quad \pi_1^*(v_0, v_1, S) = v_1 - v_0 - S - t.$$

Regime (III). Two firms coexist in the market (unique equilibrium)

If $v_0 + v_1 - S \geq 3t$ and $v_0 - 3t < v_1 - S < v_0 + 3t$, the equilibrium prices are

$$p_0^*(v_0, v_1, S) = \frac{v_0 - v_1 + S + 3t}{3}, \tag{1}$$

$$p_1^*(v_0, v_1, S) = \frac{v_1 - v_0 - S + 3t}{3}. \tag{2}$$

Both firms make positive sales. The marginal consumer is characterized by $\hat{x}(p_0^*, p_1^*) =$

$\frac{1}{2} + \frac{v_0 - v_1 + S}{6t}$ *and has a positive surplus. The equilibrium profits are*

$$\pi_0^*(v_0, v_1, S) = \frac{1}{2t} \left(\frac{v_0 - v_1 + S + 3t}{3} \right)^2,$$

$$\pi_1^*(v_0, v_1, S) = \frac{1}{2t} \left(\frac{v_1 - v_0 - S + 3t}{3} \right)^2.$$

Regime (IV). Two firms coexist in the market (multiple equilibria)

If $v_0 + v_1 - S < 3t$, then there is a continuum of equilibria. The equilibrium prices, indexed by $\alpha \in [0, 1]$, are

$$p_0^*(v_0, v_1, S) = \frac{(3 - \alpha)v_0}{3} - (1 - \alpha) \left(t - \frac{v_1 - S}{3} \right) \tag{3}$$

$$p_1^*(v_0, v_1, S) = \frac{(2 + \alpha)(v_1 - S)}{3} - \alpha \left(t - \frac{v_0}{3} \right). \tag{4}$$

The marginal consumer is characterized by $\hat{x}(p_0^*, p_1^*) = \hat{x}_0(p_0^*) = \hat{x}_1(p_1^*) = \frac{\alpha v_0}{3t} +$
$(1 - \alpha) \left(1 - \frac{v_1 - S}{3t} \right)$ and has no surplus. The equilibrium profits are

$$\pi_0^*(v_0, v_1, S) = \frac{p_0^*(v_0, v_1, S)(v_0 - p_0^*(v_0, v_1, S))}{t},$$

$$\pi_1^*(v_0, v_1, S) = \frac{p_1^*(v_0, v_1, S)(v_1 - p_1^*(v_0, v_1, S) - S)}{t}.$$

Regime (I) emerges when v_0 is large relative to $v_1 - S$. This occurs either when the entrant is significantly less efficient than the incumbent or when the switching cost is large, or both. Entry does not result in any consumers switching to the new supplier in this regime. However, the presence of the entrant gives consumers a higher surplus. In particular, the surplus of the consumer at $x = 1$ increases from
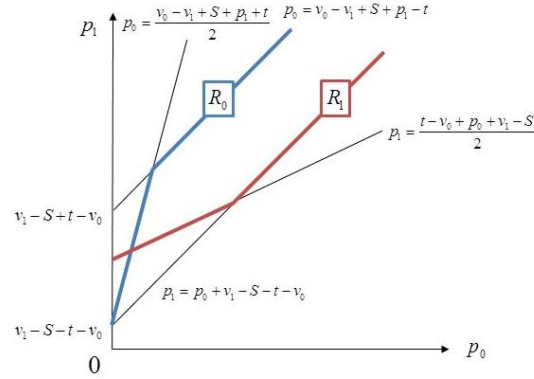
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Figure 2: Best-Response Correspondences and Equilibrium in Regime (III)

0, under the incumbent monopolist, to p_1^* after entry. The marginal consumer is indifferent between switching and not switching, and has $x = 1$.

Regime (II) emerges when v_0 is small relative to $v_1 - S$. In this case, the entrant is highly efficient, and the switching cost is sufficiently low for all consumers to switch. Again, the entrant's fixed fee is constrained because consumers do not have to switch. The consumer at $x = 0$ has a positive surplus of p_0^*.

Under regimes (III) and (IV), both firms make positive sales. Firms split the market equally when $v_0 = v_1 - S$, which is a subregime of regime (III). However, because of the switching cost, the entrant must be more efficient in order to have the same market share. The best-response correspondences and equilibrium under this regime are illustrated n Figure 2. The entrant does not reduce the final surplus by the whole amount of the switching cost because it takes into account the fact that the incumbent will also reduce its surplus in response. This is a direct result

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of strategic complementarity. For both groups of consumers, the equilibrium surplus decreases with the switching cost. However, from (1) and (2), it is easy to show that the equilibrium fee only increases for the incumbent. An increase in the switching cost leads the incumbent to charge a higher fee and to increase its market share. Thus, its profit is increasing in the switching cost. Because the entrant has a lower market share and a lower fee, its profit decreases with the switching cost.

