Social security, child allowances, and endogenous fertility*

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Abstract

Based on a simple overlapping generations model with endogenous fertility, we show that the effectiveness of social security reform and childcare support depends much on the openness of the economy, altruism, and initial fertility. For example, introducing a child allowance, which is often expected to mitigate demographic pressures, might be ineffective in a closed economy. Downsizing a social security system, even with new taxes required to compensate for existing pension liabilities, could be welfare-improving in a closed economy. Altruistic bequests tend to offset intergenerational income transfer caused by policy changes, but a higher level of initial fertility makes the outcome indeterminate.

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1. Introduction

Populations are aging rapidly in almost every industrialized country. This demographic trend has placed strong pressures on the financial viability of social security systems, because their benefits to retired generations depend heavily on working and future generations.   In many countries, the total fertility rate remains well below the level that can sustain the current population size.   There is also a growing risk that the financial positions of public pensions will deteriorate and that inequality among generations will continue to grow under an aging population.   In Japan, there is no sign of a recovery of fertility; indeed, the National Institute of Population and Social Security Research has consistently revised down its population forecasts, now estimating that the total fertility rate will recover to just 1.39 by 2050 from 1.33 in 2001 ("Population Projection for Japan: 2001-2050," January, 2002).   Lower population growth makes it more difficult for the government to maintain the current level of social security benefits without raising premiums further.

There is controversy over how to tackle these demographic pressures and make the social security system more sustainable.   Some economists propose a shift to a funded system from the current pay-as-you-go (PAYGO) one, or privatize the system along with the introduction of mandatory individual accounts (e.g. Feldstein (1995), (1998)).   Any social security reform that incorporates downsizing a PAYGO system has almost the same impact as an explicit shift to a funded system.   However, other economists argue that neither shifting to a funded system nor downsizing a PAYGO system can be Pareto-improving, because working and/or future generations have to pay new taxes to compensate for the benefits that the government has promised to pay to older generations.   This has been widely recognized as a "double-burden" problem, and has been more formally addressed by many researchers, including Breyer (1989), Geanakopolos, Mitchell and Zeldes (1998), and

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Sinn (2000).   Once a PAYGO system has been introduced, it seems to be impossible to scale it down without making any generation worse off.

If that is the case, we have to find a more realistic and efficient policy option.   One alternative reform, which seems plausible, is to provide households with financial childcare support, because children are expected to play a key role in financing social security benefits in the future.   The sustained downtrend of fertility in many countries is attributable largely to the higher opportunity cost of childcare, presumably reflecting a rise in labor force participation by highly educated women.   Thus, in recent decades, policymakers in industrialized countries have been placing more importance on childcare support, including family and child allowances.   Especially in Japan, which belongs to the group with the lowest fertility among OECD member countries and whose benefits to the "family and children" have a quite limited share of total social policy benefits, many argue for shifting benefits to childcare from income support for the elderly[1].   Moreover, there is substantial empirical literature regarding the effects of child allowances (as well as social security coverage) on fertility, including Walker (1995), Cigno and Rosati (1996), and Cigno, Casolaro and Rosati (2000), the latter two of whom suggest that financial support for childcare will have a limited impact on fertility.

To analyze the impact of social security reform and childcare support on fertility and social welfare, we need a model in which fertility is endogenously determined.   For example, Nishimura and Zhang (1992), Peters (1995), Kolmar (1997), Sinn (1998), and Cigno, Luporini and Pettini (2000) discuss this issue extensively using models with endogenous fertility.   Most recently, Groezen, Leers and Meijdam (2003) argue that

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[1] In 2000, the share of benefits to the "Family and Children" of total SS benefits was 2.9% in Japan, compared to 9.0% in Germany (1996) and 10.5% in Sweden (1996) [source: The National Institute of Population and Social Security Research, "Social Security Benefits: FY 2000," November, 2002].

Pareto-improvements can only be realized by internalizing the external effect due to the existing PAYGO scheme through the introduction of a child allowance scheme. They conclude that countries with extensive PAYGO schemes, but no child subsidy, should introduce a child allowance scheme to broaden the tax base for the PAYGO scheme. Their advice sounds not only reasonable and but also realistic, in that it is politically difficult to downsize a social security system because this comes at the cost of lower utility for at least one generation.

However, the policy impact of social security reform and childcare support is likely to depend heavily on the setup of the model. First, whether the economy is small open or closed seems to substantially affect the outcome. Groezen *et al.* (2003), who stress that a child allowance is Pareto-improving, assume a small open economy. In a closed economy and general equilibrium context, however, we have to take into account that social security reform and childcare support affect capital accumulation and thus social welfare. For example, downsizing a social security system could encourage households to save more when young and accelerate capital accumulation, which in turn adds to steady-state lifetime income and utility. Higher fertility caused by childcare support might also reduce per-capita capital stock and negatively affect social welfare, other things being equal.

Second, the altruistic behavior of parents towards their children should be considered. It is widely known that intergenerational income transfer caused by public policies is at least partially offset by altruistic bequests, as implied by the "Ricardian Equivalence Theorem." For example, public income transfer from younger to older generations via a PAYGO system tends to be offset by private income transfer from older to younger generations through altruistic bequests. Combined with altruism, endogenous fertility makes the story more complicated. Indeed, seminal works by Becker and Barro (1988) and Barro and Becker (1989), who internalize fertility in an overlapping generations model with altruism, show that

an increase in the cost of children—due for example to a tax on children or to an expanded social security system—raises social welfare, contrary to conventional wisdom, because households want to endow their children with a higher level of consumption and utility by increasing bequests.

This paper investigates how the impact of social security reform and childcare support depends on the setup of the model and the assumptions about altruism, using a simple overlapping generations model with endogenous fertility. It can help us compare different and sometime completely opposing policy implications obtained from previous studies, based on the same theoretical framework. More specifically, we compare four models: a small open economy with and without altruism and a closed economy with and without altruism. According to our analysis, we cannot rule out the possibility, for example, that downsizing a social security system is welfare-improving (even if the government levies new taxes on households to compensate for existing pension liabilities) or that introducing a child allowance does not add to social welfare, both of which are contrary to Groezen *et al.*'s argument. We also show that the initial level of fertility significantly affects the policy impact. Starting with a relatively high level of fertility in models of altruism, for example, an expanded social security system may reduce social welfare due to the negative income effect, which is contrary to Barro and Becker's argument. These results suggest that when discussing what social security reform and/or childcare support system should look like, we have to be cautious about the model specifications and the assumptions on which discussions are based.

