

**Construction of a Business Cycle Indicator in Japan:
A Dynamic Factor Model with Observable Regime Switch**

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1. Introduction

Since the burst of bubbles, Japanese economy has been stagnant and the last ten years was named 'lost decade.' It becomes extremely important under such circumstances for the Japanese economy to recognize the current business conditions in order to implement adequate economic policies. For this purpose, many efforts have been made by researchers to create business cycle indicators that can measure business conditions appropriately. Since 'the business condition' is equivocal, several indicators can coexist. As an example of such efforts, Fukuda and Onodera (2001) applied the dynamic factor model proposed by Stock and Watson (1989), (1991) to the Japanese economy and constructed the Nikkei Business Index. An extension of the dynamic factor model is to incorporate the regime switching mechanism into the model. It is based on the premise that the business cycle would behave differently in the phases of expansion and contraction. That is, there is an asymmetry in its behaviors. Such a model is extensively discussed in Kim and Nelson (1998). In Japan, applications of such a model have not appeared until quite recently, probably because of its complexity of calculation relative to new insights brought by the model (See Watanabe (2002)).

The primary purpose of this paper is to present a construction of a business cycle indicator retaining the advantages of the dynamic factor model with regime switch and improving some of its shortcomings. This paper does not intend to propose a theoretically sophisticated model, but to show an application of existing models by modifying them to meet practical demands. The approach taken here is not confined to the Japanese business cycle but commonly applicable to large number of countries. The basic idea is based on the following considerations.

The Cabinet Office (previously, the Economic Planning Agency) has been announcing the turning points of the Japanese business cycle taking various aspects of economic conditions into considerations. Apart from the precise timings of the turning points, there is little room to doubt the phases of the business cycle defined accordingly for most of the past period. In other words, there is consensus among people that business conditions in the past months would not need revisions, except in some months in the vicinity of the declared turning points. Such information of the business cycle should be made use of in the analysis. This means that it can be assumed that whether the business cycle was going up or down is known for most of the past period. In terms of econometrics, it implies that the regimes in the switching model are observable. This situation is different from those in the financial markets.

In financial markets, the markets' conditions are often classified into being bull or bear (See, e.g. Franses and van Dijk (2000), Perez-Quiros and Timmermann(2000)). Since such classification is rather subjective and ambiguous, it would be better to regard the phases as unobservable when the dynamic factor model is applied.

Based on such premises, by making use of the information of the phases, the appropriateness of the assumptions made in the model can be statistically investigated. For example, is it necessary to consider different phases in the business cycle movements and their asymmetric behaviors? What type of switching mechanism should be assumed for the phase shift?

The model proposed in this paper consists of two distinct component models. They are the dynamic factor model with two regimes and the model of the switch mechanism of the regimes. The dynamic factor model is a simplified version of Kim and Nelson's dynamic factor model with two regimes, where the regimes are assumed to be observable. The regime switching mechanism is separately estimated with logistic regression formulation, where the parameters in addition to the explanatory variables are considered to be time dependent. Within such a framework, a simple Markov switching mechanism employed in the current research is reexamined.

The construction of this paper is as follows. Section 2 explains the basic framework of the dynamic factor model with observable regimes. In Section 3, with a view to eliminating sensitivity of the model to the choice of the macro time series, a proposal is made to introduce a series of the business sentiments as one of the variables. Then the business cycle in Japan is estimated and the characteristics of the Japanese business cycle are examined. In Section 4, in order to present a particular application of the model explained in section 3, the recent announcement of the turning points in Japanese Business Cycle made by the Cabinet Office is examined in reference to the above-mentioned model. Section 5 is mainly concerned with the regime switch mechanism. The Markov property of the switching mechanism is reexamined and the switching probabilities are extended to be time dependent with time varying parameters. Section 6 briefly concludes this paper.

2. Dynamic Factor Model with Observable Regimes

Stock and Watson (1989) tried to construct a new business cycle indicator from multiple macro economic time series. The model used in Stock and Watson (1989) is generally called the Dynamic Factor Model. The formulation in this paper has a

slightly different form from theirs. For macro time series Y_{it} , let

$$\Delta Y_{it} = \gamma_{i1} \Delta C_t + \gamma_{i2} \Delta C_{t-1} + \gamma_{i3} \Delta C_{t-2} + u_{it} \quad (2.1),$$

where γ_{i1} , γ_{i2} , γ_{i3} are unknown parameters and u_{it} is a random error term. It is assumed that u_{it} 's are mutually, serially independent and normally distributed with mean 0 and variance σ^2 . (2.1) formulates how the economic time series are related to the business conditions and is called the system of observation equations. Stock and Watson (1991) considered a single factor ΔC_t on the right-hand side of (2.1), but assumed a moving average structure for the idiosyncratic error term u_{it} . (2.1) has introduced explicitly the lag structure for ΔC_t but, instead, u_{it} is simplified to be serially uncorrelated. It is necessary to confirm that the macro time series are not co-integrated before being handled. Otherwise, linear combinations of those macro time series (in their levels) would be involved on the right hand side of (2.1). In this paper, after having confirmed that the macro time series have unit roots and are not co-integrated¹, the first differences of the time series are used on the left-hand side of (2.1).