In regime (IV), the intersection of the best-response correspondences is the closed line segment between points $(p_0, p_1) = \left(\frac{2v_0}{3}, \frac{v_0}{3} + v_1 - S - t \right)$ and $\left(v_0 - t + \frac{v_1 - S}{3}, \frac{2(v_1 - S)}{3} + 2t \right)$. Among these equilibria, the most profitable for the incumbent is the one that generates the largest market share for the incumbent, $p_0^*(v_0, v_1, S) = t - \frac{v_1 - S}{3}$. This corresponds to $\alpha = 0$ in the proposition and is at the lower right end of the relevant line segment in Figure 3. It is worth noting that this equilibrium coincides with the SPNE outcome were prices to be determined sequentially and were the incumbent to choose first. This is because the best-response correspondence of the entrant (the second mover) is kinked at this point, at which prices change from strategic substitutes to strategic complements. The equilibrium reflects the strategic substitute nature of the strategies. When the switching cost increases, the surplus of the incumbent's customers increases, whereas that of the entrant's customers decreases. Equations (3) and (4) clearly show that the equilibrium fees for both firms decrease with the switching cost. When switching costs increase, the entrant's equilibrium share decreases, and its fixed fee decreases. Hence, the entrant's profit unambiguously decreases with the switching cost. An increased
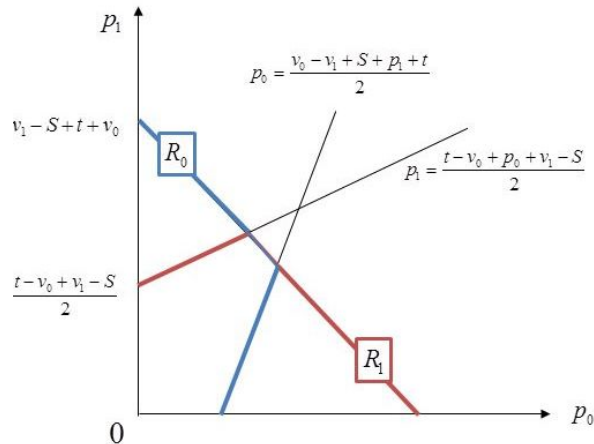
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Figure 3: Best-Response Correspondences in Regime (IV)

switching costs reduces fees but raises the incumbent's market share. Thus, if the fee is relatively high, incumbent profits increase with the switching cost.

If, in addition to assumption (M), we also assume that the entrant is sufficiently efficient, i.e., $v_1 - S \geq 2t$, then regime (IV) never occurs, and the equilibrium is unique.

3 Equilibrium Investment

In this section, we consider the equilibrium choices of technology improvement and switching cost. Firm 0 can either invest to increase v_0 and improve the current standard, or invest in a complementary technology and develop the installed base of the standard. This would increase the switching cost, S. Firm 1 invests in its

own technology. The technology improvement induced by the investment is Δ_i, $i = 0, 1$. Thus, given an existing quality level of $\overline{v}$, investment raises the quality level to $v_i = \overline{v} + \Delta_i$. To simplify the analysis, we assume $\overline{v} \geq 3t$, which is stronger than assumption (M).

Specifically, we assume that the indirect utility function takes the following form:

$$v_i = \overline{v} + \Delta_i, \quad i = 0, 1.$$

Cost of investment is

$$C_0(\Delta_0, S) = \frac{\delta(\Delta_0 + S)^2}{2}, \qquad C_1(\Delta_1) = \frac{\Delta_1^2}{2},$$

where δ is the investment efficiency parameter. The expected payoffs are

$$\Pi_0(\Delta_0, \Delta_1, S) = \pi_0^*(\overline{v} + \Delta_0, \overline{v} + \Delta_1, S) - C_0(\Delta_0, S),$$

$$\Pi_1(\Delta_0, \Delta_1, S) = \pi_1^*(\overline{v} + \Delta_0, \overline{v} + \Delta_1, S) - C_1(\Delta_1).$$

$\pi_0(\cdot)$ and $\pi_1(\cdot)$ are defined by Proposition 1 for each regime. If there is no investment ($\Delta_0 = \Delta_1 = S = 0$), qualities $v_0 = v_1 = \overline{v}$ and regime (III) prevail. It is legitimate to consider firm 0 choosing $\Delta \equiv S + \Delta_0$ to maximize profit because Δ_0 and S are symmetric in this setting. Once firm 0 has made its investment choice, two regimes are possible: $\Delta \equiv \Delta_0 + S > 3t$ (regime (I), Figure 4) and $\Delta \equiv \Delta_0 + S < 3t$ (regime (III), Figure 5).

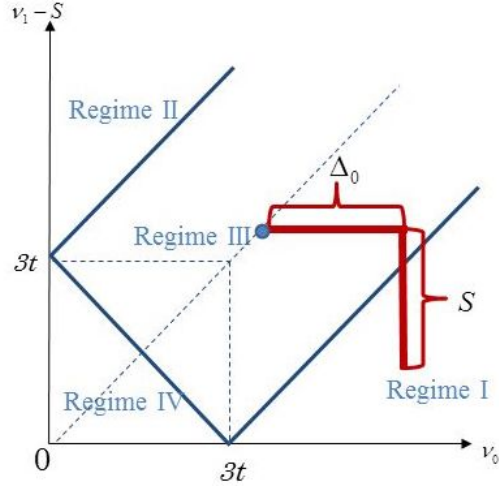In the regime (I) subgame, depending on firm 1's investment choice, either
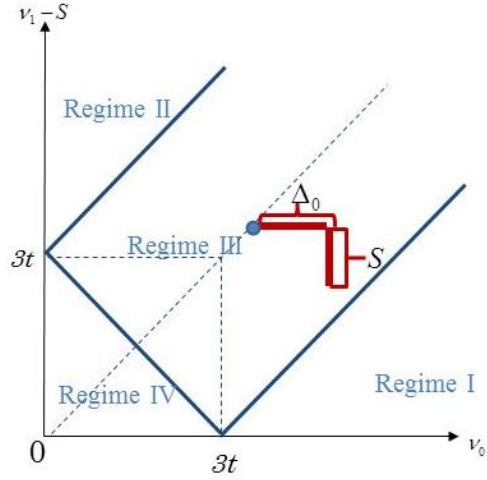
Figure 4: subgame ($\Delta > 3t$)   Figure 5: subgame ($\Delta < 3t$)

regime (I), regime (II), or regime (III) prevails. The next lemma shows the final outcome under regime (I) .