Another feature of our analysis is that it assumes the households (parents) are aware of the effects of aggregate fertility (via social security, child allowances, and taxes) on their own budget sets and take this effect into account when making decisions. We assume that each (symmetric) household maximizes its utility taking the behavior of other households as

given, and that under this Nash equilibrium the government looks for a socially optimal policy. This approach is useful to grasp the "free rider" or "moral hazard" problem caused by a social security system; for instance, if there is a social security system, people have fewer incentives to bear children than otherwise, because they can rely on social security benefits, which are financed by other people's children. Our approach is comparable to that of Cigno, Luporini and Pettini (2000), who interpret the relationship between the government and households as a "principal-agent" relationship, with the government in the role of the principal and households in the role of agents. They assume that households take the tax-benefit system (and thus other households' decisions) as given and look for a socially optimal solution. Their analysis is, however, based on a partial equilibrium approach implicitly assuming a small open economy.

The remainder of this paper is arranged as follows. Section 2 presents the theoretical framework on which our analysis is based. Starting with a benchmark model, which assumes a small open economy with no altruism, we present four different models and discuss how the policy impact relies on the setup of the model and the assumptions. Section 3 presents some numerical simulations that illustrate the conclusions in Section 2, some of which are theoretically indeterminate and depend on the parameters of the model. Section 4 gives a summary.

2. Theoretical framework

2.1 A small open economy with no altruism

Let us consider a simple overlapping generations model of two periods (working and retirement periods) and two generations (working and retirement generations) in a small, open economy. We start with a benchmark model with no altruism: the household cares

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only about its own utility and leaves no altruistic bequests. Each household consumes c_1 and c_2, in each period, respectively, and it bears and cares for n children during the working period. Because we consider a household in individual terms, the size of the population remains unchanged if n is equal to unity, grows (shrinks) if n is greater (less) than unity. So n-1 can be interpreted as the rate of population growth rate per generation, and $2n$ corresponds to the total fertility rate.

We assume that the household's utility is determined solely by levels of consumption in each period and the number of children, not by the children's "quality" such as represented by the education with which they are endowed[2]. For simplicity and convenience, we assume that a representative household has a Cobb-Douglas utility function such as

$$u = a \ln n + b \ln c_1 + g \ln c_2, \quad a + b + g = 1, \quad 0 < a, b, g < 1. \tag{1}$$

This setup of the household's utility is essentially the same as the one used in Kato (1999), Groezen *et al.* (2003), and elsewhere. We take this utility function of a representative household as a proxy for (per-capita) social welfare.

Then, consider the household's budget constraint. Denote w, r, and z as the wage rate, the interest rate, and the cost of childcare per child, respectively, and assume that the household takes these variables as given exogenously in a small, open economy, and that the capital market is perfect. There is a PAYGO system, under which the government levies a lump-sum social security tax, p, per household on the younger generation, and gives a lump-sum social security tax to the older generation that has retired. Under this system, total social security benefits are set to be equal to social security tax revenues in

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[2] Peters (1995) includes education in the household's utility function as the altruistic behavior and shows that subsidies which ease the financial burden of a child's education lead to a higher average level of productive skills and thus enhance national income.

each period.

The household determines the levels of consumption in each period and the number of children to maximize its utility, given the average number of children, $\bar{n}$, of the other households in the economy. In addition, we assume that the household is aware of the effects of aggregate fertility on its own budget constraint, and takes its effect into account when making decisions: the more children other households have, the more social security benefits after retirement each household expects to get. Assuming tentatively that the working generation consists of m symmetric households, the budget constraint of each household is given by

$$zn + c_1 + \frac{c_2}{1+r} = w - p + \frac{(m-1)\bar{n} + n}{(1+r)m} p \qquad (2)$$

The last term on the right hand side is the present discount value of the social security tax received after retirement, with $[(m-1)\bar{n} + m]/m$ being the average number of children per household given the average number of children of the other households.

If the number of households, m, is very large, the budget constraints (2) can be condensed to

$$zn + c_1 + \frac{c_2}{1+r} = w - p + \frac{p}{1+r}\bar{n}. \qquad (3)$$

This budget constraint confirms that the average number of children of other households affects each household's budget: other households having more children increases each household's lifetime disposable income through the social security tax. In this sense children are "public goods" that have positive externalities, as already pointed out by many researchers including Nishimura and Zhang (1992), Cigno (1993) and Folbre (1994).

The optimal number of children for each household, given the number of children of other households, is given by

$$n = \frac{a}{z}\left[w - p + \frac{p}{1+r}\bar{n} \right],$$

which maximizes utility (2) under the budget constraint (3). Because the households are symmetric, the average number of children per household in this economy is given by a Nash equilibrium such that

$$n^* = \frac{a(1+r)(w-p)}{(1+r)z - ap} \tag{4}$$

which is obtained assuming that $n = \bar{n}$ and that $(1+r)z > ap$ to ensure that the number of children is positive. We can easily show from (4) that higher social security coverage will reduce fertility as far as $(1+r)z > aw$, in line with the results of an empirical analysis by Cigno and Rosati (1996) and Cigno, Casolaro and Rosati (2000).

The conventional theory of public goods tells us that a socially optimal level of a public good cannot be achieved by private optimization due to the "free rider" or "moral hazard" problem. This holds in the case of fertility, too. The socially optimal number of children, denoted as n^{**}, given the social security tax, is obtained to maximize (1) under the social budget constraint:

$$\left(z - \frac{p}{1+r} \right)n + c_1 + \frac{c_2}{1+r} = w - p,$$

where we assume that $\bar{n} = n$ in (3). Simple calculations show that the socially optimal number of children is equal to

$$n^{**} = \frac{a(1+r)(w-p)}{(1+r)z - p}, \tag{5}$$

assuming that $(1+r)z > p$. Comparing (4) and (5), it is clear that the privately optimal number of children, n^*, is smaller than the socially optimal one, n^{**}, insofar as there is a social security system. If there is a social security system, people have weaker incentives

to bear children than otherwise, because they can rely on social security benefits, which are financed by other people's children. Indeed, we can easily show that a higher social security tax widens the gap between privately and socially optimal numbers of children.