On the other hand, ΔC_t obeys the following state transition equations provided that the behavioral patterns are different in the two phases of the business cycle:

$$\begin{aligned} \Delta C_t &= \varphi_0 + \varphi_1 \Delta C_{t-1} + \varphi_2 \Delta C_{t-2} + \varphi_3 \Delta C_{t-3} + \varepsilon_t && \text{if } t \text{ is in an expansion period} \\ &= \varphi_0' + \varphi_1' \Delta C_{t-1} + \varphi_2' \Delta C_{t-2} + \varphi_3' \Delta C_{t-3} + \varepsilon_t' && \text{if } t \text{ is in a contraction period.} \end{aligned} \quad (2.2)$$

where ε_t and ε_t' are distributed as $N(0, \sigma^2)$ and $N(0, \sigma'^2)$, respectively. For identification purpose, σ^2 is set to one and $\sigma'^2 = (1+\eta)^2$ without loss of generality.

As was mentioned above, each period can be classified either as an expansion period or a contraction period according to the turning points announced by the Cabinet Office (previously by the Economic Planning Agency). Following this classification, it can be judged from the data if the business cycle takes different patterns in the two phases. If not, of course, there is no need to consider the switch between them. A dummy variable to express the contraction period is introduced and, then, (2.2) can be expressed as follows:

¹ Fukuda and Onodera (2001) reported that the macro time series used in this paper have unit roots and the hypotheses of no-co-integration are accepted.

$$C_t = (\varphi_0 + \phi_0 D_t) + (\varphi_1 + \phi_1 D_t)C_{t-1} + (\varphi_2 + \phi_2 D_t)C_{t-2} + (\varphi_3 + \phi_3 D_t)C_{t-3} + (1 + \eta D_t)\varepsilon_t \quad (2.3)$$

where $D_t = 0$ if t is in an expansion period
 $= 1$ if t is in a contraction period

and $\varphi_0' = \varphi_0 + \phi_0$, $\varphi_1' = \varphi_1 + \phi_1$, $\varphi_2' = \varphi_2 + \phi_2$, $\varphi_3' = \varphi_3 + \phi_3$.

In the formulation (2.2), different error terms are assumed in order to take heteroskedasticity into consideration. Note, however, that the type of heteroskedasticity considered here is the difference of the two unconditional variances. The difference of conditional variances is not investigated here, though similar discussions are possible.

Let $\mathbf{y}_t = (\Delta Y_{1t}, \Delta Y_{2t}, \dots, \Delta Y_{Kt})'$, $\mathbf{f}_t = (\Delta C_t, \Delta C_{t-1}, \Delta C_{t-2})'$, $\boldsymbol{\phi} = (\phi_1, \phi_2, \dots, \phi_K)'$,
 $\mathbf{u}_t = (u_{1t}, u_{2t}, \dots, u_{Kt})'$, $\boldsymbol{\varphi} = (\varphi_0 + \phi_0 D_t, 0, 0)'$, $\boldsymbol{\varepsilon}_t = (\varepsilon_t, 0, 0)'$ and

$$H = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \vdots & & \\ \gamma_{K1} & \gamma_{K2} & \gamma_{K3} \end{bmatrix} \quad F = \begin{bmatrix} \varphi_1 + \phi_1 D_t & \varphi_2 + \phi_2 D_t & \varphi_3 + \phi_3 D_t \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \sigma_K^2 \end{bmatrix}.$$

Then, (2.1) and (2.3) are expressed as follows:

$$\begin{aligned} \mathbf{y}_t &= \boldsymbol{\varphi} + H \mathbf{f}_t + \mathbf{u}_t \quad \mathbf{u}_t \sim N(0, \Omega) \\ \mathbf{f}_t &= F \mathbf{f}_{t-1} + (1 + \eta D_t) \boldsymbol{\varepsilon}_t \end{aligned} \quad (2.4)$$

(2.4) is called the state space representation and its estimation can be carried out recursively following the Kalman=Filtering algorithm.²

The Regime Switch Model takes the viewpoint that the business cycle has

² Strictly speaking, if the turning points were determined depending on the macro variables appeared in (2.1), the estimation of (2.3) may be biased due to simultaneity. In practice, the Cabinet office have been determining the turning points by taking various aspects of the economy into consideration and such bias is expected to be small and negligible.

different patterns in its different phases of expansion and contraction. If the phase of the business condition were unknown (even after a certain period), the estimation of the dynamic factor model would require complicated computations.