Lemma 1. *In the regime (I) subgame ($\Delta > 3t$), firm 1 invests nothing, and its payoff is zero. Then, the final outcome is regime (I).*

The next lemma shows that in the regime (III) subgame, either regime (II) or (III) prevails.

Lemma 2. *In the regime (III) subgame ($\Delta < 3t$), if $\delta > \frac{1}{3t}$, then firm 1's optimal investment results in regime (III). Otherwise, firm 1 invests so that the final outcome is regime (II).*

If the final outcome is regime (II), firm 0's payoff will be negative because it makes no profit. From the two lemmas, we obtain the next proposition.

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Proposition 2. *In equilibrium (in the SPNE), if $\delta \leq \frac{1}{3t}$, firm 0's investment is $\Delta^* > 3t$, firm 1 invests $\Delta_1^* = 0$, and the final outcome is regime (I) (upgrading and deterrence). If $\delta > \frac{1}{3t}$, firm 0's investment is $\Delta^* < 3t$, firm 1 invests $\Delta_1^* > 0$, and the final outcome is regime (III) (coexistence).*

If investment costs are low, there is upgrading without entry, but high investment costs lead to coexistence. Because of symmetry, investment costs are low for both incumbent and entrant. However, the incumbent can invest in the switching cost, which is a more efficient way of gaining a relative advantage, and is thus able to deter entry.

3.1 Welfare Analysis

When the switching cost is reduced, part of R_1 moves upward. However, increases in marginal costs of production, c_i, move part of R_i downward. Within regime (III), a reduction in S unambiguously increases the consumer surplus. However, if this results in S being higher than c_i, the total effect might be to reduce the consumer surplus. This is because the equilibrium might then change from one regime to another because of parameter changes. As a result, it is more useful to analyze welfare in the space between v_0 and $v_1 - S$.

The equilibrium consumer surplus and producer surplus for each of the four regimes (defined in Proposition 1) are summarized below. In regime (IV), under which there are multiple equilibria, we choose the one that yields the highest payoff for the incumbent ($\alpha = 0$). The iso-consumer surplus lines are shown in
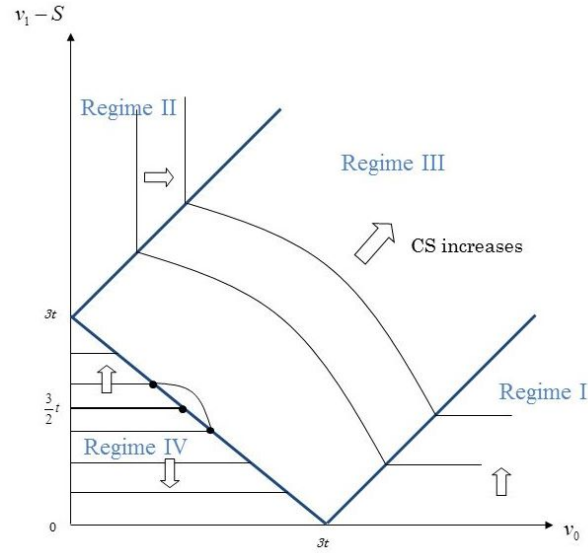
Figure 6: Iso-Consumer Surplus Curve

Figure 6.

	Consumer Surplus	Producer Surplus
I	$v_1 - S + \frac{t}{2}$	$v_0 - v_1 + S - t$
II	$v_0 + \frac{t}{2}$	$v_1 - S - v_0 - t$
III	$\frac{(v_0 - v_1 + S)^2}{36t} + \frac{v_1 - S + v_0}{2} - \frac{5}{4}t$	$\frac{(v_0 - v_1 + S)^2}{9t} + t$
IV	$\frac{1}{2t}\left\{(t - \frac{v_1 - S}{3})^2 + \left(\frac{v_1 - S}{3}\right)^2\right\}$	$v_0 - t - \frac{(v_0 - 2t)(v_1 - S)}{3t} + \frac{(v_1 - S)^2}{9t}$

Table 1: Consumer and Producer Surpluses by Regime

In both regimes (I) and (II), consumers are served by only one of the firms. In regime (I), the incumbent is the sole supplier, and thus consumers cannot switch

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to another supplier. However, the fee that they pay reflects the switching cost: the higher the cost, the greater the fee that the incumbent can charge. Thus, the consumer surplus is decreasing in the switching cost. Because the switching cost is "collected" by the incumbent, its profit is increasing in the switching cost. The sum of the two surpluses, however, does not depend on the switching cost, which effectively determines the share of the total surplus that accrues to consumers and the incumbent. Recall that although the entrant does not actually sell anything, entry increases the consumer surplus. Thus, high switching costs syphon off some of the benefit to the incumbent.

Because everyone switches in equilibrium in regime (II), switching costs are incurred. In this case, firm 1 must bear the switching cost to attract all consumers in the market. Thus, although the consumer surplus is independent of the switching cost, the producer surplus decreases in the switching cost.

In regime (III), the switching cost is anticompetitive in the standard sense: the consumer surplus decreases and the producer surplus increases in the switching cost. This is because the switching cost reduces the temptation to cut prices and therefore decreases the consumer surplus. In addition to reducing competition, within the model, although the switching cost is paid, it is not collected by anyone. This also contributes to reducing the social surplus when there are higher switching costs.