Now, what does the socially optimal policy that addresses this public-good nature of fertility look like? The first-best solution seems to be to remove the social security system, that is, make p equal to zero. This is true within the assumptions of our model, which offers no rationale for introducing a social security system in the first place. However, there can be several arguments in favor of a PAYGO system: neglecting purely distributional objectives, important candidates are insurance against not having children (Sinn (1998)), intergenerational risk-sharing (Smith (1982)), imperfect capital markets, and guaranteeing a "minimum standard of living" for the elderly. However, whatever the rationale may be, removing (or reducing) a social security system cannot be Pareto-improving, once it has been introduced in a small open economy and the government has net pension liabilities. If the government removes a social security system, the current younger and future generations have to pay additional taxes to compensate for what the government has already promised to pay to those who have paid social security taxes.

We can easily confirm that downsizing a social security system cannot be Pareto-improving in our simple model. Assume that the government plans to reduce the social security system by f x100% ($0 \leq f \leq 1$). Then, the government has to cover f x100% of the inherited obligation to existing retirees, that is, $fn*p$. The government compensates for this by issuing public debt, which has to be redeemed by the current younger and future generations. If the government imposes a lump-sum tax, t, on these generations when they are young, the budget constraint for each household in those generations becomes

$$zn + c_1 + \frac{c_2}{1+r} = w - t - (1-\boldsymbol{f})p + \frac{(1-\boldsymbol{f})p}{1+r}\,\overline{n}, \qquad (3)'$$

so that the privately optimal number of children, $n^{*'}$, is given in a Nash equilibrium by

$$n^{*'} = \frac{\boldsymbol{a}(1+r)[w - t - (1-\boldsymbol{f})p]}{(1+r)z - \boldsymbol{a}(1-\boldsymbol{f})p}. \qquad (6)$$

Meanwhile, the government has to make the present value of tax revenues equal to the inherited obligation to the current older generation, that is,

$$n^*\left[1 + \frac{n^{*'}}{1+r} + \left(\frac{n^{*'}}{1+r}\right)^2 + \cdots\right]t = \boldsymbol{f}n^* p \quad \text{or} \quad t = \frac{(1+r-n^{*'})\boldsymbol{f}p}{1+r}. \qquad (7)$$

Combining (6) and (7), we get

$$n^{*'} = \frac{\boldsymbol{a}(w-p)(1+r)}{(1+r)z - \boldsymbol{a}p} = n^*, \quad t = \frac{(1+r)z - \boldsymbol{a}w}{(1+r)z - \boldsymbol{a}p}\boldsymbol{f}p. \qquad (8)$$

Hence, the number of children remains unchanged after downsizing a social security system. We can also show that levels of consumption in both periods remain the same as those under a social security system due to the unchanged budget constraint, indicating that downsizing a social security system cannot affect the household's behavior and utility.

The second-best solution for the government appears to be to encourage the households to bear children by giving them a child allowance, so as to raise fertility to its socially optimal level. Assume that the government provides a lump-sum allowance, s, per child, and finances it with a lump-sum tax levied on each household to make allowance payments and tax revenues balanced in each generation. This child allowance scheme just provides for income transfers within the same generation and does not affect the government's budget constraint on net income. The household's new budget constraint is given by

$$(z-s)n + c_1 + \frac{c_2}{1+r} = w - p + \left(\frac{p}{1+r} - s\right)\bar{n}. \qquad (3)"$$

The third term of the right hand side shows that the average number of children has both positive and negative externalities: it raises disposable lifetime income via a social security system and at the same time lowers it via a child allowance scheme. Hence, intuition tells us that the optimal policy should make these opposing externalities cancel out each other, as shown below.

The optimal number of children for each household, with the number of children of other households taken as given, is calculated in the same way as (4) in the Nash equilibrium:

$$n^* = \frac{\boldsymbol{a}(1+r)(w-p)}{(1+r)z - (1-\boldsymbol{a})(1+r)s - \boldsymbol{a}p}, \qquad (9)$$

where we assume a positive denominator—that is, $z > (1-\boldsymbol{a})s + \boldsymbol{a}p/(1+r)$—meaning that the net childcare cost, which is defined as gross childcare cost minus weighted average of a child allowance and social security tax, is positive. To make the privately optimal level of fertility equal to the socially optimal level, which is given by (5), the government has to set

$$s = \frac{p}{1+r}. \qquad (10)$$

This result is exactly the same as that obtained by Groezen *et al.* (2003), who correctly explain that the optimal subsidy to (tax on) the parents is equal to the present value of a child's contribution (costs) to the intergenerational redistribution scheme during his working life. We can reach this conclusion even if we start with a situation where we have a child allowance and no social security system: to offset the externality of a child allowance, the second-best policy is to introduce a social security system that satisfies (10).

The optimal level of a child allowance can be obtained in another way: first, the

government allows each household to choose its own optimal level of children and consumption in both periods (given the social security and child allowance schemes); then, the government searches for the optimal level of child allowance that maximizes the household's utility. This is similar to the mechanism presented by Cigno, Luporini and Pettini (2000). They model the relationship between the government and the parents as a "principal-agent" relationship, and assume that the government (as principal) aims to maximize social welfare taking decisions by parents (as agents) as given[3]. In our model, the optimal levels of consumption for the household are calculated as

$$c_1{}^* = \frac{\boldsymbol{b}(1+r)(w-p)(z-s)}{(1+r)z - (1-\boldsymbol{a})(1+r)s - \boldsymbol{a}p}, \quad c_2{}^* = \frac{\boldsymbol{g}(1+r)^2(w-p)(z-s)}{(1+r)z - (1-\boldsymbol{a})(1+r)s - \boldsymbol{a}p}. \qquad (11)$$

Plugging (9) and (11) into (1) yields the household's indirect utility function, denoted as v, which is given by

$$
\begin{aligned}
v = \ln(w-p) + (1-\boldsymbol{a})\ln(z-s) + (1+\boldsymbol{g})\ln(1+r) \\
- \ln[(1+r)z - (1-\boldsymbol{a})(1+r)s - \boldsymbol{a}p] + const.
\end{aligned}
\qquad (12)
$$

The government aims to choose the level of child allowance that maximizes this privately maximized utility. Differentiating (12) with respect to s yields:

$$\frac{\partial v}{\partial s} = \frac{(1-\boldsymbol{a})\boldsymbol{a}[p - (1+r)s]}{(z-s)[(1+r)z - (1-\boldsymbol{a})(1+r)s - \boldsymbol{a}p]}, \qquad (13)$$

which becomes zero when $s = p/(1+r)$, confirming that condition (10) maximizes social welfare[4].