Let us briefly consider how the inferences would be different when the regimes are observable in the dynamic factor models with regime switch. Let Y^t and S^t be a set of observations available at time t on the macro time series and on the states and $\theta = (\theta_1, \theta_2)$ be an unknown vector consisting of parameters of the model, where θ_1 corresponds to the parameters specifying the dynamic factor model and θ_2 corresponds to those describing the switching mechanism. Assume that θ_1 and θ_2 do not have common elements. The likelihood function for θ can be expressed as follows:

$$\begin{aligned}
f(Y^t, S^t; \theta) &= f(y_t, s_t | Y^{t-1}, S^{t-1}; \theta) f(Y^{t-1}, S^{t-1}; \theta) \\
&= f(y_t | s_t, Y^{t-1}, S^{t-1}; \theta_1) f(s_t | Y^{t-1}, S^{t-1}; \theta_2) f(Y^{t-1}, S^{t-1}; \theta) \\
&\quad \dots\dots \\
&= \Pi \{ f(y_t | s_t, Y^{t-1}, S^{t-1}; \theta_1) f(s_t | Y^{t-1}, S^{t-1}; \theta_2) \} \\
&= \Pi f(y_t | s_t, Y^{t-1}, S^{t-1}; \theta_1) \Pi f(s_t | Y^{t-1}, S^{t-1}; \theta_2)
\end{aligned}$$

When the states are observable, it is sufficient to consider the first product term for the inference of θ_1 , and it is equivalent to the conditional likelihood of y_t on the state. In case of necessity, the inference on θ_2 can be implemented separately.

When the states are not observable, the likelihood function is not separable. That is,

$$\begin{aligned}
f(Y^t; \theta) &= \int \dots \int f(Y^t, S^t; \theta) ds_t \dots ds_1 \\
&= \int \dots \int f(y_t, s_t | Y^{t-1}, S^{t-1}; \theta) f(Y^{t-1}, S^{t-1}; \theta) ds_t \dots ds_1 \\
&= \int \dots \int f(y_t | s_t, Y^{t-1}, S^{t-1}; \theta_1) f(s_t | Y^{t-1}, S^{t-1}; \theta_2) f(Y^{t-1}, S^{t-1}; \theta) ds_t \dots ds_1 \\
&= \int \dots \int \Pi f(y_t | s_t, Y^{t-1}, S^{t-1}; \theta_1) \Pi f(s_t | Y^{t-1}, S^{t-1}; \theta_2) ds_t \dots ds_1.
\end{aligned}$$

Therefore, in order to estimate θ_1 , an explicit formulation of the switching mechanism is inevitable. Hamilton (1989) described the mechanism as a Markov process. Even after a certain model is assumed for the switch mechanism, the computational burden for integration increases exponentially, which requires a proper approximation approach. Kim and Nelson (1998) introduced the procedures of estimation using approximations and/or the Bayesian MCMC estimation approach.

3. Use of Business Sentiments

The dynamic factor model tries to extract a common movement from several macro time series and its application can be regarded as an extension of the factor analysis to the time series data. One of the disadvantages of the dynamic factor model inherited from the factor analysis is its sensitivity to the selection of the macro time series. For example, when two series are selected from the money market, the resulting business cycle indicator reflects the business condition in the money market twice as strong as that in the market where only one series is selected. It is also difficult to obtain a consensus regarding the choice of macro economic variables adopted in (2.1). Usually, a number of variables are listed as candidate series and possible combinations among them are tried until the 'best' combination is selected. In Japan, the current coincident index published by the Cabinet Office (previously by the EPA) is constructed from 11 series listed in Table 1 and they are naturally considered as the candidates.

Stock and Watson (1989) used the following four time series in order to extract a common factor C_t (ΔC_t) that seemed acceptable in general as representing every aspect of the U.S. economy. They are: 1) Industrial Production, 2) Total Personal Income less Transfer Payment, 3) Total Manufacturing and Trade Sales, 4) Employees on Non-agricultural Payrolls. The Nikkei Business Index uses the corresponding four time series selected from the 11 series listed in Table 1. They are: a) Index of Industrial Production, b) Index of Non Scheduled Working Hours (Manufacturing), c) Index of Sales in Wholesale Trade, d) Effective Job Offer Rate. In the experiment carried out below, e) Index of Sales in Small and Medium-sized Enterprises (Manufacturing) is added as the fifth time series. Those five series are depicted in Figure 1.