In regime (IV), although the switching cost increases the consumer surplus, it is questionable whether this is procompetitive. In this regime, an increase in the switching costs increases the surplus for consumers who buy from the incum-
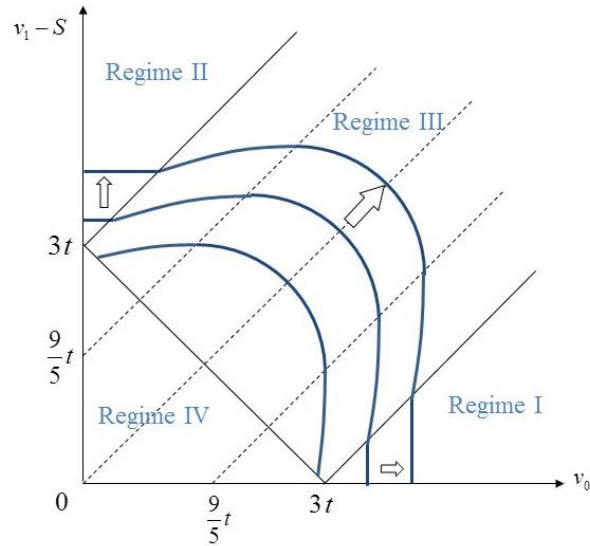
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Figure 7: Iso-Social Surplus Curve

bent and reduces the surplus for those buying from the entrant. In addition, the proportion of those buying from the incumbent increases. This increases the total consumer surplus. An increase in switching costs increases consumer welfare by skewing the surplus distribution so that there are more people in the higher surplus consumer group (which benefits) and fewer in the lower surplus consumer group (which is disadvantaged). The producer surplus decreases for a similar reason.

The iso-social surplus curves are presented in Figure 7, which shows the social benefits of equalizing $v_1 - S$ and v_0. Given some reduction in the switching cost, technologies that change the marginal cost may improve welfare. Furthermore, if costs are allocated carefully, there may be a distributional gain. For instance, unless S is reduced to zero, allocations that increase c_0 more than c_1 may equate $v_1 - S$ and v_0. Recall that in regime (III),the consumer surplus decreases in S.

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In some regions of regime (III), the social surplus may increase with S if gains in the producer surplus are sufficiently large. This occurs when $v_1 - S \leq -\frac{9}{5}t + v_0$. In these regions, firm 0 is significantly more efficient, which gives it substantial market power. In this case, whereas increasing the switching cost barely hurts consumers at the margin, producers gain significantly.

4 Policy Implications

We have shown that a firm or patent pool with a stake in the current standard either upgrades it or invests in the installed base to deter the entry of another standard when the technology is in its infancy. When the technology is in its infancy, the cost of innovation is low not only for the incumbent but also for the entrant. In this case, investing in the installed base to increase consumer switching costs is a viable strategy for deterring entry. As the technology matures and innovation costs increase, the incumbent no longer deters entry, and different standards coexist in the market.

The fact that replacement never occurs in equilibrium implies that if the existing standard's consortium is stable, then it is difficult for a new entrant to supplant the standard. Even when the technology matures, the best that an entrant can achieve is coexistence. While stability seems a desirable policy objective for a consortium in its own right, it goes a surprisingly long way toward preserving standard dominance. Members should take into account such dynamic benefits of consortium unity when designing consortium rules and licensing terms.

From the viewpoint of a potential entrant, having superior technology is not sufficient to replace an existing standard. An entrant should consider actively seeking to break up the existing consortium. One way of achieving this might be to use part of the existing standard technology to design a new standard. In such an endeavor, dynamic considerations would be important. That is, the value of cooperating with part of the existing coalition would be the potential expansion of the market that could be achieved by replacing the current standard. Members of the existing coalition would have to compare the dynamic benefits obtained from sharing the market with the costs of being replaced.

Our analysis suggests that competition and standardization policies should take account of the technology life cycle. Although the persistence of a single standard may be consistent with high-level upgrades, it may also be the result of high switching costs or anticompetitive behavior. The latter is likely to reduce the consumer surplus. In this case, antitrust intervention might be warranted. However, our analysis also shows that as the technology matures, entry and the coexistence of standards are likely to occur without policy intervention. Because market mechanisms for restoring competition exist, there may be no need for policies to promote entry.

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Appendix

Derivation of Demand under Assumption (M)

We define the benchmarks, $\hat{x}_0(p_0)$, $\hat{x}_1(p_1)$, and $\hat{x}(p_0, p_1)$, by

$$v_0 - p_0 - t\hat{x}_0(p_0) = 0, \quad v_1 - p_1 - S - t(1 - \hat{x}_1(p_1)) = 0, \qquad (5)$$

$$v_0 - p_0 - t\hat{x}(p_0, p_1) = v_1 - p_1 - S - t(1 - \hat{x}(p_0, p_1)). \qquad (6)$$

All consumers to the left (right) of $\hat{x}_0(p_0)$ ($\hat{x}_1(p_1)$) derive positive utility from buying from firm 0 (firm 1). All consumers to the left (right) of $\hat{x}(p_0, p_1)$ derive greater utility from buying from firm 0 (firm 1). By definition, it must be that either (i) $\hat{x}_0(p_0) < \hat{x}(p_0, p_1) < \hat{x}_1(p_1)$, or (ii) $\hat{x}_0(p_0) \geq \hat{x}(p_0, p_1) \geq \hat{x}_1(p_1)$. In case (i), there is an interval of consumers in the middle that do not buy at all. In case (ii), all consumers buy, and there are three possibilities: all buy from firm 0 if $\hat{x}(p_0, p_1) \leq 0$; all buy from firm 1 if $\hat{x}(p_0, p_1) \geq 1$; and otherwise, both firms make positive sales. We have $\hat{x}_0(p_0) = (v_0 - p_0)/t$, $1 - \hat{x}_1(p_1) = (v_1 - S - p_1)/t$, and $\hat{x}(p_0, p_1) = (v_0 - p_0 - v_1 + S + p_1 + t)/2t$.