Finally, we can check whether or not raising a social security tax can improve welfare in this model. Starting with no child allowance (s=0), differentiating (12) with respect to p

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[3] It should be noted that Cigno *et al.* added an extra dimension to the problem by recognizing that some parental actions are not observable, which is not covered by our paper.

[4] It can be easily shown that the second-order conditions are satisfied. Also note that a child allowance cannot be justified if there is no social security system.

and using (9), we get

$$\frac{\partial v}{\partial p} = \frac{aw - (1+r)z}{(w-p)[(1+r)z - ap]} = \frac{n^* - (1+r)}{(1+r)(w-p)}. \tag{14}$$

This implies that raising a social security tax cannot be Pareto-improving if $n^* < 1+r$, which is likely to be the case under an aging population. In the same way, we can show that raising a social security tax lowers fertility if $n^* < 1+r$ due to a reduction in the household's net income. This is the well-known conclusion obtained from a conventional life-cycle model.

2.2 A closed economy with no altruism

The discussions in the previous section indicate that in a small, open economy with no altruism, i) removing (reducing) a social security system with a debt policy that compensates for the inherited obligation cannot be Pareto-improving nor affect fertility, ii) a child allowance can internalize the externality of a social security system and enhance social welfare by raising fertility, and iii) raising the social security tax reduces both social welfare and fertility in an aging society.

In a closed economy, however, the story appears to become more complicated because capital accumulation is endogenous. For instance, a child allowance, which aims to encourage households to bear children, could reduce their savings due to the increased relative price of future consumption. Lower household savings decelerate capital accumulation, and eventually subdue steady-state levels of lifetime income and utility. This section assesses the impact of a social security reform and child allowance scheme in a closed economy, where capital accumulates through household savings and affects household income, while keeping the assumption of no altruism intact.

We continue to assume that the household takes wage rate, interest rate, and

average number of children of other households as given and maximizes its utility with no bequests.   Then, the privately optimal levels of children and consumption are determined in the same way as given by (9) and (11), respectively.   In the steady state and the Nash equilibrium (with $\bar{n} = n$), per-capita capital stock, k, is given by

$$k = \frac{w - p - ns - (z - s)n - c_1}{n} = \frac{w - p - c_1}{n} - z = \frac{g(z - s)}{a} - \frac{p}{1 + r} \tag{15}$$

using (9) and (11) for rearrangement and assuming that capital stock is fully depreciated within one generation.

As usual, firms act competitively, hiring labor to the point where the marginal product of labor is equal to the wage, and renting capital to the point where the marginal product of capital is equal to the interest rate.   Also, if the production function displays constant returns to scale with respect to capital and labor, we know

$$w = f(k) - kf'(k), \ r = f'(k), \ f'(k) > 0, \ f''(k) < 0. \tag{16}$$

In addition, if the initial level of per capita capital stock is so low that the interest rate exceeds the rate of population growth[5]—which looks plausible under an aging population— then we can show the result through simple and tedious calculations:

$$\frac{dv}{dk} > \frac{ag[s - p/(1+r)]r'}{(1+r)z - (1-a)(1+r)s - ap} \geq 0, \tag{17}$$

where the second inequality holds as far as $s \leq p/(1+r)$ [see **Appendix A** for proof].   In this case, capital accumulation is desirable because it adds to social welfare.

Now, let us investigate whether or not downsizing a social security system can be Pareto-improving.   As in the case of a small open economy, the government plans to reduce social security tax and compensate for inherited pension obligations by issuing

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[5] This condition is expressed as: $z > (1-a)s + aw/(1+r)$, which assures that $n^* < 1 + r$ in (9) and also that the denominator in (17) is positive (assuming that $p < w$).

public debt, which has to be redeemed by a lump sum tax levied on the current younger and future generations.   Then, we get the same solutions for the number of children and the size of the lump sum tax as given by (8), taking the wage rate and interest rate as given, and we can also show that levels of consumption in both periods are unaffected by the reform, given the levels of capital stock and, correspondingly of wage and interest rates.

What would happen if downsizing a social security system affects capital accumulation?   If the social security tax is cut by fx100%, the current younger and future generations have to pay the newly introduced tax, t, when young.   Importantly, this new tax is smaller than the reduction in the social security tax as far as $p<w$, because from (8) we get

$$t - fp = -\frac{a(w-p)fp}{(1+r)z - ap} < 0.$$

Hence, they can save more when young than under the initial social security system and thus accelerate capital accumulation.   Indeed, when the social security tax is cut by fx100%, the new steady-state level of per-capita capital is calculated as

$$k = \frac{gz}{a} - \frac{(1-f)p}{1+r} > \frac{gz}{a} - \frac{p}{1+r},$$

where the right hand side indicates the level of per-capita capital under the initial social security system (assuming no child allowance ($s=0$) in (15)).   Hence, reducing the social security tax can be Pareto-improving by encouraging capital accumulation even if its direct impact on utility is fully offset by the new tax, in contrast to the case of a small, open economy[6].   In addition, downsizing a social security system raises fertility, because accelerated capital accumulation increases the household's disposable income, thereby

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[6] Implicitly assuming a small open economy, Feldstein (1995) (1998) showed that shifting to an investment-based (funded) system raises efficiency if three conditions are met: the marginal product of capital exceeds the implicit return in a social security system, and the marginal product of capital exceeds the rate at which future consumption is discounted, and the economy is growing.

enabling the household to spend more on childcare.

What affect has a child allowance? In a small open economy, fertility has only a positive externality and public intervention that internalizes it can be justified. In a closed economy, however, the outcome becomes ambiguous. From (15) and (16), introducing a child allowance tends to reduce the rate of capital accumulation, since

$$\frac{dk}{ds} = -\frac{g}{a}\left[1 - \frac{pr'}{(1+r)^2}\right]^{-1} < 0,\tag{18}$$

where $r' = dr/dk < 0$. A higher child allowance subdues household savings and decelerates capital accumulation. In addition, differentiating the household's indirect utility function, (12), with respect to s yields

$$\frac{dv}{ds} = \frac{\partial v}{\partial s} + \frac{dv}{dk}\frac{dk}{ds},\tag{19}$$

where the first term of the right hand side indicates the direct impact of a child allowance assuming no change in capital accumulation and the second term indicates the indirect impact through capital accumulation. The first term is zero when $s = p/(1+r)$ and positive when s is smaller than $p/(1+r)$, as seen from (13) in a small open economy, whereas the second term is negative as far as $s \leq p/(1+r)$ from (17) and (18).