The Cabinet Office also publishes the leading index constructed from 12 series. Among the 12 series, a notable series is the Index of Business Outlook Judgment of Small Enterprises, that is, the business sentiments held by the managers of small and medium sized enterprises. This series is obtained only on a quarterly basis, whereas the others are monthly. It is important to use monthly series in order to obtain a timely signal of the business conditions. Nevertheless, the series of the business outlook judgment was adopted with an expectation to reflect the business conditions of the small and medium sized industries properly and promptly. In general, if a business indicator largely deviates from business sentiments, it is the business

indicator rather than the business sentiments that is to be scrutinized and, if necessary, revised. By including the business sentiment as a component, it is expected that the resulting indicator would be stabilized. Following such considerations, the series reflecting the business sentiment is included as a component series in the experiment of creating a new indicator. As the series for this end, the judgments on the current business atmosphere for all industries compiled in the Short-term Economic Survey of Enterprises in Japan compiled by the Bank of Japan are used in this article. Figure 2 depicts smooth and periodic movements of the series. The use of business sentiments for construction of a business cycle indicator is also discussed in a different context in Kanoh (1990) and Kanoh and Saito (1994).

As stated above, the data of the business sentiments are obtained on a quarterly basis, whereas the others are monthly. A certain device is necessary to incorporate such series with different frequencies into the framework of the dynamic factor model. First, the following mechanism is assumed for the level of the business sentiments.

$$M_t = \mu_0 + \gamma_0 C_t + \eta_t \quad .$$

Then,

$$\begin{aligned} \Delta_3 M_t &= M_t - M_{t-3} = \gamma_0 \{ C_t - C_{t-3} \} + \{ \eta_t - \eta_{t-3} \} \\ &= \gamma_0 \{ \Delta C_t + \Delta C_{t-1} + \Delta C_{t-2} \} + \eta_t' \quad . \end{aligned}$$

Let D_{M_t} be a dummy variable that takes the value 1 when M_t is observed and 0 otherwise. Then the observation equation for M_t may be expressed as

$$D_{M_t} \Delta_3 M_t = D_{M_t} \gamma_0 \{ \Delta C_t + \Delta C_{t-1} + \Delta C_{t-2} \} + D_{M_t} \eta_t' \quad . \quad (3.1)$$

The equation (3.1) is added as an extra series in (2.1). At the same time, the dummy variable is also introduced to the corresponding element of the variance-covariance matrix Ω . That is, the final diagonal element of Ω is expressed as $D_{M_t} \sigma_{\eta}^2$ and the other additional off-diagonal elements are set as zero for simplicity. Then, in the state space representation, \mathbf{y}_t , \mathbf{H} , \mathbf{F} and \mathbf{Q} are respectively defined as follows:

$$\mathbf{y}_t = (\Delta Y_{1t}, \Delta Y_{2t}, \dots, \Delta Y_{Kt}, D_{M_t} \Delta_3 M_t)' \quad , \quad \mathbf{Q} = (\sigma_1^2, \sigma_2^2, \dots, \sigma_K^2, \sigma_{\eta}^2)'$$

$$H = \begin{bmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} \\ \gamma_{21} & \gamma_{22} & \gamma_{23} \\ \vdots & & \\ \gamma_{K1} & \gamma_{K2} & \gamma_{K3} \\ \gamma_0 \mathbf{D}_{M_t} & \gamma_0 \mathbf{D}_{M_t} & \gamma_0 \mathbf{D}_{M_t} \end{bmatrix} \quad F = \begin{bmatrix} \varphi_1 + \phi_1 \mathbf{D}_t & \varphi_2 + \phi_2 \mathbf{D}_t & \varphi_3 + \phi_3 \mathbf{D}_t \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$$\Omega = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & \sigma_K^2 & 0 \\ 0 & 0 & 0 & 0 & \sigma_M^2 \end{bmatrix}$$

Such formulation implies that when M_t is observed, (2.1) consists of $(K+1)$ equations and if not, it consists of K equations by eliminating the final equation. Though the error term η_t' is serially correlated, it is assumed independent for further simplicity in this paper.³

Figure 3 and Figure 4 show the estimated Japanese business cycle for two cases when four series among the above-mentioned five series are used with and without the business sentiments series, respectively. It is noticed that the extracted business cycle show considerable variability and that the variability is reduced to a certain level by introducing the business sentiments series.

Table 2 summarizes the estimation results and Figure 5 shows the resulting Japanese business cycle using the five series with the business sentiment series. Among the dummies in the transition equation, only the intercept dummy (α_0) is significant. The difference of the error variance (η) is minor and statistically insignificant. It is therefore justifiable to consider different models in the different phases of the business cycle, but complicated asymmetries need not be taken into consideration such as the asymmetries with respect to AR coefficients and the error variances. Though the asymmetry in its conditional variances is not investigated here as mentioned above, it is unlikely to be able to detect such asymmetry judging from the figures regarding the unconditional variances. In the observation equations, the estimated coefficients of each variable show different lagged dependences on the

³ A couple of models were estimated with serially correlated errors but the results were not essentially different. The filtering technique for the series with different frequencies is also discussed in Mariano and Murasawa (2000), where the GDP series is used as the quarterly data.

business conditions.