Proof of Proposition 1

The problem is to find the p_0 that maximizes

$$
\pi_0 = \begin{cases}
\pi_0^A = \frac{p_0(v_0 - p_0)}{t} & \text{for } v_0 - p_0 \leq t - v_1 + S + p_1, \\[2mm]
\pi_0^B = \frac{p_0(v_0 - p_0 - v_1 + S + p_1 + t)}{2t} & \text{for } t - v_1 + S + p_1 < v_0 - p_0 \leq t + v_1 - S - p_1, \\[2mm]
\pi_0^C = p_0 & \text{for } t + v_1 - S - p_1 < v_0 - p_0.
\end{cases}
$$

Straightforward but tedious calculation yields the next lemma.

Lemma 3. *Firm 0's best-response correspondence $p_0 = R_0(p_1)$ is as follows.*

(1) If $t < v_0/3$, then

$$
R_0(p_1) = \begin{cases}
v_0 - v_1 + S + p_1 - t & \text{for } v_1 - S - p_1 \leq v_0 - 3t, \\[2mm]
\frac{v_0 - v_1 + S + p_1 + t}{2} & \text{for } v_1 - S - p_1 \geq v_0 - 3t.
\end{cases}
$$

(2) If $t > v_0/3$, then

$$
R_0(p_1) = \begin{cases}
v_0 + v_1 - S - p_1 - t & \text{for } v_1 - S - p_1 \leq t - \frac{v_0}{3}, \\[2mm]
\frac{v_0 - v_1 + S + p_1 + t}{2} & \text{for } t - \frac{v_0}{3} \leq v_1 - S - p_1.
\end{cases}
$$

(3) If $t = v_0/3$, then

$$
R_0(p_1) = \frac{v_0 - v_1 + S + p_1 + t}{2} \text{for all } v_1 - S - p_1 \geq 0.
$$

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Firm 1's best-response correspondence is obtained similarly and differs only because the switching cost must be taken into account in the profit function. By using the same argument applied to firm 0, the problem for firm 1 is to choose p_1 to maximize

$$\pi_1 = \begin{cases} \pi_1^A = \frac{p_1(v_1 - S - p_1)}{t} & \text{for } v_1 - S - p_1 \leq t - v_0 + p_0, \\[2mm] \pi_1^B = \frac{p_1(t - v_0 + p_0 + v_1 - S - p_1)}{2t} & \text{for } t - v_0 + p_0 < v_1 - S - p_1 \leq t + v_0 - p_0, \\[2mm] \pi_1^C = p_1 & \text{for } t + v_0 - p_0 < v_1 - S - p_1. \end{cases}$$

Lemma 4. *Firm 1's best-response correspondence* $p_1 = R_1(p_0)$ *is as follows.*

(1) If $t < (v_1 - S)/3$, then

$$R_1(p_0) = \begin{cases} \frac{v_1 - S}{2} \text{ or } p_0 - t - v_0 + v_1 - S & \text{for } v_0 - p_0 \leq t - \frac{v_1 - S}{2}, \\[2mm] p_0 - t - v_0 + v_1 - S & \text{for } t - \frac{v_1 - S}{2} < v_0 - p_0 \leq v_1 - S - 3t, \\[2mm] \frac{t - v_0 + p_0 + v_1 - S}{2} & \text{for } v_1 - S - 3t < v_0 - p_0. \end{cases}$$

(2) If $t > (v_1 - S)/3$, then

$$R_1(p_0) = \begin{cases} \frac{v_1 - S}{2} & \text{for } p_0 - t - v_0 + v_1 - S \leq t - \frac{v_1 - S}{2} \\[2mm] v_1 - S - t + v_0 - p_0 & \text{for } t - \frac{v_1 - S}{2} < v_0 - p_0 \leq t - \frac{v_1 - S}{3}, \\[2mm] \frac{t - v_0 + p_0 + v_1 - S}{2} & \text{for } t - \frac{v_1 - S}{3} \leq v_0 - p_0. \end{cases}$$

(3) If $t = (v_1 - S)/3$, then

$$R_1(p_0) = \frac{t - v_0 + p_0 + v_1 - S}{2} \text{ for all } v_0 - p_0 \geq 0.$$

In case (1), the value of $R_1(p_0)$ for $v_0 - p_0 \leq t - (v_1 - S)/2$ is $(v_1 - S)/2$ if $\pi_1^A(\frac{v_1 - S}{2}) \geq \pi_1^B(p_0 - t - v_0 + v_1 - S)$, and the value is $p_0 - t - v_0 + v_1 - S$ otherwise. It is unambiguously the case that $R_1(p_0) > v_0 - p_0$, which guarantees that this segment of the best-response function never contains the Nash equilibrium (in pure strategies). Because of the switching cost, firm 1 may not always want to sell to all consumers not buying from firm 0. However, because of assumption (M), firm 0 takes any opportunity to sell to a consumer who does not buy from firm 1. Using the best-response correspondences, we can characterize the Nash equilibrium prices and allocations.

For both firms, there is a case (case (2) for both) for which strategies can be strategic complements. Competition based on fixed fees is effectively competition based on prices that are strategic substitutes: when a rival firm lowers its fee, the firm's optimal response is to lower its fee. That is, when its rival increases demand, each firm finds it profitable to reduce its fee and to increase demand (to get back some of the lost demand caused by the rival lowering its fee). In doing so, each firm must forgo some of the surplus previously collected from its captive consumers. However, in case (2), if $v_1 - S - p_1 \leq t - \frac{v_0}{3}$, then in response to its rival's fee reduction, firm 0 finds it optimal to increase its own fee (and to lose further demand) to extract more surplus from its captive consumers. For

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this to be optimal, the reduction in demand induced by the fee increase must be small relative to the surplus; i.e., transportation cost (t) must be sufficiently large, which is the condition for case (2) to prevail. In addition, the marginal consumer's surplus must be small enough that it is not worth retaining that consumer ($v_1 - S - p_1 \leq t - \frac{v_0}{3}$). A similar argument holds for firm 1's strategic complementarity.