Hence, the child allowance scheme that satisfies $s = p/(1+r)$ can no longer be the second-best policy in a closed economy, unlike the case of a small open economy: the optimal level of a child allowance should be lower than $p/(1+r)$. Furthermore, we cannot rule out the possibility that introducing a child allowance *reduces* social welfare. The negative impact of a childcare allowance on capital accumulation could dominate its total impact on the household's utility, by more than offsetting its direct impact on the household's utility. Section 3 examines this possibility, based on numerical simulations. In addition, regarding the impact of a child allowance on fertility, a reduction of the net childcare cost *per*

se encourages the household to bear children, but the negative income effect due to decelerated capital accumulation is expected to at least partially offset that effect.

Finally, we can show that in this closed economy with no altruism, raising a social security tax cannot be justified, because from (15) and (16) we get

$$
\frac{dk}{dp} = -\frac{g}{a} \left[1 - \frac{pr'}{(1+r)^2} \right] < 0,
$$

which indicates that raising a social security tax decelerates capital accumulation and thus subdues social welfare. Combined with the direct, negative impact given by (14), this confirms that raising social security tax reduces social welfare. The negative income effect is also likely to reduce fertility. It should be noted, however, that these results depend crucially on the assumption that households care only about the quantity of children. If they care also about the welfare (quality) of children or that they can provide for old age through intra-family arrangements, then the effect on capital accumulation will be ambiguous. In the subsequent two sections, we introduce altruistic bequests into the model and analyze how they affect the conclusions.

2.3 A small open economy with altruism

Assume that households care about the welfare of their future generations, as well as themselves. The well-known "Ricardian Equivalence Theorem" argues that individuals offset the change in public income transfers such as net contributions of social security by an offsetting change in altruistic bequests, leaving net transfers between generations unaffected. Does this argument still hold in this model with endogenous fertility? And, is introducing a child allowance still the second-best policy under a social security system?

We now change the household's utility function by introducing altruism, that is, assuming that the household cares about its children's welfare by weighting the children's

utility in its own utility function, and denote the utility function of the altruistic household as U.
To make the calculations simple, set up U as

$$U = a \ln n + b \ln c_1 + g \ln c_2 + \frac{n^e}{1+d} U_{+1},$$
$$a + b + g = 1, \quad 0 < a, b, g < 1, \quad 0 \le e, d,$$

where U_{+1} is the per-capita utility of the children, e indicates the elasticity of "taste" for

children with respect to the number of children, and d is the rate to discount the children's

utility.   The number of children affects the household's utility in two ways: directly (the first

term) and indirectly by multiplying the child's utility by n^e (the fourth term).   This setup of

the household's utility function, which is a simplified version of Barro and Becker (1988), can

be easily compared to the model with no altruism, (1).

First, let us consider the case of a small open economy, where wage rate and interest

rate are exogenously given, and capital accumulation is neglected.   To make the system

dynamically stable, we have to assume that $n^e < 1+d$ hereafter (this condition can be

transformed to $(1+r)^e < 1+d$, as discussed below).   In addition, assume that the

household receives bequests from its parents and leaves bequests to its children during the

working period, so the budget constraint in each period is expressed as

$$c_1 = w + (1+r_-)b_- - (z - s + b)n - p - s\overline{n} - x, \tag{20-1}$$

$$c_2 = (1+r)x + p\overline{n}, \tag{20-2}$$

where b is per-capita bequests left to the children, b_- is bequests left by the parents, and x is

savings.   Bequests, like savings, bear the interest rate, r, and r_- denotes the interest rate on

bequests received from the parent generation.   Each household maximizes its utility, U,

under the above budget constraints, giving wage rate, interest rate, and number of children

of other households.   For simplicity we neglect corner solutions, assuming that households

are allowed to leave negative bequests (as well as make negative savings).

Differentiating U with respect to x, b, and n gives

$$\frac{\partial U}{\partial x} = -\frac{\boldsymbol{b}}{c_1} + \frac{\boldsymbol{g}(1+r)}{c_2} = 0, \tag{21-1}$$

$$\frac{\partial U}{\partial b} = -\frac{\boldsymbol{b}n}{c_1} + \frac{n^e}{1+\boldsymbol{d}}\frac{\boldsymbol{b}(1+r)}{c_{1,+1}} = 0, \tag{21-2}$$

$$\frac{\partial U}{\partial n} = \frac{\boldsymbol{a}}{n} - \frac{\boldsymbol{b}(z-s+b)}{c_1} + \frac{\boldsymbol{e}n^{e-1}}{1+\boldsymbol{d}}U_{+1} = 0, \tag{21-3}$$

where $c_{1,+1}$ is the children's consumption during the working period.   Let us concentrate on a Nash equilibrium in a steady state, where $\bar{n} = n$, $b_{-1}=b$, $r_{-1}=r$, $c_1=c_{1,+1}$, and $c_1=c_{2,+1}$.   Then, it follows from (21-2) that

$$n^* = \left(\frac{1+r}{1+\boldsymbol{d}}\right)^{\frac{1}{1-e}}, \tag{22}$$

which means that in an open economy the number of children depends solely on the (world's) interest rate and the degree of altruism concerning children, as already pointed out in Becker and Barro (1988).   If there is a perfect arbitrage between human capital and non-human capital, the interest rate can be interpreted as a rate of return on human capital. Consequently, it makes sense that a higher interest rate encourages the household to increase the number of children.   The net cost of childcare, which reflects social security and childcare support, is neutral to fertility due to the household's altruistic income transfer. Moreover, if (22) holds, the assumption that $n^e < 1+\boldsymbol{d}$, which makes the system dynamically stable, is transformed into the inequality $(1+r)^e < 1+\boldsymbol{d}$, which in turn means $n < 1+r$.

In a Nash equilibrium in a steady state, the budget constraints of the household, (20-1) and (20-2), are aggregated with some rearrangement into

$$c_1 + \frac{c_2}{1+r} + zn = w + (1+r-n)\left(b - \frac{p}{1+r}\right). \tag{20'}$$

This budget constraint (20)', together with the first-order conditions (21-1), (21-3), and (22), yields privately optimal solutions for c_1, c_2, and b, given the social security tax and child allowance, as well as the number of children, which is independently determined by (22).