4. Determination of the turning point in Japan

The estimation of the dynamic factor model with regime switch is greatly simplified by using the information of regimes. This is because estimation of the switching mechanism can be avoided. On the other hand, without introducing the switching mechanism, it is not possible to predict the future regime, and without the prediction, the use of the model seems to be extremely limited. Before considering the switching mechanism, this section presents an application of the dynamic factor model to the Japanese economy in order to show usefulness of its own. Note that the applicability of the dynamic factor model combined with the model describing the switching mechanism is as broad as that of the conventional dynamic factor models with unobservable regime switch.

It is usually the case that the turning point is not officially announced until more than a year later. This is because the government must be prudent in its judgment of the turning points, considering its impact on various political and economic aspects. In December 2001, the Cabinet Office announced that the peak of the 13th cycle was identified as October 2000. The identification was, however, provisional and would be possibly followed by an amendment in a year later. At the same time, the Cabinet Office changed the provisional turning points previously determined and finalized them. Concretely, the peak of the 12th cycle was changed from March 1997 to May 1997 and the trough of the 13th cycle was changed from April 1999 to January 1999⁴. By this, the expansion period of the 13th cycle became, though tentatively, one year and 9 months that was the shortest after the Second World War. This story is visualized in Figure 6.

The dynamic factor model explained above can be used to evaluate such identification of the turning points. The numerical experiment was carried out as follows. First, for the 12th cycle, the provisional turning points previously determined were assumed to be correct. That is, the peak was March 1997 and the trough was April 1999. Also assume that the change of the phase happened once at most (once or none) during the period between April 1999 and May 2001⁵. It would be difficult to imagine that the government would announce more than one peak during such a short

⁴ By this time, people's interest in the turning points of the 12th cycle must have eroded.

⁵ This was the maximum data set available at the time of analysis.

period. Based on such assumptions, the timing of the change was investigated. The dummy variable (D_t) expressing the phases of the business cycle was extended after April 1999 to May 2001 depending on the hypothesized timing of the peak that was changed from May 1999 to May 2001. Two extreme cases with no peaks, that is, the case when the business cycle was in a contraction phase for the whole period and the case when it was in an expansion phase for the period between April 1999 to May 2001 were also investigated. Based on the whole sample from January 1980 to May 2001, the model was re-estimated using the augmented dummy variable. The log-likelihood values were calculated as depicted in Figure 7.

The X-axis denotes the last month of the consecutive expansion and the Y-axis denotes the value of the log-likelihood function. The figure shows that the maximum value of the log-likelihood is attained when the economy was regarded as being in an expansion phase until July 2000, implying that the turning point was August 2000. Judging from the likelihood values, the case where the economy was going up with no turning point until May 2001 and the case where the economy was going down until May 2001, ignoring the fact that April 1999 was the bottom, were both statistically unacceptable. Note also that October 2000, as was announced by the Cabinet Office, was unlikely to be the peak.

Then, a similar calculation was made with the assumption that the trough of the 12th cycle was January 1999 as the Cabinet Office finalized. The Figure 8 shows the resulting likelihood values. Further, the same calculation was made with the peak of the 12th cycle at May 1997, whose results are depicted in Figure 9. From these experiments, the following facts are recognized. Regardless of the preceding turning points of the 12th cycle, the peak of the 13th cycle is considered to be August 2000. It is better to change the trough of the 12th cycle from April 1999 to January 1999 as the Cabinet Office announced. However, the change of the peak of the 12th cycle is not justified judging from the values of the log-likelihood. The logic behind this amendments was not given. Judging from these findings, the Cabinet Office might have intended to keep the expansion period, that is the shortest after the Second World War, as long as possible. It would be recommended for the Office to make prompt and timely decisions as well as to make the process of determining turning points transparent to the public in order not to invite undue skepticism⁶.

⁶ In July 2002, when the then latest version of this paper was prepared, the Japanese government upgraded its view of the economy in its July report for the fourth time in five months. If the Japanese economy had already bottomed out, the Cabinet Office will announce officially the timing of the recent turning points in the coming year. Following the same procedure explained in the context, assuming that the peak of the 13th cycle was

Note finally that such determination of the turning point is not clear, though not impossible, using the conventional dynamic factor model with regime switch when regimes are unobservable.