Proof of Lemma 1

First, we consider firm 1's response when firm 0's investment is sufficiently high ($\Delta_0 + S > 3t$). In this case, firm 1 must exit the market unless it can improve the quality of its product sufficiently. We consider the optimal investments in equilibrium. To determine firm 0's strategy, we must consider firm 1's response.

Firm 1 does not invest ($\Delta_1 = 0$)

When firm 1 does not invest to improve product quality, the outcome is in region 1. Then, the producers' profits are given by

$$\pi_0 = \Delta_0 + S - t - \frac{\delta(\Delta_0 + S)^2}{2},$$

$$\pi_1 = 0.$$

The optimal switching cost S^* and the optimal degree of quality improvement Δ_0^* solve the following:

$$\max_{\Delta_0, S} \pi_0 = \Delta_0 + S - t - \frac{\delta(\Delta_0 + S)^2}{2}$$

$$s.t. \Delta_0 + S \geq 3t.$$

We define the Lagrangian

$$L_0 = \Delta_0 + S - t - \frac{\delta(\Delta_0 + S)^2}{2} + \lambda(\Delta_0 + S - 3t).$$

We can consider firm 0 choosing $\Delta \equiv S + \Delta_0$ to maximize profit because Δ_0 and S are symmetric in this setting. Then, the Kuhn–Tucker conditions are

$$\frac{\partial L_0(\Delta)}{\partial \Delta} = 1 - \delta\Delta + \lambda = 0, \ \Delta \frac{\partial L_0(\Delta)}{\partial \Delta} = 0,$$

$$\frac{\partial L_0(\Delta)}{\partial \lambda} = \Delta - 3t > 0, \ \lambda \geq 0, \ \lambda \frac{\partial L_0(\Delta)}{\partial \lambda} = 0.$$

First, we consider the case in which $\Delta > 0$, $\lambda = 0$ when $\delta > 1/3t$, which gives

$$\Delta^* = 3t.$$

The optimal profits in this region are thus given by

$$\pi_0^* = 2t - \frac{9t^2\delta}{2}, \quad \pi_1^* = 0.$$

Second, we consider the case in which $\Delta > 0$, $\lambda > 0$ when $\delta \leq 1/3t$, which gives

$$\Delta^* = \frac{1}{\delta}.$$

The optimal profits in this region are thus given by

$$\pi_0^* = \frac{1}{2\delta} - t, \quad \pi_1^* = 0.$$

Firm 1 tries to move to region 3 $(\Delta - 3t < \Delta_1 < \Delta + 3t)$

When firm 1 invests in quality improvement and tries to move to region 3, producers' profits are given by

$$\pi_0 = \frac{(\Delta - \Delta_1 + 3t)^2}{18t} - \frac{\delta\Delta^2}{2}, \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}.$$

We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2}$$

$$s.t. \Delta_1 + 3t \geq \Delta \geq \Delta_1 - 3t.$$

We define the Lagrangian

$$L_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta\Delta_1^2}{2} + \lambda_1(\Delta - \Delta_1 + 3t) + \lambda_2(\Delta_1 - \Delta + 3t).$$

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The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = \frac{(\Delta_1 - \Delta + 3t)}{9t} - \delta \Delta_1 - \lambda_1 + \lambda_2 = 0, \ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$

$$\frac{\partial L_1}{\partial \lambda_1} = \Delta - \Delta_1 + 3t > 0, \ \lambda_1 \frac{\partial L_1}{\partial \lambda_1} = 0,$$

$$\frac{\partial L_1}{\partial \lambda_2} = \Delta_1 - \Delta + 3t > 0, \ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0.$$

We consider the case in which $\Delta_1 \geq 0$, $\lambda_1 = \lambda_2 = 0$ when $\delta < 1/9t$, which gives

$$\Delta_1^* = \frac{\Delta - 3t}{1 - 9t\delta}.$$

The optimal profits in this region are thus given by

$$\pi_1^* = -\frac{\delta(\Delta - 3t)^2}{2(1 - 9t\delta)}.$$

When $\delta < 1/9t$, firm 1's equilibrium profit is negative. Thus, firm 1 does not choose this strategy.

Firm 1 tries to move to region 2 $(\Delta_1 \geq \Delta + 3t)$

When firm 1 invests sufficiently in quality improvement and tries to move to region 2, producers' profits are given by

$$\pi_0 = -\frac{\delta \Delta^2}{2}, \pi_1 = \Delta_1 - \Delta - t - \frac{\delta \Delta_1^2}{2}.$$

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We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \Delta_1 - \Delta - t - \frac{\delta \Delta_1^2}{2}$$

$$s.t. \Delta_1 \geq \Delta + 3t.$$

We define the Lagrangian

$$L_1 = \Delta_1 - \Delta - t - \frac{\delta \Delta_1^2}{2} + \lambda(\Delta_1 - \Delta - 3t).$$

The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = 1 - \delta \Delta_1 - \lambda = 0, \ \ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$

$$\frac{\partial L_1}{\partial \lambda} = \Delta_1 - \Delta - 3t > 0, \ \ \lambda \frac{\partial L_1}{\partial \lambda} = 0.$$