Now, let us discuss the impact of social security reform and childcare support on social welfare. First, consider downsizing a social security system. Assuming again that the government cuts the social security tax by f x100% and imposes a lump sum tax, t, on the current younger and future generations to finance the inherited obligation, the household's budget constraint is revised to

$$c_1 = w + (1 + r_-)b_- - (z + b)n - (1 - f)p - t - x, \qquad (20\text{-}1)'$$

$$c_2 = (1 + r)x + (1 - f)pn, \qquad (20\text{-}2)'$$

which makes the marginal conditions (21) unchanged (assuming $s=0$), thus the number of children remains the same as in (22). Then, the level of a lump sum tax, t, is given as $(1 + r - n^*)fp/(1 + r)$, as in (7), thus the lifetime budget constraint also remains the same as in (20)'. Accordingly, downsizing a social security system cannot be Pareto-improving.

Instead, there is a possibility that raising a social security tax is Pareto-improving. To illustrate this, we take the special case of $e = 0$, where (21-3) becomes

$$c_1^* = \frac{bn^*(1 + r - n^*)}{a(1 + r) - n^*}\left[\frac{(1 + r)z - w}{1 + r - n^*} + \left(\frac{p}{1 + r} - s\right)\right], \qquad (23)$$

using (20)' and (21-1). Differentiating (23) with respect to p, we have

$$\frac{dc_1^*}{dp} = \frac{bn^*(1 + r - n^*)}{(1 + r)[a(1 + r) - n^*]} \qquad (24)$$

which becomes positive if $n^* < a(1 + r)$, that is, $a(1 + d) > 1$, from (22) assuming that $e = 0$. Considering that social welfare is determined solely by c_1, expanded social security can raise the level of the household's consumption, and thus utility, if fertility is initially quite

"low" relative to the interest rate[7].

The intuitive reason for this result is that a higher social security tax raises the net cost of childcare under an aging population with $n^* < 1 + r$, which in turn encourages the household to endow future generations with a higher level of consumption and utility by increasing bequests. Indeed, we can confirm that a higher social security tax can lead to higher bequests, because we get from (20)', (21-1)', and (23)

$$\frac{db^*}{dp} = \frac{a(1+r-n^*)}{(1+r)[a(1+r)-n^*]},$$

which becomes positive if $n^* < a(1+r)$. It should be noted, however, that if the initial number of children is not so small—that is, $a(1+r) < n^* < 1+r$—then raising the social security tax cannot be justified because its negative income effect dominates the overall impact on consumption and utility under an aging population. The effectiveness of social security reforms thus relies on the initial level of fertility.

In the same manner, we can show the possibility that introducing a childcare allowance reduces social welfare. In the special case of $e = 0$, we get

$$\frac{dc_1^*}{ds} = -\frac{bn^*(1+r-n^*)}{a(1+r)-n^*}, \quad \frac{db^*}{ds} = -\frac{(1-a)n^*}{a(1+r)-n^*}.$$

Thus, if the initial level of fertility is so low that $n^* < a(1+r)$, introducing a childcare allowance reduces the net cost of childcare, which makes the household endow future generations with a lower level of consumption and utility by reducing bequests. If $n^* > a(1+r)$, we will have the reversed results. In what follows, let us assume that $e = 0$ for simplicity, because key results remain the same for a positive e.

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[7] Note here that the assessment of the level of initial fertility depends on the value of a : the more importance the household puts on the number of children in its utility function, the more the household is likely to consider that fertility is low. This seems to intuitively make sense.

2.4 A closed economy with altruism

Finally, let us assume the household's altruistic behavior in a closed economy. The household's budget constraints (20) and the first-order conditions (21) remain the same as in the case of a small, open economy. We now take into account capital accumulation, where per-capita capital stock is determined by savings and bequests such that

$$k = \frac{w+(1+r)b-p-ns-(z-s+b)n-c_1}{n} + b = \frac{w+(1+r)b-p-nz-c_1}{n}$$

in the steady state.

As discussed in the case of a small open economy, downsizing a social security system does not affect the household's decisions on consumption, fertility, and bequests, given per-capita capital, because both the marginal conditions for utility maximization and budget constraint remain the same. However, this reform will accelerate capital accumulation, because the household can save more when young than before the reform. Assuming that wage and interest rate are held constant, the household can make additional savings, which are equal to the difference between the existing social security tax and the new tax. An increase in per capita capital stock leads to a reduction in the interest rate, which in turn reduces the number of children because of (22). Regarding the impact on social welfare, accelerated capital accumulation is likely to add to the household's disposable income and utility, but that positive effect on social welfare is expected to be at least partially offset by the negative effect of a reduction in the number of children. Consequently, it is difficult to algebraically assess the overall net impact, so we estimate it based on the numerical calculations in the next section.

As in the case of a small open economy, there is a possibility that raising a social security tax raises social welfare and reduces fertility, if we start with low fertility. **Appendix**

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B shows that if $n^* < a(1+r)$, we can see the positive impact of SS on capital accumulation in a closed economy, which was pointed out by Barro and Becker (1989). This result contradicts the conventional view that a PAYGO social security system decelerates capital accumulation. In our model, a higher social security tax leads to more altruistic bequests, which more than offset a reduction in household savings due to an increase in social security tax payments, therefore, it reduces the number of children. If we start with high fertility so that $n^* > a(1+r)$, however, such is not the case and we cannot rule out the possibility that a higher social security tax subdues capital accumulation, in line with conventional life-cycle models with no altruism. In addition, we cannot know *a priori* the overall net effect of a higher social security tax on social welfare in both cases, because capital accumulation and fertility affect the household utility in opposite ways.

The effect of introducing a child allowance also depends on the initial level of fertility. Appendix B shows that if $n^* < a(1+r)$, a child allowance decelerates capital accumulation, and thus raises the interest rate. If $n^* > a(1+r)$, we will have ambiguous results. It should also be noted that if we start with high fertility, a higher social security tax and a higher child allowance have opposing effects on capital accumulation and fertility. As in the case of social security reform, the overall net impact on social welfare is indeterminate.

2.5 Summary

Table 1 summarizes the expected impacts of each policy reform under an aging population, where "Child allowance," "SS down," and "SS up" mean introducing a child allowance, and downsizing and expanding a social security system, respectively. The key results are summarized in what follows.

First, in a small open economy with no altruism, reducing a social security system with a debt policy that compensates for existing net liabilities is neutral to both social welfare

and fertility.   By contrast, introducing a child allowance can internalize the externality of a social security system and enhance social welfare by raising fertility.   A rise in a social security tax cannot be justified under an aging society, because it reduces net lifetime income and social welfare.   Its negative income effect also reduces fertility.