5. Examination of the Switching Mechanism

When the states are observable, the dynamic factor model can be estimated without incorporating the mechanism of the switch. Such mechanism can be considered separately based on the past regimes. This is an advantage of the dynamic factor model with observable regime switch over the conventional model. In the conventional dynamic factor model with Markov Switch, a simple first order Markov process is tacitly assumed for the regime switch. It is desirable to test the appropriateness of the assumption from the data. Hamilton (1996) proposed a specification test of the Markov-Switching for a single time series. Kim and Nelson (2001) proposed tests for Markov-Switching within the framework of the dynamic factor model using the Bayesian Gibbs Sampler. Here, the hypotheses to be tested are more specific and various. By separating the switch mechanism, the tests are expected to be more powerful. For example, isn't it necessary to assume a higher order Markov process rather than the simple first order Markov process? Isn't the transition probability a function of exogenous variables? Most importantly, isn't the function of the exogenous variables time variant? This section, by fully taking advantage of our model, tries to formulate the switching mechanism in the Japanese business cycle.

Let S_t be a random variable denoting the state of the regime at time t and $P(S_t = j)$ be the probability of the regime being at state j . In order to simplify the arguments, $P(S_t = j)$ is formulated as a logistic function of other exogenous or predetermined variables. For example, when the past values of S_t , that is, $S_{t-1}, S_{t-2}, \dots, S_{t-J}$ are taken as such variables, the above probability can be expressed as:

$$P(S_t = j) = \exp(a + b_1 S_{t-1} + b_2 S_{t-2} + \dots + b_J S_{t-J}) / \{ 1 + \exp(a + b_1 S_{t-1} + b_2 S_{t-2} + \dots + b_J S_{t-J}) \}. \quad (5.1)$$

By testing the significance of b_1, \dots, b_J , the dependency of the state probability on the past states can be checked. If $b_1 \neq 0, \dots, b_j \neq 0, b_{j+1} = \dots = b_J = 0$, the Markov process has j -th order. If $b_1 = \dots = b_J = 0$, the change of the regime happens independently from the

October 1999, it is found that the bottom of this cycle was in December 2001.

past regimes.

Table 3 shows the estimation results for a few Markov processes with different orders. When more than one state variables are introduced, the estimates of coefficients and corresponding t-values are unstable because of the multi-co-linearity. However, judging from the summary statistics listed in Table 3, it is seen that the state probability at t depends only on the state at t-1. This means that conventional modeling of the business cycle movement using a simple first order Markov process is appropriate. Note that this does not exclude the possibility of the dependence on other exogenous variables. A natural extension is to include exogenous variables and the products of them with S_t . The product terms are included to take the asymmetry of the influence of Z_{it} into consideration. That is,

$$P(S_t=j) = \exp(a+bS_{t-1} + c_i Z_{it} + d_k S_{t-1} Z_{kt}) / \{ 1 + \exp(a+bS_{t-1} + c_i Z_{it} + d_k S_{t-1} Z_{kt}) \}. \quad (5.2)$$

Then, the transition probabilities are calculated as:

$$\begin{aligned} P(S_t = 1 \mid S_{t-1} = 1) &= \exp(a + b + c_i Z_{it} + d_k Z_{kt}) / \{ 1 + \exp(a + b + c_i Z_{it} + d_k Z_{kt}) \} \\ P(S_t = 1 \mid S_{t-1} = 0) &= \exp(a + c_i Z_{it}) / \{ 1 + \exp(a + c_i Z_{it}) \} \\ P(S_t = 0 \mid S_{t-1} = 1) &= 1 / \{ 1 + \exp(a + b + c_i Z_{it} + d_k Z_{kt}) \} \\ P(S_t = 0 \mid S_{t-1} = 0) &= 1 / \{ 1 + \exp(a + c_i Z_{it}) \} \end{aligned}$$

Table 4 shows the estimation results of the logit model with different formulations. In the experiments, the Diffusion Index (coincident indicator) and the official discount rates are used as Z_{it} .

An apparent characteristic of the business cycle data is that the regime switch does not happen frequently. Out of 234 months, the switch happened only in 8 months. For such data, if S_t were predicted as the same value of S_{t-1} , such prediction would be mostly correct. However, it would be useless for the prediction of the turning points. Taking this into consideration, the evaluation of the model must be based on both goodness of fit to the sample data and prediction behavior at the turning points depending on the purpose of modeling. In this paper, in order to measure the goodness of the models, the following statistic PE is calculated for referential purpose in addition to the likelihood and BIC.

$$PE = [\sum |y_t - \Pr(y_t \mid y_{t-1} = y_t)| / \#(y_t = y_{t-1}) + \sum |y_t - \Pr(y_t \mid y_{t-1} = 1 - y_t)| / \#(y_t = 1 - y_{t-1})] / 2$$

In the statistic PE, the average of the prediction errors at the turning points are separately taken. For the business cycle data set, PE heavily penalizes the prediction error at the turning points by treating the two prediction errors with an equal weight. The results are summarized in the last column of Table 4. As a general tendency seen from the table, use of S_{t-1} as one of the explanatory variables increases the likelihood value but increases PE at the same time. That is, there seems to be a trade off between goodness of fit to the sample data and goodness of prediction at the turning points. Though it is not reported here, the asymmetry of the transition probability was not seen since the cross terms were mostly insignificant.