First, we consider the case in which $\Delta_1 \geq 0$, $\lambda > 0$ when $\max\{3t, \frac{1}{\delta} - 3t\} < \Delta$, which gives

$$\Delta_1^* = \Delta^* + 3t.$$

The optimal profits in this region are thus given by

$$\pi_1^* = 2t - \frac{\delta(\Delta^* + 3t)^2}{2}.$$

Second, we consider the case in which $\Delta_1 \geq 0$, $\lambda = 0$ when $3t < \Delta \leq \frac{1}{\delta} - 3t$, which gives

$$\Delta_1^* = \frac{1}{\delta}.$$

The optimal profits in this region are thus given by

$$\pi_1 = \frac{1}{2\delta} - \Delta - t.$$

Optimal investment in this region

We can now consider firm 0's optimal investment in this region. Firm 1 has no incentive to move to region 3 because its profit is negative. Firm 0 prefers region 1 to region 2. We can easily show that firm 1 has no incentive to move to region 2 given firm 0's optimal investment in region 1. Therefore, in this region, firm 0 tries to maximize profit in region 1, and firm 1 does not invest in equilibrium.

Proof of Lemma 2

In this case, firm 0 invests little ($\Delta_0 + S \leq 3t$), and firm 1 can stay in the market unless firm 0 substantially improves product quality. To consider firm 0's strategy, we must take into account firm 1's response.

Firm 1 tries to stay in region 3 $(\Delta - 3t < \Delta_1 < \Delta + 3t)$

When firm 1 invests in quality improvement and tries to stay in region 3, producers' profits are given by

$$\pi_0 = \frac{(\Delta - \Delta_1 + 3t)^2}{18t} - \frac{\delta \Delta^2}{2}, \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta \Delta_1^2}{2}.$$

We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta \Delta_1^2}{2}$$

$$s.t. \Delta_1 + 3t \geq \Delta \geq \Delta_1 - 3t.$$

We define the Lagrangian

$$L_1 = \frac{(\Delta_1 - \Delta + 3t)^2}{18t} - \frac{\delta \Delta_1^2}{2} + \lambda_1(\Delta - \Delta_1 + 3t) + \lambda_2(\Delta_1 - \Delta + 3t).$$

The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = \frac{(\Delta_1 - \Delta + 3t)}{9t} - \delta \Delta_1 - \lambda_1 + \lambda_2 = 0, \ \Delta_1 \frac{\partial L_1}{\partial \Delta_1} = 0$$

$$\frac{\partial L_1}{\partial \lambda_1} = \Delta - \Delta_1 + 3t > 0, \ \lambda_1 \frac{\partial L_1}{\partial \lambda_1} = 0,$$

$$\frac{\partial L_1}{\partial \lambda_2} = \Delta_1 - \Delta + 3t > 0, \ \lambda_2 \frac{\partial L}{\partial \lambda_2} = 0.$$

We consider the case in which $\Delta_1 \geq 0$, $\lambda_1 = \lambda_2 = 0$ when $\delta > 1/9t$, which gives

$$\Delta_1^* = \frac{3t - \Delta}{9t\delta - 1}.$$

Firm 0 takes into account firm 1's strategy to maximize profit. Then, the optimal values of Δ^* are the solutions to

$$\max_{\Delta} \pi_0 = \frac{\left(\Delta - \frac{3t-\Delta}{9t\delta-1} + 3t\right)^2}{18t} - \frac{\delta\Delta^2}{2}$$

$$s.t. \Delta \geq \frac{3t - \Delta}{9t\delta - 1} - 3t,$$

$$\frac{3t - \Delta}{9t\delta - 1} \geq \Delta - 3t.$$

In this section, we focus on the inner solution. Then, in equilibrium, the optimal investments are

$$S^* = \Delta_0^* = \frac{3t(9t\delta - 2)}{2(81t^2\delta^2 - 27t\delta + 1)}, \quad \Delta_1^* = \frac{9t(3t\delta - 1)}{81t^2\delta^2 - 27t\delta + 1}.$$

The optimal profits in this region are thus given by

$$\pi_0^* = \frac{t(9t\delta - 2)^2}{2(81t^2\delta^2 - 27t\delta + 1)}, \quad \pi_1^* = \frac{81t^2\delta(3t\delta - 1)^2(9t\delta - 1)}{2(81t^2\delta^2 - 27t\delta + 1)^2}.$$

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We must check that the following conditions are satisfied in equilibrium:

$$\Delta^* < 3t \iff \frac{9t(9t\delta - 1)(1 - 3t\delta)}{81t^2\delta^2 - 27t\delta + 1} < 0,$$

$$\Delta^* \geq \frac{3t - \Delta^*}{9t\delta - 1} - 3t \iff \frac{3t(9t\delta - 1)(9t\delta - 2)}{81t^2\delta^2 - 27t\delta + 1} \geq 0,$$

$$\frac{3t - \Delta^*}{9t\delta - 1} \geq \Delta^* - 3t \iff \frac{81t^2\delta(3t\delta - 1)}{81t^2\delta^2 - 27t\delta + 1} \geq 0.$$

The satisfaction of these conditions requires

$$\text{sign}\left(81t^2\delta^2 - 27t\delta + 1\right) = \text{sign}(9t\delta - 2) = \text{sign}(3t\delta - 1).$$

We consider the case in which all signs are positive. (When all signs are negative, it is not possible to satisfy all conditions.) We can rewrite the conditions as follows:

$$\text{sign}\left(81t^2\delta^2 - 27t\delta + 1\right) > 0 \iff \frac{9t\delta}{(9t\delta - 1)^2} < 1,$$

$$\text{sign}(9t\delta - 2) > 0 \iff \frac{2}{9t} < \delta,$$

$$\text{sign}(3t\delta - 1) > 0 \iff \delta > \frac{1}{3t}.$$

The term $9t\delta/(9t\delta - 1)^2$ is a decreasing function of δ when $\delta > 1/9t$. In addition, $9t\delta/(9t\delta - 1)^2$ is smaller than 1 when $\delta = 1/3t$. Therefore, all conditions are satisfied when δ exceeds $1/3t$.