Second, however, we have less obvious results of policy reforms, once we assume a closed economy where capital accumulation is endogenous.   Reducing a social security system can be Pareto-improving, even if it requires a lump-sum tax to compensate for the inherited obligation, because it allows households to save more and accelerates capital accumulation.   Its positive income effect is also expected to raise fertility.   On the other hand, the positive effect of a child allowance on social welfare will be at least partially offset by its negative impact on the level of per-capita capital stock.   The level of a child allowance should be set at a lower level than in the case of a small open economy, or it should not be introduced at all.   Even the possibility that a child allowance reduces fertility cannot be ruled out.   A rise in a social security tax fails to raise both social welfare and fertility, as in the case of a small open economy.

Third, when we introduce altruism into the model, the policy impact of each policy will substantially change, reflecting altruistic private transfers, which tend to offset intergenerational transfers caused by social policies.   In a small open economy with altruism, neither changing the social security tax nor introducing a child allowance can affect fertility, which is determined solely by (world's) interest rate and degree of altruism towards children.   Regarding the impact on social welfare, reducing a social security tax cannot be Pareto-improving as in the case of no altruism.

In the models of altruism, we have to take another factor into account: the initial level of children.   Starting with relatively low fertility, raising a social security tax improves social welfare, because it encourages the household to increase altruistic bequests and eventually

raise steady-state levels of consumption.   A child allowance also lowers social welfare,

because it makes the household endow future generations with less consumption and utility

by reducing bequests.   By contrast, when starting with relatively high fertility we are likely to

have reversed results, because the income effect of policy reform tends to dominate.

Fourth, in a closed economy with altruism, we have less clear results.   Downsizing a

social security system, even accompanied by a lump-sum tax to compensate for the

inherited obligation, accelerates capital accumulation via more household savings.   This

effect leads to lower fertility, which by itself reduces the household's utility and makes the

overall net effect on social welfare mixed.   The results of raising a social security tax and of

introducing a child allowance depend on the initial level of fertility.   Starting with low fertility,

a higher social security tax accelerates capital accumulation and reduces fertility, and a

higher child allowance decelerates capital accumulation and raises fertility.   Starting with

higher fertility, we may have the results reversed.   In addition, we can say little about what

the net effect on social welfare looks like in any case.

3. Simulations

3.1 Assumptions

In this section, we illustrate the impact of social security reform and child support on social

welfare and fertility for two models (a model with no altruism and a model with altruism)

under two economies (an open economy and a closed economy), starting with the initial

situation in which a PAYGO social security system has already been incorporated.   In

addition, we compare two cases that start with different levels of fertility in the case of

models with altruism.   The discussions in Section 2 imply that the policy results depend

crucially on the setup of the model and the economy, as well as the initial level of fertility.

We first set up the initial condition by providing parameters with tentative plausible values, as summarized in Table 2. For the utility function of a Cobb-Douglas type, we take two sets of weights: (A) $a = 2/3$, $b = g = 1/6$ and (B) $a = b = g = 1/3$, with the former (latter) putting a higher (lower) weight on the number of children. The production function is $y = k^q$, with the share of capital income, q, being equal to 1/3. The lump-sum social security tax is set at 20% of the initial wage, and a child allowance is not introduced initially. For models with altruism, we set the elasticity of "taste" for children with respect to the number of children, e, equal to 0 to make the simulations consistent with the discussions in sections 2.3 and 2.4. And, we assume that discount rate, d, to discount the child's utility is equal to one. Then, gross cost of childcare, z, is left fixed. We first set the initial value of n as 0.65, reflecting the fact that the total fertility rate, which is equivalent to $2n$ in our model, is now approaching 1.3 in Japan. Next, we run simulations and search for the value of z that makes n equal to 0.65. The value of z depends on the set of parameters a, b, and g ? that is, (A) or (B)? as well as whether there is altruism, as reported in terms of the ratio to the initial wage rate in Table 2[8].

For the models with altruism, we have to characterize the initial level of fertility. Because we assume that $e = 0$, $d = 1$, and $n_0=0.65$, we have $1 + r_0 = n_0(1 + d) = 0.65 x (1 + 1) = 1.3$ from (22). Hence, the initial level of fertility is characterized as "low" in case (A), because $n_0 - a(1 + r_0) = 0.65 - 2/3 x 1.3 \cong -0.217 < 0$. By contrast, the initial fertility is characterized as "high" in case (B), because $n_0 - a(1 + r_0) = 0.65 - 1/3 x 1.3 \cong 0.217 > 0$, even though the initial level of fertility is 0.65 in both cases. This difference matters when assessing the policy impact, as implied in the

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[8] It may be more realistic to assume that childcare cost is an increasing function of wage rate. But, we fix its value, because we are interested in the basic feature of our model and how it is affected by policy changes and assumptions.

discussions in Section 2.　In addition, the initial values for endogenous variables, which are to be solved by simulations based on the above-mentioned parameters, are reported in Tables 3 and 4 (see below).

3.2 Steady state comparisons

We concentrate on steady-state comparisons before and after reforms, neglecting the transition process following them[9].　In each model, we consider three policy options: (i) introducing a child allowance, which is equivalent to 1% of the initial wage rate, (ii) reducing social security tax by 1% of the initial wage; and, (iii) raising social security tax by 1% of the initial wage rate.　We make these three policy options correspond to "Child allowance," "SS down," and "SS up," respectively.

Tables 3 and 4 summarize the simulation results, with the former for models with no altruism and the latter for those with altruism.　Each table reports two cases, (A) and (B), of different sets of $a, b,$ and g.　To begin with, let us look at the results for the most basic model (small open and no altruism) in Table 3.　We find that, in both cases of (A) and (B), a child allowance adds to the household's utility, as it increases the number of children and mitigates the negative impact of it not existing, in line with the argument presented in 2.2. However, the improvement of utility looks quite limited, because lowered consumption levels due to the substitution effect largely offset the positive effect of increased fertility[10]. Reducing a social security tax has no impact on social welfare and utility, because it does not affect the household's disposable income due to the new tax.　Finally, raising a social

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[9]　See Oshio (2002) regarding the dynamic process after social security reform and introduction of a child allowance.