The best formulation in terms of goodness of fit to the sample data is found to be No. 12; that is, a linear combination of S_{t-1} and D_t . The results shown in Table 4 implies that when the emphasis is placed upon the fit to the sample data or the confirmation of the past turning points, for which the conventional Markov regime model has been used, it would be essential to include S_{t-1} . Even in that case, however, it is recommended to include another variable like D_t besides S_{t-1} . When the purpose is to predict the turning points, it is essential to rely on exogenous variables. In general, however, the improvements brought by the exogenous variables introduced here are marginal and further research is necessary on the selection of variables. In the following analysis, the analysis is confined to the formulation of No. 12.

When exogenous variables are not considered, the transition probabilities are constant through time. In the above formulation, since Z_{it} depends on time, the transition probabilities change through time accordingly. Kim and Nelson (1999) dealt with the time-varying transition probabilities in this sense. In this paper, time dependency is extended to the parameters. For example, a certain government policy may be an important factor for the regime switch and its effectiveness may change from period to period. In order to capture such a time-varying transition mechanism, one possibility is to formulate the mechanism in the following way.

In (5.2), let $\theta_t = (a, b, c_1, \dots, d_1, \dots)$ be an M dimensional vector of coefficient parameters and assume that θ_t evolves according to the following AR(1) model:

$$\theta_t = \xi + A \theta_{t-1} + \varepsilon_t \quad \varepsilon_t \sim N(0, \Omega) \quad (5.3),$$

where A is a coefficient matrix. When $A=I$ and $\xi=0$, (5.3) corresponds to a random walk model. When $A=0$ and $\Omega=0$, (5.3) reduces to a constant parameter model. (5.3) is regarded as the state transition equation. Here the observation equation is expressed as a likelihood function:

$$f(y_t; \boldsymbol{\theta}_t) = [\exp(\boldsymbol{\theta}_t' \mathbf{x}_t) / \{1 + \exp(\boldsymbol{\theta}_t' \mathbf{x}_t)\}]^{y_t} [1 / \{1 + \exp(\boldsymbol{\theta}_t' \mathbf{x}_t)\}]^{(1-y_t)} \quad (5.4),$$

where y_t takes either 0 or 1. Note here that (5.4) is a non-linear function of $\boldsymbol{\theta}_t$.

Let $\hat{\boldsymbol{\theta}}_t$ denote the estimate of $\boldsymbol{\theta}_t$, $\Sigma_{t|t}$ denote the estimate of the variance-covariance matrix using the available information at time t , and $\Sigma_{t|t-1}$ denote the estimate of the variance-covariance matrix at time $t-1$. From (5.3) and (5.4), the posterior distribution of $\boldsymbol{\theta}_t$ given $\{y_t, Y^{t-1}\}$ is proportional to:

$$h(\boldsymbol{\theta}_t) = \exp\left\{-\frac{1}{2}(\boldsymbol{\theta}_t - \boldsymbol{\xi} - A_{t-1})' \Sigma_{t|t-1}^{-1} (\boldsymbol{\theta}_t - \boldsymbol{\xi} - A_{t-1})\right\} \frac{[\exp(\boldsymbol{\theta}_t' \mathbf{x}_t)]^{y_t} [1 / \{1 + \exp(\boldsymbol{\theta}_t' \mathbf{x}_t)\}]^{(1-y_t)}}{[\exp(\boldsymbol{\theta}_t' \mathbf{x}_t)]^{y_t} [1 / \{1 + \exp(\boldsymbol{\theta}_t' \mathbf{x}_t)\}]^{(1-y_t)}} \quad (5.5).$$

By approximating this posterior distribution by a normal distribution, the recursive estimation is made possible as follows.

Let $g(\boldsymbol{\theta}_t)$ be the first derivative of $\log\{h(\boldsymbol{\theta}_t)\}$ with respect to $\boldsymbol{\theta}_t$ and $\hat{\boldsymbol{\theta}}_t$ be the solution of $g(\boldsymbol{\theta}_t) = 0$. Concretely, $g(\boldsymbol{\theta}_t)$ is expressed as:

$$g(\boldsymbol{\theta}_t) = -\Sigma_{t|t-1}^{-1} (\boldsymbol{\theta}_t - \boldsymbol{\xi} - A_{t-1}) + [y_t - \exp(\boldsymbol{\theta}_t' \mathbf{x}_t) / \{1 + \exp(\boldsymbol{\theta}_t' \mathbf{x}_t)\}] \mathbf{x}_t = 0. \quad (5.6)$$