Firm 1 tries to move to region 2 $(\Delta_1 > \Delta + 3t)$

When firm 1 invests in quality improvement and tries to move to region 2, producers' profits are given by

$$\pi_0 = -\frac{\delta\Delta^2}{2}, \pi_1 = \Delta_1 - \Delta - t - \frac{\delta\Delta_1^2}{2}.$$

We must consider firm 1's strategy. The optimal values of Δ_1^* are the solutions to

$$\max_{\Delta_1} \pi_1 = \Delta_1 - \Delta - t - \frac{\delta\Delta_1^2}{2}$$

$$s.t.\Delta_1 \geq \Delta + 3t.$$

We define the Lagrangian

$$L_1 = \Delta_1 - \Delta - t - \frac{\delta\Delta_1^2}{2} + \lambda(\Delta_1 - \Delta - 3t).$$

The Kuhn–Tucker conditions are

$$\frac{\partial L_1}{\partial \Delta_1} = 1 - \delta\Delta_1 - \lambda = 0, \ \Delta_1\frac{\partial L_1}{\partial \Delta_1} = 0,$$
$$\frac{\partial L_1}{\partial \lambda} = \Delta_1 - \Delta - 3t > 0, \ \lambda\frac{\partial L_1}{\partial \lambda} = 0.$$

First, we consider the case in which $\Delta_1 \geq 0, \lambda > 0$ when $\frac{1}{\delta} - 3t < \Delta < 3t$, which gives

$$\Delta_1^* = \Delta^* + 3t.$$

The optimal profits in this region are thus given by

$$\pi_1^* = 2t - \frac{\delta(\Delta + 3t)^2}{2}.$$

Second, we consider the case in which $\Delta_1 \geq 0, \lambda = 0$ when $\Delta < \max\{3t, \frac{1}{\delta} - 3t\}$, which gives

$$\Delta_1^* = \frac{1}{\delta}.$$

The optimal profits in this region are thus given by

$$\pi_1 = \frac{1}{2\delta} - \Delta - t.$$

Optimal investment in this region

We can now consider firm 0's optimal investment in this region. Firm 0 prefers region 3 to region 2. Therefore, both firms invest and stay in region 3 when $\delta > 1/3t$. Otherwise, region 2 defines the equilibrium.

Proof of Proposition 2

We can now consider optimal investment.

If quality improvement is costly $(\delta > 1/3t)$

When firm 0's investment is not sufficiently high $(\Delta_0 + S \leq 3t)$, the region defines the equilibrium. When firm 0's investment is sufficient $(\Delta_0 + S > 3t)$,

the equilibrium is defined by region 1. We can easily show that, in this case, firm 0 makes more profit under region 3 than under region 1. Thus, firm 0 tries to stay in region 3.

If quality improvement is not costly ($\delta \leq 1/3t$)

When firm 0's investment is not sufficiently high ($\Delta_0 + S \leq 3t$), the equilibrium is located in region 2. When firm 0 does invest sufficiently ($\Delta_0 + S > 3t$), region 1 defines the equilibrium. Thus, firm 0 tries to invest enough to prevent firm 1's entry.

Derivation of the Consumer Surplus

In a mature industry, the consumer surplus for the four regimes is given below.

$$
\text{Regime (I)} : CS_I = \int_0^1 (v_0 - p_0^* - tx)dx = v_1 - S + \frac{t}{2},
$$

$$
\text{Regime (II)} : CS_{II} = \int_0^1 (v_1 - S - p_1^* - t(1-x))dx = v_0 + \frac{t}{2},
$$

$$
\text{Regime (III)} : CS_{III} = \int_0^{\hat{x}(p_0^*,p_1^*)} (v_0 - p_0^* - tx)dx + \int_{\hat{x}(p_0^*,p_1^*)}^1 (v_1 - S - p_1^* - t(1-x))dx
$$

$$
= \frac{(v_0 - v_1 + S)^2}{36t} + \frac{v_1 - S + v_0}{2} - \frac{5}{4}t,
$$

$$
\text{Regime (IV)} : CS_{IV} = \int_0^{\hat{x}(p_0^*,p_1^*)} (v_0 - p_0^* - tx)dx + \int_{\hat{x}(p_0^*,p_1^*)}^1 (v_1 - S - p_1^* - t(1-x))dx
$$

$$
= \frac{1}{2t}\left\{\left(t - \frac{v_1 - S}{3}\right)^2 + \left(\frac{v_1 - S}{3}\right)^2\right\}.
$$

Derivation of the Iso-Social Surplus Curves

These curves are obtained from the expressions below.

$$\text{Regime (I)} : SS = CS + PS = -\frac{1}{2}t + v_0,$$

$$\text{Regime (II)} : SS = -\frac{1}{2}t + v_1 - S.$$

For the remaining regimes, by using the following partial derivatives, we obtain

$$\text{Regime (III)} : \frac{\partial SS}{\partial v_0} = \frac{1}{2}t + \frac{5v_0 - v_1 + S}{18t},$$

$$\frac{\partial SS}{\partial (v_1 - S)} = \frac{1}{2}t - \frac{5(v_0 - v_1) + S}{18t},$$

$$\text{Regime (IV)} : \frac{\partial SS}{\partial v_0} = \frac{3t - v_1 + S}{3t},$$

$$\frac{\partial SS}{\partial (v_1 - S)} = \frac{4(v_1 - S) - 3v_0 - 3t}{9t}.$$