[10]　The optimal level of the child allowance (not reported in the table), which corresponds to the second-best condition of $s = p/(1+r)$, is calculated to be about 4.9% and 8.3% of the initial wage rate, in the case of (A) and (B), respectively.

security tax reduces social welfare and fertility, because it reduces the household disposable

income under $n < 1 + r$. These results are all consistent with the arguments in the Section

2.3.

A closed economy with capital accumulation presents a quite different picture of

policy results, even with the assumptions of no altruism unchanged. First, a child

allowance reduces both social welfare and fertility in case (A), while it raises fertility, but

reduces social welfare in case (B). As suggested by the discussions in 2.4, the explanation

lies with capital accumulation: per-capita capital stock drops due to reduced household

savings, which in turn lowers the household's income. This negative effect of a child

allowance on the household income partly offsets its direct positive impact on fertility in case

(B) and more than offsets it in case (A). By contrast, curtailing a social security tax raises

social welfare, mainly because it stimulates the pace of capital accumulation, even with the

additional tax to compensate net pension liabilities. In this case, fertility also recovers due

to an increase in the household's income, which in turn helps utility to recover. Finally, a

higher social security reduces both social welfare and fertility due to subdued capital

accumulation.

Next, let us move to Table 4, in which altruistic bequests are incorporated. In this

table, we can compare two cases that start with low and high levels of fertility. We first

confirm that in a small open economy a policy option cannot affect fertility, which remains

unchanged from the initial level. A lower social security tax also does not affect social

welfare, because its positive impact on capital accumulation is neglected in a small open

economy and bequests remain unchanged due to no change in the budget constraint.

The impact of introducing a child allowance and raising a social security tax

depends much on initial fertility. Starting with low fertility, a child allowance reduces social

welfare because it lowers altruistic bequests, and thus consumption. Starting with high

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fertility, by contrast, a child allowance raises levels of consumption and social welfare, as the lowered cost of childcare raises the household's disposable income and leaves it more room to increase consumption and bequests. Raising a social security tax, which adds to net childcare costs, affects social welfare in the opposite way. It raises altruistic bequests and social welfare if we start with a low level of fertility, while it leads to the opposite results if we start with a high level of fertility.

In a closed economy with altruism, downsizing a social security system reduces fertility regardless the initial level of fertility, because it accelerates capital accumulation and reduces interest rate. We cannot tell *a priori* whether or not the net impact of downsizing a social security system on social welfare will be positive or negative, because lowered fertility and raised consumption tend to offset each other. Our simulations show, however, that it will be negative if we start with low fertility and that it will be positive with high fertility. Introducing a child allowance and raising a social security tax affect social welfare and fertility in completely opposite ways, and the initial level of fertility reverses their impacts in each policy. Starting with low fertility, introducing a child allowance lowers social welfare and raises fertility, while raising a social security tax raises social welfare and lowers fertility. And, raising a social security tax leads to the opposite results. Starting with high fertility, we have the reverse outcome.

4. Concluding Remarks

We have investigated the impact of social security reform and childcare support using a simple overlapping generations model with endogenous fertility. Our analysis shows that the policy impact on social welfare and fertility depends heavily on assumptions about the openness of the economy, altruism, and initial fertility. For example, introducing a child

allowance, which is often expected to mitigate demographic pressures on the social security system, could turn out to be ineffective in a closed economy, where capital accumulation is taken into account. Downsizing a social security system, which is often considered not to be Pareto-improving due to the additional tax to compensate for existing pension liabilities, could raise welfare by encouraging household savings in a closed economy. Furthermore, the initial level of fertility tends to make the outcome indeterminate in models with altruism: starting with high fertility, the income effect of the policy reform tends to more than offset private transfers through altruistic bequests which was encouraged by the reform. These results imply that we should have empirical knowledge about the openness of the economy, the degree of altruism, and the assessment of the initial fertility, when discussing policy measures to make the social security system more sustainable under an aging population.

Appendix A: proof for (17).

Differentiating (12) with respect to *k* yield

$$\frac{dv}{dk} = \frac{w'}{w-p} + \frac{(1+\boldsymbol{g})r'}{1+r} - \frac{[z-(1-\boldsymbol{a})s]r'}{(1+r)z-(1-\boldsymbol{a})(1+r)s-\boldsymbol{a}p},$$

where $w' = dw/dk > 0.$ Assume that $n^* < 1+r$, then from (9) we get $\boldsymbol{a}w < (1+r)[z-(1+r)s].$ Plug this into the above equation to get

$$\frac{dv}{dk} > \frac{\boldsymbol{a}w' - [z-(1-\boldsymbol{a})s]r'}{(1+r)z-(1-\boldsymbol{a})(1+r)s-\boldsymbol{a}p} + \frac{(1+\boldsymbol{g})r'}{1+r}.$$

Then rearranging this using (14) and taking the relation that $w' + kr' = 0,$ we get (17).

Appendix B Full impact of social security reform and a child allowance on capital accumulation in a closed economy with altruism.

Per-capita capital stock is given by

$$k = \frac{n* + g(1+r)}{a(1+r) - n*} z - \frac{1-b}{a(1+r) - n*} w + \frac{bn* + g(1+r)}{a(1+r) - n*}\left(\frac{p}{1+r} - s\right)$$

$$= \frac{1 + g(1+d)}{a(1+d) - 1} z - \frac{(1-b)(1+d)}{a(1+d) - 1}\frac{w}{1+r} + \frac{b + g(1+d)}{a(1+d) - 1}\left(\frac{p}{1+r} - s\right)$$

from (20)', (21-1), (22), (24), and (27), assuming that $e = 0$ in a closed economy with altruism. Differentiating this with respect to *s* and *p*, respectively, we get

$$\frac{dk}{dp} = \frac{[b + g(1+d)](1+r)}{[a(1+d) - 1](1+r)^2 + (1-b)(1+d)(1+r)w' + \{[b + g(1+d)]p - (1-b)(1+d)w\}r'},$$

$$\frac{dk}{ds} = -(1+r)\frac{dk}{dp}.$$

Now, if $a(1+d) > 1$ —that is, $n* < a(1+r)$ —then we know the sign of { } in the denominator of *dk/dp* as

$$[b + f(1+d)]p - (1-b)(1+d)w < [b + g(1+d) - (1-b)(1+d)]w = [b - a(1+d)]w < 0,$$

meaning that the denominator of *dk/dp* is positive. Thus, a higher social security tax accelerates capital accumulation, while a higher child allowance decelerates it. On the contrary, if $a(1+d) < 1$, the signs of *dk/dp* and *dk/ds* are indeterminate.

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