Also let $\Sigma_{t|t}$ be defined as:

$$\Sigma_{t|t}^{-1} = -[g'(\boldsymbol{\theta}_t)]|_{\boldsymbol{\theta}_t = \hat{\boldsymbol{\theta}}_t} = \Sigma_{t|t-1}^{-1} + \exp(\hat{\boldsymbol{\theta}}_t' \mathbf{x}_t) / \{1 + \exp(\hat{\boldsymbol{\theta}}_t' \mathbf{x}_t)\}^2 (\mathbf{x}_t \mathbf{x}_t') \quad (5.7)$$

Further, for the next period,

$$\Sigma_{t+1|t} = A_{t|t} \Sigma_{t|t} A_{t|t}' + \Omega. \quad (5.8)$$

Combining equations from (5.6) to (5.8), the coefficients can be estimated recursively starting from given initial values $\hat{\boldsymbol{\theta}}_0$ and $\Sigma_{0|0}$. The details of derivations of the above formulas are explained in Kanoh & Li (1990). If the observation y_t has probit formulation, the recursive estimation can be implemented by using the Gibbs sampler. Here we preferred a simple approximation method rather than relying on the Bayesian computational approach in order to maintain the simplicity of the calculation.

In order to estimate the parameters $\boldsymbol{\xi}$, A and Ω , the log likelihood function:

$$\mathcal{L}(Y^t; \theta_t, \xi, A, \Omega) = [y_t - \alpha_{t-1}'x_t - \log\{1 + \exp(\alpha_{t-1}'x_t)\}] \quad (5.9)$$

is maximized with respect to them. Note that α_{t-1} is used to evaluate (5.9).

In the experiment here, the analysis is confined to the best-fitted model No.12 selected from the formulations in Table 4. The matrices A and Ω are assumed to be diagonal. Concretely, $\theta_t = (c_{1t}, c_{2t})'$ where c_{1t} and c_{2t} are the coefficients of S_{t-1} and D_t , respectively. In the initial estimation, the variance of ε_{1t} , i.e. σ_1^2 , corresponding to c_{1t} was estimated nearly equal to zero and, at the same time, the convergence was not achieved. This is due to the characteristic of the data that the regime switch rarely happens. Then, a_{11} (the first diagonal element of A) and σ_1^2 were set to zero and the model was estimated again. That is, only the coefficient of D_t was regarded as time varying. In this case, the maximum value of (5.9) was found to be -30.63 . The likelihood test statistic for testing $H_0: A = \Omega = 0$ is defined as:

$$L = -2 \{ \mathcal{L}(Y^t; \theta_t | a_{22} = \sigma_2^2 = 0) - \mathcal{L}(Y^t; \theta_t | a_{22} = a_{22}^{opt}, \sigma_2^2 = \sigma_2^{2\ opt}) \},$$

and L distributes according to chi square distribution with degrees of freedom 2 under H_0 . Under H_0 , the value of the log-likelihood was -33.21 . Then $L = 5.16$ and H_0 was rejected at 10% significance level with the critical value 4.61 but accepted at 5% significance level with the critical value 5.99, implying that the time-dependency of the switch probabilities was not strongly supported but its possibility remains. For referential purpose, the transition of θ_t is depicted in Figure 10 and the probabilities of being in expansion regimes calculated from the time-varying model and the constant parameter model are depicted in Figure 11. Further research is needed on time-dependency of the parameters since such a conclusion hinges upon the formulation employed here as well as the sample period.

6. Conclusion

This paper presented a construction of a business cycle indicator within the framework of the dynamic factor model with regime switch. By making use of the observations on the regimes, the dynamic factor model could be estimated independently of the switch mechanism, which greatly reduced the necessary computation. It was then pointed out that the business indicators derived from the

dynamic factor model were sensitive to the choice of macro economic time series. Introduction of the business sentiments as one of the variables could stabilize the resulting indicators. By fitting the model to the Japanese data, it was observed that the Japanese business cycle had an asymmetry in its expansion and contraction. This asymmetry was, however, only with respect to the levels of the cycle in the two phases. More complicated asymmetries such as asymmetries with respect to the coefficients of AR structure or with respect to the variances of the error terms were not observed. Then, in order to show a practical use of the dynamic factor model with observable regime switch, an evaluation using the model was made on the determination of the recent turning points in the Japanese business cycle announced by the Cabinet Office. It should not be misunderstood that the application of the dynamic factor model with observable regime switch was not confined to this but is as broad as the dynamic factor model with unobservable regimes.

The switching mechanism was separately considered. It was found that the switching mechanism could not be described by a simple first order Markov process. The transition probabilities depended on an exogenous variable as well as the preceding regime. If the transition probabilities were formulated this way, the time dependency of the parameters might not be necessarily taken into consideration, though this point needs further investigation.

As complicated econometric models are employed in empirical researches, the assumptions made behind the models tend to be ignored or become difficult to check. In an effort to obtain robust conclusions, this paper tried to build a model by making the most of the available information and scrutinized the assumptions implicitly made behind the model.

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